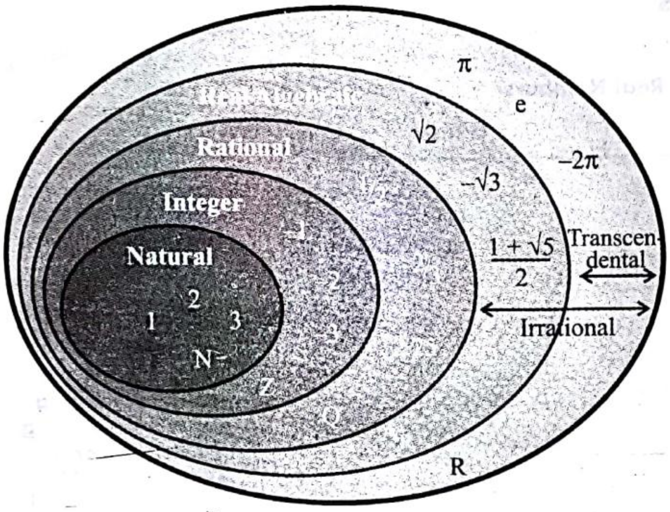
PART A



CHAPTER 1

Number System

INTRODUCTION

We all know the numbers. We are playing with the numbers since our childhood. All the numbers which we studied till now are rational numbers. We also studied the representation of rational numbers on number line and about their basic algebraic operations like addition, subtraction, multiplication, division, L.C.M., H.C.F., etc of rational numbers.

In this chapter we shall learn a new type of numbers called Irrational numbers. We will also learn the representation of irrational numbers on the number line and basic operations related to irrational numbers. All the rational and irrational numbers taken together are known as real Numbers.

Actually ***Real Numbers*** *are the numbers which are really exist. It may be +ve, −ve or 0 (zero).*

CLASSIFICATION OF REAL NUMBERS

Real Numbers

- **Rational Numbers**
  - **$\frac{p}{q}$ Form**
    $\frac{p}{q}$, where p and q are integers and $q \neq 0$
    Examples : $\frac{5}{8}, \frac{-4}{3}, \frac{2}{-7}$, 3,0, – 4 etc.
  - **Decimal Form**
    - **Terminating Decimal**
      In which the number of digits after decimal point is finite.
      Examples : 4.024 5.008, 0.23 etc.
    - **Non-terminating Repeating Decimal Form**
      In which number of digits after decimal point is infinite but after decimal point a group of digit (s) (one or more than one digits) repeats continuously at the end Examples : 5.4141....., – 0.2383838...., 0.061245245.... etc These irrational numbers can be written by putting a bar or recurring above the first group of digit(s) after decimal point which repeats further. For examples 5.4141.... = $5.\overline{41}$, 0.2383838... = $0.2\overline{38}$, 0.061245245..... = $0.061\overline{245}$ etc.
- **Irrational Numbers**
  - **$\frac{p}{q}$ Form**
    $\frac{p}{q}$, where $q \neq 0$ and at least one of p and q is not an integer
    Examples : $\frac{5}{4.2}, \frac{0.51}{6}, \frac{-8.02}{2.3}$ etc.
  - **Non-terminating and Non-repeating Decimal Form**
    In which number of digits after decimal point is infinite but there is no group of digit(s) (one or more than one digits) Which repeats continuously at the end. Examples : 501.060060006..., 23.1424434444...., 0.009191191119..., –7.401002003....., etc.

From the above classification of real numbers, it is clear that both rational and irrational numbers combined together are called real numbers i.e. each rational number is a real number as well as each irrational number is also a real number. Sum, difference, product or quotient [provided denominator not equal to 0 (zero)] of two rational, two irrational or one rational and one irrational number is also a real number.

Note :

(i) All integers are rational numbers.

(ii) The square root of every perfect square number is rational. eg. $\sqrt{4}=2, \sqrt{9}=3, \sqrt{16}=4$ etc are all rational numbers.

(iii) The square root of any positive number which is not a perfect square is an irrational number.
E.g.: $\sqrt{5}, \sqrt{3}, \sqrt{10}, \sqrt{12}, \sqrt{3.4}, \sqrt{0.748}$ etc.

(iv) π is an irrational number, which is actually the ratio of circumference to the diameter of a circle i.e. $\pi = \frac{c}{d}$, where c and d are the circumference and diameter of a circle. Approximate value of π is taken as $\frac{22}{7}$ or 3.14

METHOD TO FIND A GIVEN NUMBER OF RATIONAL NUMBERS BETWEEN TWO GIVEN DISTINCT RATIONAL NUMBERS IN THE FORM $\frac{p}{q}$

There are two simple methods to find a given number of rational numbers between two given rational numbers.

METHOD-I

Let $\frac{a}{b}$ and $\frac{c}{d}$ be two distinct rational numbers. To find n rational numbers between $\frac{a}{b}$ and $\frac{c}{d}$ we proceed as follows :

(i) If denominator b and d are not equal then find the L.C.M. of b and d. Let the L.C.M of b and d be l.

Multiply both numerator and denominator of $\frac{a}{b}$ by $\frac{l}{b}$ and both numerator and denominator of $\frac{c}{d}$ by $\frac{l}{d}$, so that denominator of both the given rational numbers become equal i.e. $\frac{a}{b} = \frac{a\times\frac{l}{b}}{b\times\frac{l}{b}} = \frac{a\times\frac{l}{b}}{l}$ and $\frac{c}{d} = \frac{c\times\frac{l}{d}}{d\times\frac{l}{d}} = \frac{c\times\frac{l}{d}}{l}$.

If b and d are equal, then go to the step (ii) directly.

(ii) If the difference of numerators of rational numbers with equal denominator is not greater than n, then multiply both numerator and denominator of both the rational numbers having equal denominator by a suitable natural number k so that the difference of numerator of resulting rational numbers become more than n but the its denominator remains equal.

$$\frac{a}{b} = \frac{a\times\frac{l}{b}}{l} = \frac{a\times\frac{l}{b}\times k}{l\times k} \quad \frac{c}{d} = \frac{c\times\frac{l}{d}}{l} = \frac{c\times\frac{l}{d}\times k}{l\times k}$$

If the difference of numerators of rational numbers with equal denominator is greater than n, then go to the step (iii) directly.

(iii) (a) If $\frac{a\times\frac{l}{b}\times k}{l\times k} < \frac{c\times\frac{l}{d}\times k}{l\times k}$, then

$\frac{a\times\frac{l}{b}\times k+1}{l\times k}, \frac{a\times\frac{l}{b}\times k+2}{l\times k}, \frac{a\times\frac{l}{b}\times k+3}{l\times k}, \ldots\ldots\ldots\ldots, \frac{a\times\frac{l}{b}\times k+n}{l\times k}$ are n rational numbers between given two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$.

(b) If $\frac{c\times\frac{l}{d}\times k}{l\times k} < \frac{a\times\frac{l}{b}\times k}{l\times k}$, then $\frac{c\times\frac{l}{d}\times k+1}{l\times k}, \frac{c\times\frac{l}{d}\times k+2}{l\times k}, \frac{c\times\frac{l}{d}\times k+3}{l\times k}, \ldots\ldots, \frac{c\times\frac{l}{d}\times k+n}{l\times k}$ are n rational numbers between given two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$.

Note : There may be some other set of n rational numbers between the given two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$.

Illustration 1: Find five rational numbers between $\frac{5}{6}$ and $\frac{8}{9}$.

SOLUTION :

(i) Denominator 6 and 9 are not equal. Hence, first we small find the L.C.M. of 6 and 9.

$$\begin{array}{r|l} 3 & 6,9 \\ \hline & 2,3 \end{array} \qquad \text{L.C.M} = 3 \times 2 \times 3 = 18$$

Multiply both numerator and denominator of $\frac{5}{6}$ by $\frac{18}{6}$ and both numerator and denominator of $\frac{8}{9}$ by $\frac{18}{9}$, we get

$$\frac{5}{6} = \frac{5 \times \frac{18}{6}}{6 \times \frac{18}{6}} = \frac{15}{18}$$

$$\frac{8}{9} = \frac{8 \times \frac{18}{9}}{9 \times \frac{18}{9}} = \frac{16}{18}$$

(ii) Difference of the numerators of $\frac{15}{18}$ and $\frac{16}{18}$

$= 16 - 15 = 1$, which is not greater than 5 (i.e. the number of rational numbers which is to be find out)

So we multiply both numerator and denominator of both the rational numbers $\frac{15}{18}$ and $\frac{16}{18}$ by a suitable natural number 6, so that the difference of the numerator of both the rational numbers become greater than 5 but its denominator remains equal.

$$\frac{5}{6} = \frac{15}{18} = \frac{15 \times 6}{18 \times 6} = \frac{90}{108}$$

$$\frac{8}{9} = \frac{16}{18} = \frac{16 \times 6}{18 \times 6} = \frac{96}{108}$$

Note : By multiplying both numerator and denominator of both $\frac{15}{18}$ and $\frac{16}{18}$ by any natural number less than 6, we can not get the difference of numerator greater than 5 but by multiplying by any natural number greater than 5 we get the difference of numerator greater than 5.

(iii) Since $\frac{90}{108} < \frac{96}{108}$, therefore $\frac{91}{108}, \frac{92}{108}, \frac{93}{108}, \frac{94}{108}$ and $\frac{95}{108}$ are one set of five rational numbers between two given rational numbers $\frac{5}{6}$ and $\frac{8}{9}$.

Note : If we were multiplied both numerator and denominator of $\frac{15}{18}$ and $\frac{16}{18}$ by a natural number greater than 6, say 7, then

$\frac{5}{6} = \frac{15}{18} = \frac{15 \times 7}{18 \times 7} = \frac{105}{126}$ and $\frac{8}{9} = \frac{16}{18} = \frac{16 \times 7}{18 \times 7} = \frac{112}{126}$

The difference of numerator of $\frac{105}{126}$ and $\frac{112}{126}$ is still greater than 5 and $\frac{105}{126} < \frac{112}{126}$.

Therefore there are many sets of five rational numbers between the given two rational numbers $\frac{5}{8}$ and $\frac{8}{9}$.

For example $\frac{106}{126}, \frac{107}{126}, \frac{108}{126}, \frac{109}{126}, \frac{110}{126}$,

$\frac{107}{126}, \frac{108}{126}, \frac{109}{126}, \frac{110}{126}, \frac{111}{126}$ and $\frac{106}{126}, \frac{108}{126}, \frac{109}{126}, \frac{110}{126}, \frac{111}{126}$ are some sets of 5 required rational numbers between given two rational numbers $\frac{5}{6}$ and $\frac{8}{9}$.

Illustration 2 : Find six rational numbers between $\frac{4}{7}$ and $\frac{1}{7}$.

SOLUTION : Denominator of both the given rational numbers $\frac{4}{7}$ and $\frac{1}{7}$ are equal.

Now the difference of numerator = 4 – 1 = 3, which is not greater than 6 (i.e. the number of rational numbers is to be find out)

On multiply both numerator and denominator of $\frac{4}{7}$ and $\frac{1}{7}$ by 3, we get

$$\frac{4}{7}=\frac{4\times 3}{7\times 3}=\frac{12}{21}$$

$$\frac{1}{7}=\frac{1\times 3}{7\times 3}=\frac{3}{21}$$

Now the difference of numerator of $\frac{12}{21}$ and $\frac{3}{21}$ = 12 – 3 = 9, which is greater than 6.

Also $\frac{3}{21}<\frac{12}{21}$ therefore $\frac{4}{21},\frac{5}{21},\frac{6}{21},\frac{7}{21},\frac{8}{21}$ and $\frac{9}{21}$ are one set of six rational numbers between $\frac{4}{7}$ and $\frac{1}{7}$.

Illustration 3: Find four rational numbers between $\frac{1}{7}$ and $\frac{2}{3}$.

SOLUTION :

Method - I

Denominator of given two rational numbers are not equal.

L.C.M. of 7 and 3 = 7 × 3 = 21

$$\frac{1}{7}=\frac{1\times\frac{21}{7}}{7\times\frac{21}{7}}=\frac{3}{21}$$

$$\frac{2}{3}=\frac{2\times\frac{21}{3}}{3\times\frac{21}{3}}=\frac{14}{21}$$

Difference of numerator of $\frac{3}{21}$ and $\frac{14}{21}$ = 14 – 3 = 11, which is greater than 4 (i.e. the number of rational numbers is to be find out).

Also $\frac{3}{21}<\frac{14}{21}$, therefore one set of four rational numbers between given two rational numbers are $\frac{4}{21},\frac{5}{21},\frac{6}{21}$ and $\frac{7}{21}$.

Method-II

Let the given two rational numbers be a and b.

Also suppose $d=\frac{b-a}{n+1}$, where n is the number of rational numbers we need to find between a and b.

Then, first rational no. $= a + d$

second rational no. $= a + 2d$

3rd rational no. $= a + 3d$

----

n^{th} rational no. $= a + nd$

Illustration 4 : The five rational numbers between $\frac{5}{6}$ and $\frac{8}{9}$. Using the above method – II

SOLUTION : Here $a = \frac{5}{6}$, $b = \frac{8}{9}$ and $n = 5$

$$d = \frac{b-a}{n+1} = \frac{\frac{8}{9}-\frac{5}{6}}{5+1} = \frac{\frac{8\times 2-5\times 3}{18}}{6} = \frac{\frac{1}{18}}{6} = \frac{1}{18\times 6} = \frac{1}{108}$$

Hence five rational numbers between $\frac{5}{6}$ and $\frac{8}{9}$ are :

$$a+d = \frac{5}{6}+\frac{1}{108} = \frac{5\times 18+1}{108} = \frac{91}{108}$$

$$a+2d = \frac{5}{6}+\frac{2}{108} = \frac{5\times 18+2}{108} = \frac{92}{108}$$

$$a+3d = \frac{5}{6}+\frac{3}{108} = \frac{5\times 18+3}{108} = \frac{93}{108}$$

$$a+4d = \frac{5}{6}+\frac{4}{108} = \frac{5\times 18+4}{108} = \frac{94}{108}$$

And $a+5d = \frac{5}{6}+\frac{5}{108} = \frac{5\times 18+5}{108} = \frac{95}{108}$

DECIMAL REPRESENTATION OF RATIONAL NUMBERS IN THE FORM $\frac{p}{q}$.

Every rational number in the form $\frac{p}{q}$ can be expressed to its equivalent decimal form.

For example $\frac{1}{5} = 0.2$

$\frac{1}{3} = 0.333..... = 0.\overline{3}$ etc

Generally we use long division method to get the decimal form from $\frac{p}{q}$ form. For example to get the decimal form of $\frac{13}{7}$, we simply divide 13 by 7 is shows below

```
7 ) 13 ( 1.857142
     7
    ---
     60
     56
    ---
      40
      35
     ---
       50
       49
      ---
        10
         7
       ---
         30
         28
        ---
          20
          14
         ---
           6
```

$\therefore \quad \frac{13}{7} = 1.857142857142....$

$= 1.\overline{857142}$

Similarly to get the decimal form of $\frac{11}{6}$ we divide 11 by 6.

$$\begin{array}{r|l} 6 & 11 \;(1.83 \\ & \underline{6} \\ & 50 \\ & \underline{48} \\ & 20 \\ & \underline{18} \\ & 2 \end{array}$$

$\therefore\ \frac{11}{6} = 1.8333......$

$= 1.8\overline{3}$

CONVERSION OF RATIONAL NUMBER IN DECIMAL FORM TO ITS SIMPLEST $\frac{p}{q}$ FORM

Type-I : Conversion of a Terminating Decimal Number to its Simplest $\frac{p}{q}$ Form.

Step 1 : Obtain the rational number.

Step 2 : Determine the number of digits in its decimal part.

Step 3 : Remove decimal point from the given number and write 1 as its denominatorfollowed by as many zeros as the total number of digits in the decimal part of the given number.

Step 4 : Write the number obtained in step-3 in its simplest form (i.e. the form in which there is no common fator other than 1 in its numerator and denominator).

The number so obtained is the required $\frac{p}{q}$ from.

Illustration: Convert rational number 2.348 in simplest $\frac{p}{q}$ form.

***SOLUTION* :** Given rational number = 2.348

There are three digits in the decimal part.

$\therefore\quad 2.348 = \frac{2348}{1000}$

Now write $= \frac{2348}{1000}$ in its lowest form.

$2.348 = \frac{2348}{1000} = \frac{1174}{500} = \frac{587}{250}$, which is the required form

Type-II : Conversion of Non-terminating Repeating Decimal Number to its Simplest $\frac{p}{q}$ Form.

Step 1 : Put the given number equal to x and consider it as equation number (i)

Step 2 : If there are some non-repeating digits after decimal on the right hand side of the equation (i), than count the number of non-repeating digits after the decimal point. Let it be n. Otherwise go to step 4 directly.

Step 3 : Multiply both sides of equation (i) by $(10)^n$, so that on the right hand side of the decimal point only the repeating digit(s) is/are left. Consider the equation so obtained as equation number (ii).

Step 4 : Multiply both sides of the equation (ii) by $(10)^m$, where m is the number of digit(s) which repeats on the right hand side after decimal point. Consider the equation so obtained as equation number (iii).

Step 5 : Subtract the equation (ii) from equation (iii),

Step 6 : Divide both sides of the equation obtained in step-5 by the cofficient of x.

Step 7 : Write the rational number on the right hand side of equation obtained in step-6 in the simplest $\frac{p}{q}$ form. This simplest $\frac{p}{q}$ form is the required form.

Illustration 5 : Express 6.48261261261.......... in the simplest $\frac{p}{q}$ form.

SOLUTION : Let $x = 6.48261264261......$

$\Rightarrow x = 648.\overline{261}$ ...(i)

Since there are two non-repeating digits (48) on the right hand side of equation (i), therefore we multiply both sides of equation (i) by $(10)^2$ i.e. 100, we get

$100x = 648.\overline{261}$ ...(ii)

Since there are three repeating digits (261), therefore multiply both sides of equation (ii) by $(10)^3$ i.e. 1000, we get

$100000x = 648261.\overline{261}$ ... (iii)

Note : $648.\overline{261}$ can be written as $648.261\overline{261}$ also.

Subtracting equation (ii) from (iii), we get $90900x = 648261 - 648$

$\Rightarrow 90900x = 647613$ ...(iv)

Divide both side of equation (iv) by the coefficient 90900 of x in equation (iv), we get

$$\frac{90900x}{90900} = \frac{647613}{90900}$$

$$\Rightarrow x = \frac{647613}{90900}$$

$$\Rightarrow x = \frac{71957}{10100} \text{ (in simplest form)}$$

Hence, $6.48261261261.................. = \frac{71957}{10100}$, which is the required $\frac{p}{q}$ form.

Illustration 6 : Express $0.\overline{52}$ in the $\frac{p}{q}$ form.

SOLUTION : Let $x = 0.\overline{52}$

$x = 0.525252$ ...(i)

There is no non-repeating digit after decimal point on the right hand side in equation (i).

Number of repeating digits after the decimal point on the right hand side of equation (i) is 2. Hence, multiplying both sides of equation (i) by $(10)^2$ i.e. 100

$100x = 52.525252.....$ ...(ii)

Subtract (i) from (ii), we get

$$\begin{array}{r} 100x = 52.5252..... \\ x = 0.525252... \\ \hline 99x = 52 \end{array}$$

$$x = \frac{52}{99}$$

$$\therefore \quad 0.\overline{52} = \frac{52}{99}$$

Illustration 7 : Express $2.\overline{34}$ in the $\frac{p}{q}$ form.

SOLUTION : Let $x = 2.\overline{34}$

$x = 2.343434$ ...(i)

There is no non-repeating digit after decimal point in the right hand side of equation (i).

Number of repeating digits after the decimal point is 2

Multiplying both sides of (i) by 100

$100x = 234.3434$ ...(ii)

Subtracting equation (i) form (ii), we get

$$\begin{aligned} 100x &= 234.3434.....\\ x &= 2.3434........\\ \hline 99x &= 232.0 \end{aligned}$$

$\Rightarrow \quad x = \dfrac{232}{99}$

$2.\overline{34} = \dfrac{232}{99}$ is the required $\dfrac{p}{q}$ form.

Illustration 8 : Express 0.2434343......... in the form of $\dfrac{p}{q}$.

SOLUTION : Let $x = 0.2434343$ ......

$x = 0.2\overline{43}$ ...(i)

Here, a digit 2 does not repeat after decimal point.

So, we multiply equation (i) by 10, we get

$10x = 2.4343$ ...(ii)

Since there are two repeating digits (43) after decimal point in the right side of equation (ii).

So, multiplying (ii) by $(10)^2$ i.e. 100, we get

$1000x = 243.4343$ ...(iii)

Substract (ii) from (iii), we get

$$\begin{aligned} 1000x &= 243.4343......\\ 10x &= \quad 2.4343......\\ \hline 990x &= \quad 241 \end{aligned}$$

$x = \dfrac{241}{990}$

$\Rightarrow \quad 0.2434343........... = \dfrac{241}{990}$ is the required $\dfrac{p}{q}$ form.

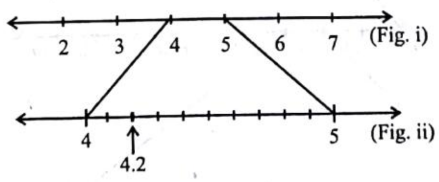
REPRESENTATION OF REAL NUMBERS IN DECIMAL FORM (UPTO A CERTAIN NUMBER OF DIGITS) ON THE REAL NUMBER LINE USING THE PROCESS OF SUCCESSIVE MAGNIFICATION

In this section, we get the knowledge of representation of decimal number on the number line by representation of different types of decimal numbers on the number line.

(i) Representation of 4.2 on the number line

Clearly 4.2 lies. between 4 and 5 in fig. (i).

Take a magnified look of the line segment between 4 and 5 and divide it into 10 equal parts and mark each point of division between 4 and 5 as shown in the fig. (ii).



We can see in fig. (ii), 4.2 is represented by the second mark of division after 4 in between 4 and 5.

(ii) Representation of 4.83 on the number line

4.83 will lies in between 4 and 5. In a closer look 4.83 lies in between 4.8 and 4.9 as 4.8 is equal to 4.80 and 4.9 is equal to 4.90 Hence first we draw a number line indicating all integral point on it (fig. (i)). Then after, we find the points on the number line representing 4.8 and 4.9 by dividing the line segment between 4 and 5 in to 10 equal parts using successive magnification (fig. (ii)).

Take a magnified look of the line segment between 4.8 and 4.9 and divide it into 10 equal parts. Then third marked point of division shows 4.83 (fig. (iii)).

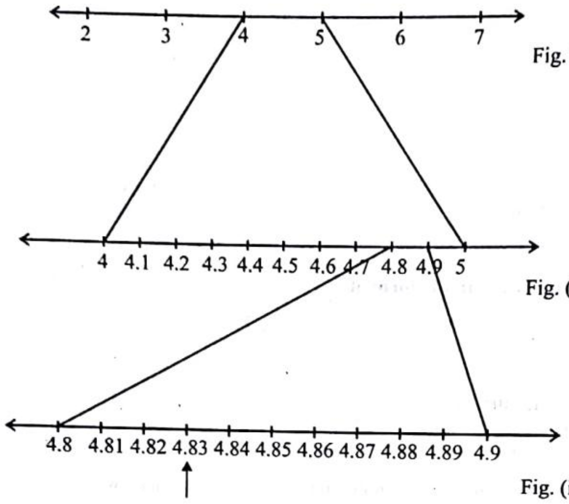


Fig. (i)

Fig. (ii)

Fig. (iii)

The method of representing a real number in decimal form (up to a certain number of digits) on the number line using the process of magnification is explained above. Now we are able to represent any real number in decimal form (up to a certain number of digits) on the number line using process of successive magnification.

Lets represent some more real numbers in decimal form (up to a certain number of digits) on the number line without explaining the method in detail.

(iii) Representation of 6.328 on the number line

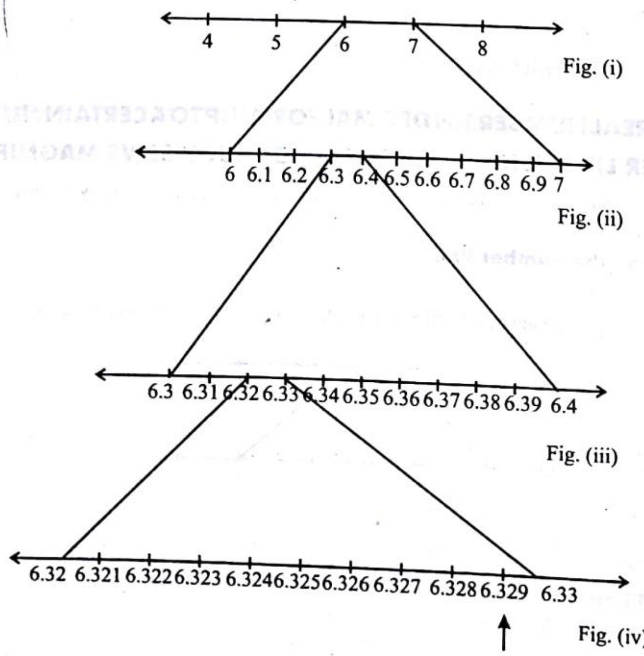


Fig. (i)

Fig. (ii)

Fig. (iii)

Fig. (iv)

(iv) Representation of $4.6\bar{2}$ on the number line up to five decimal places

$4.6\bar{2} = 4.62222$ (up to 5 decimal places)

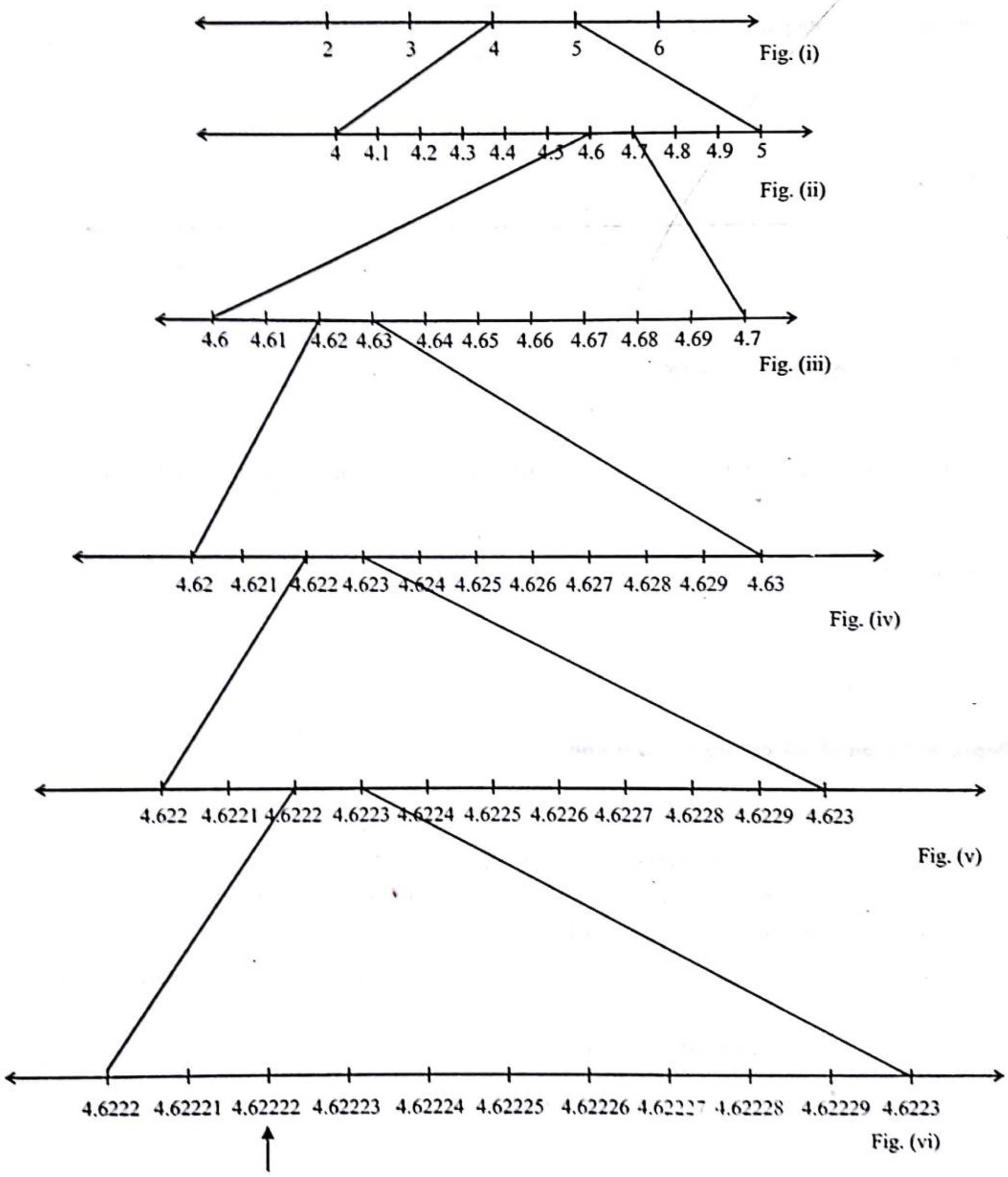


Fig. (i) Fig. (ii) Fig. (iii) Fig. (iv) Fig. (v) Fig. (vi)

REPRESENTATION OF IRRATIONAL NUMBERS IN THE FORM $\frac{p}{q}$ ON THE NUMBER LINE

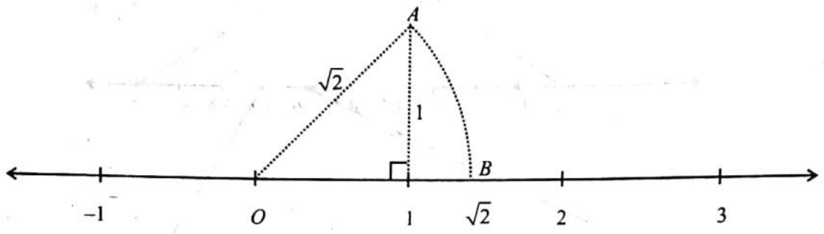
To represent an irrational number in the form $\frac{p}{q}$, we use the Pythagoras theorem of a right angle triangle, according to which, in a right angled triangle, the square of the hypotenuse is equal to the sum of the square of the other two sides.

i.e. $(\text{Hypotenus})^2 = (\text{Base})^2 + (\text{Perp})^2$

In this section, we get the knowledge of representation of irrational numbers on the number line by representation of different types of irrational numbers on the number line.

(i) Representation of $\sqrt{2}$ on the number line

Draw a number line indicating all integral points on it.



Since $\sqrt{2} > 0$

Hence, $\sqrt{2}$ lies right side of O on the number line.

Now, $\sqrt{2} = \sqrt{(1)^2 + (1)^2}$

So, if we draw a right angled triangle whose base and perpendicular each is of unit length, then length of its hypotenuse is $\sqrt{2}$ units.

Draw a perpendicular line segment of unit length above the number line at the point representing 1 on it. Let the top-point of this perpendicular line segment be A. Clearly, $OA = \sqrt{2}$ units.

Now draw an arc with O as centre and OB as radius intersecting the number line at a point B on the right side of 1 on the number line. Hence, $OB = \sqrt{2}$ unit.

Therefore point B on the number line represent $\sqrt{2}$.

(ii) Representation of $\sqrt{3}$ on the number line

$\sqrt{3} = \sqrt{(\sqrt{2})^2 + 1^2}$

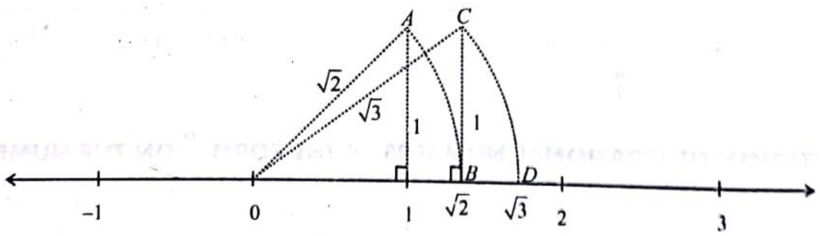
Mark a point B on the number line which represent $\sqrt{2}$ on the number line as discussed above.

Draw a perpendicular line segment of unit length above the number line at the point representing $\sqrt{2}$ on it.

Let top point of this perpendicular line segment be C. Clearly $OC = \sqrt{3}$ units.

Taking O as centre and OC as radius, draw an arc intersecting the number line a point D on the right side of $\sqrt{2}$ on the number line.

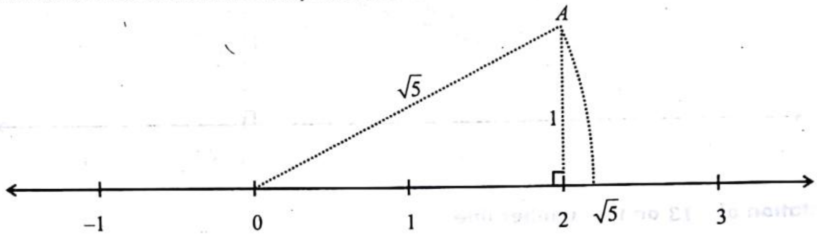
Clearly point D on the number line represents the number $\sqrt{3}$.



(iii) Representation of $\sqrt{5}$ on the number line

$\sqrt{5} = \sqrt{2^2 + 1^2}$

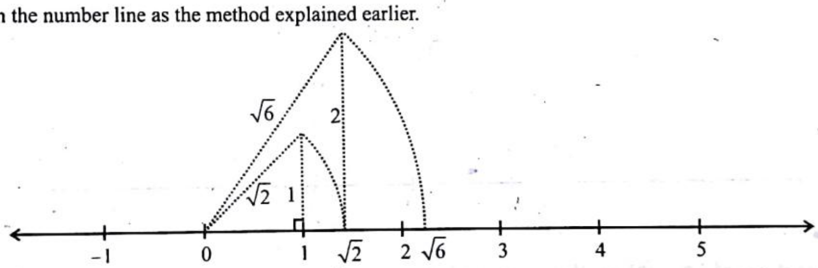
Now mark $\sqrt{5}$ on the number line as the methods explained above.



(iv) Representation of $\sqrt{6}$ on the number line

$\sqrt{6} = \sqrt{2^2 + (\sqrt{2})^2}$

Now mark $\sqrt{6}$ on the number line as the method explained earlier.

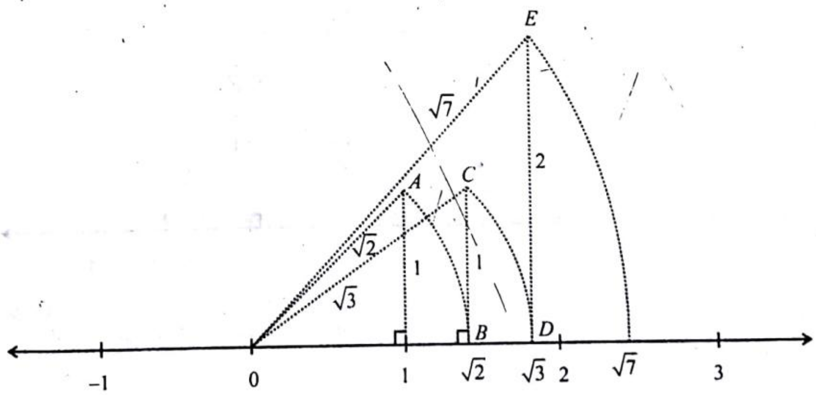


(v) Representation of $\sqrt{7}$ on the number line

$\sqrt{7} = \sqrt{(\sqrt{6})^2 + 1^2}$

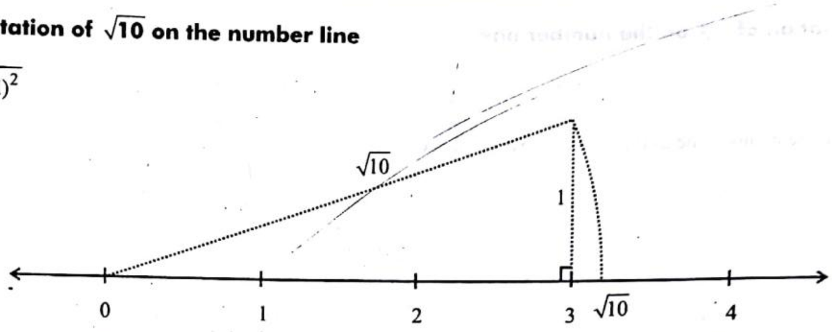
or $\quad \sqrt{7} = \sqrt{(\sqrt{3})^2 + (2)^2}$

Take $\sqrt{7} = \sqrt{\sqrt{3}^2 + (2)^2}$



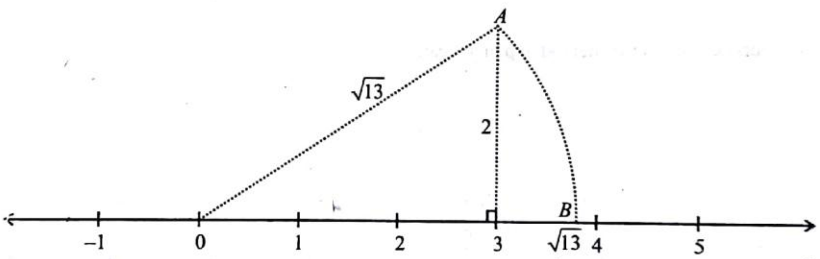
(vi) Representation of $\sqrt{10}$ on the number line

$\sqrt{10} = \sqrt{(3)^2 + (1)^2}$



(vii) Representation of $\sqrt{13}$ on the number line

$\sqrt{13} = \sqrt{3^2 + 2^2}$

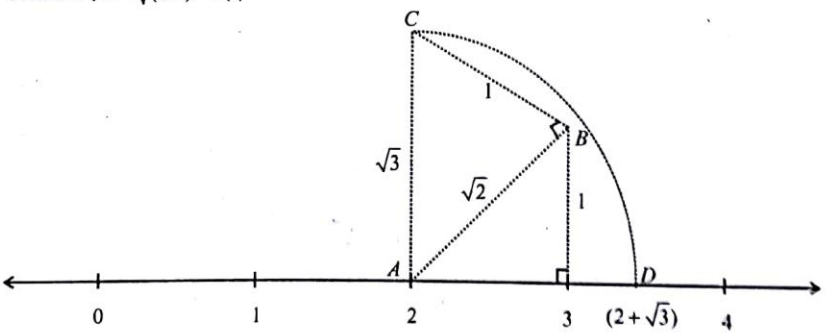


(viii) Representation of $(2+\sqrt{3})$ on the number line

Draw a number line indicating all integral points on it. Let the point representing 2 on the number line be A. Draw a line segment of unit length perpendicular and above the number line at a point representing 3 on it. Let the top of this line segment be B. Join A to B. Clearly $AB = \sqrt{2}$ units, because $\sqrt{2} = \sqrt{(1)^2 + (1)^2}$

Now draw a line segment BC perpendicular to AB at B as shown in the figure. Join A to C.

Clearly $AC = \sqrt{3}$ because $\sqrt{3} = \sqrt{(\sqrt{2})^2 + (1)^2}$

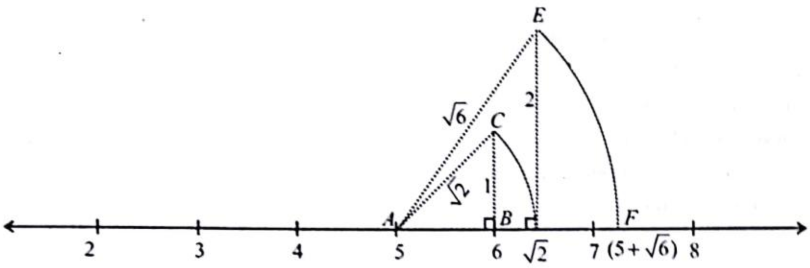


Now with centre A and radius AC, draw an arc intersecting the number line at a point D on the right side of 2. Clearly point D represent $(2+\sqrt{3})$ on the number.

(ix) Representation of $(5+\sqrt{6})$ on the number line

$$\sqrt{2} = \sqrt{(1)^2 + (1)^2}$$

$$\sqrt{6} = \sqrt{\sqrt{2}^2 + (2)^2}$$



TO PROVE $\sqrt{2}$ IS AN IRRATIONAL NUMBER

This is proved by the method of contradiction.

Let us suppose, if possible that $\sqrt{2}$ is not an irrational number i.e. $\sqrt{2}$ is a rational number. Hence it can be represented in the form of $\frac{p}{q}$ i.e. $\sqrt{2} = \frac{p}{q}$; where p and q are integers but $q \neq 0$.

Let $\frac{p}{q}$ is in simplest form hence p and q are co-primes i.e. they have no common factor other than 1.

Squaring both the sides, we get $2 = \frac{p^2}{q^2}$

$\Rightarrow \quad p^2 = 2q^2 \qquad$ ...(i)

Since RHS of (i) is twice the square of an integer,

Hence, RHS of (i) is even

$\therefore \quad p^2$ is even

Hence P is also an even integer.

Let $p = 2m$, where m is an integer.

$p^2 = 4m^2$

Putting the value of p^2 in (i),

$4m^2 = 2q^2$

$q^2 = 2m^2 \qquad$ ...(ii)

Since RHS of (ii) is an even integer, hence L.H.S. of it i.e. q^2 is also even integer.

Therefore, q is also an even integer.

Hence, p and q both are even integers.

But two even integers have always a common factor 2, which contradicts our assumption that p and q are co-primes.

Hence, $\sqrt{2}$ is an irrational number.

TO PROVE $\sqrt{3}$ AND $\sqrt{5}$ ARE IRRATIONAL NUMBERS

Let us suppose if possible that $\sqrt{3}$ is not an irrational number i.e. $\sqrt{3}$ is a rational number.

Hence $\sqrt{3}$ can be represented in the form of $\frac{p}{q}$.

i.e. $\sqrt{3} = \frac{p}{q}$, where p and q both are integers but $q \neq 0$.

Let $\frac{p}{q}$ is in simplest form, hence p and q are co-prime i.e. they have no common factor other than 1.

Squaring both sides, we get

$$3 = \frac{p^2}{q^2} \Rightarrow p^2 = 3q^2 \qquad \text{...(i)}$$

Since q is an integer, therefore q^2 is also an integer.

$\Rightarrow$ $3q^2$ is an integral multiple of 3.

$\Rightarrow$ p^2 is also an integral multiple of 3.

Hence p is also an integral multiple of 3.

Therefore we can assume that $p = 3m$, where m is an integer.

Now putting the value of p in equation (i), we get

$9m^2 = 3q^2, \Rightarrow q^2 = 3m^2 \Rightarrow q^2$ is an integral multiple of 3.

Hence q is an integral multiple of 3.

Since both p and q are integral multiple of 3, therefore they have a common factor 3, which contradict our assumption that p and q are co-primes. Hence $\sqrt{3}$ is an irrational number.

In the same way, we can prove that $\sqrt{5}$ is also an irrational number.

TO PROVE $(2+\sqrt{3})$ IS AN IRRATIONAL NUMBER

Method-I : 2 is a rational number

$\sqrt{3}$ is an irrational number

$(2+\sqrt{3})$ is the sum of a rational and an irrational numbers. Hence, according to the properties of irrational numbers $(2+\sqrt{3})$ is an irrational number.

Method II : Let us suppose if possible that $2+\sqrt{3}$ is not an irrational number i.e. $2+\sqrt{3}$ is a rational number, then it can be represented in the form of $\frac{p}{q}$ where p and q are intergers but $q \neq 0$.

i.e. $2+\sqrt{3} = \frac{p}{q}$

$$\sqrt{3} = \frac{p}{q} - 2 = \frac{p-2q}{q} \qquad \text{...(i)}$$

Since p and q are integer so $(p-2q)$ will also be, an integer, hence $\frac{p-2q}{q}$ is a rational number.

But $\sqrt{3}$ is an irrational.

This contradicts our assumption that $(2+\sqrt{3})$ is a rational number.

Hence, $(2+\sqrt{3})$ is an irrational number

TO PROVE $\sqrt{n}$ IS NOT A RATIONAL NUMBER. IF n IS NOT A PERFECT SQUARE

Let n be a whole number but not a perfect square

If possible, let $\sqrt{n}$ be the rational number, then it can be represented in the form of $\frac{p}{q}$, where p and q are integers but $q \neq 0$. Also suppose $\frac{p}{q}$ is in simplest form, hence p and q are co-prime i.e. they have no common factor other than 1.

Now, $\frac{p}{q} = \sqrt{n}$

Squaring both the sides we get $\frac{p^2}{q^2} = n$

$p^2 = nq^2$ ...(i)

Since n is a factor of p^2

Hence, n is a factor of p.

Hence, we can suppose that $p = nm$, where m is an integer.

Putting the value of p in equation (i), we get

$n^2m^2 = nq^2$

$q^2 = nm^2$

$\therefore$ n is a factor of q^2

$\Rightarrow$ n is a factor of q. ...(ii)

Hence, n is a factor of both p and q. This contradicts our assumption that p and q does not have any common factor.

This means our assumption that $\sqrt{n}$ is a rational number is wrong. Hence $\sqrt{n}$ is an irrational number, where n is not a perfect square.

Note : Using the above result, we can prove that $\sqrt{7}, \sqrt{10}, \sqrt{11}$ etc are all irrational numbers.

PROPERTIES OF IRRATIONAL NUMBERS

(i) Negative of an irrational number is also an irrational number

For example: $\sqrt{5}$ is an irrational number, because $\sqrt{5}$ is an irrational number.

(ii) The sum or difference of a rational number and an irrational number is an irrational.

For example : $2+\sqrt{3}$, $2-\sqrt{3}$, $\sqrt{3}-2$ are irrational numbers because 2 is a rational number and $\sqrt{3}$ is an irrational number.

(iii) The product or quotient of a non-zero rational number with an irrational number is an irrational.

For example : $2\sqrt{3}, \frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{2}$ are irrational numbers because 2 is a rational number and $\sqrt{3}$ is an irrational number.

(iv) The sum, difference, product and quotient of two irrational numbers may be rational or irrational.

For examples :

(i) $(2+\sqrt{3})$ and $(2-\sqrt{3})$ are two irrational numbers but their sum $= 2+\sqrt{3}+2-\sqrt{3} = 4$, is a rational number.

(ii) $(2+\sqrt{3})$ and $(-2+\sqrt{3})$ are two irrational numbers. Their sum $= (2+\sqrt{3})+(-2+\sqrt{3}) = 2\sqrt{3}$, is an irrational number

(iii) $(2+\sqrt{3})$ and $(-2+\sqrt{3})$ are two irrational numbers but their difference $= (2+\sqrt{3})-(-2+\sqrt{3}) = 4$, is a rational number.

(iv) $(2+\sqrt{3})$ and $(2-\sqrt{3})$ are two irrational numbers. Their difference $= (2+\sqrt{3})-(2-\sqrt{3}) = 2\sqrt{3}$ is also an irrational number.

(v) $\sqrt{2}$ and $\sqrt{8}$ are two irrational numbers but their product $= \sqrt{2}\times\sqrt{8} = \sqrt{16} = 4$ or -4, is a rational number

(vi) $\sqrt{2}$ and $\sqrt{3}$ are two irrational numbers. Their product $= \sqrt{2}\times\sqrt{3} = \sqrt{6}$, is an irrational number

(vii) $\sqrt{8}$ and $\sqrt{2}$ are irrational numbers but their quotient $= \frac{\sqrt{8}}{\sqrt{2}} = \sqrt{\frac{8}{2}} = \sqrt{4} = 2$ is a rational number

(viii) $(2+\sqrt{3})$ and $(2-\sqrt{3})$ are irrational numbers. Their quotient

$$= \frac{2+\sqrt{3}}{2-\sqrt{3}} = \frac{(2+\sqrt{3})\times(2+\sqrt{3})}{(2-\sqrt{3})\times(2+\sqrt{3})} = \frac{(2+\sqrt{3})^2}{4-3}\ \frac{4+3+4\sqrt{3}}{1} = 7+4\sqrt{3}$$, is an irrational number.

SURDS

Any irrational number of the form '$\sqrt[n]{a}$' is given a special name, called Surd. 'a' is called radicand, it should always be a rational number.

Also the symbol '$\sqrt[n]{\ }$' is called the radical sign and the index 'n' is called order of the surd.

$\sqrt[n]{a}$ is read as nth root of 'a' and can also be written as $(a)^{\frac{1}{n}}$.

Example : $\sqrt[3]{4}$, $\sqrt[5]{17}$, $(12)^{\frac{1}{5}}$ etc are surds.

TYPES OF SURDS

(i) Pure surd A surd which has unity only as its rational factor, the other factor being irrational. For example $\sqrt{3}, \sqrt{15}, \sqrt[4]{3}, \sqrt[6]{8}$ etc.

(ii) Mixed surds: A surd which has a rational factor other than unity, the other factor being irrational, is called a mixed surd.

For example - $2\sqrt{3}$, $2\sqrt[3]{5}$, $-6\sqrt[4]{8}$ etc.

(iii) Simple surd: A surd consisting of a single term is called a simple surd.

For example $\sqrt{3}$, $3\sqrt[4]{5}$, $\frac{7}{2}\sqrt[3]{6}$ etc.

(iv) Compound Surd: An algebraic combination of two or more surds is called a compound surd.

For example $(\sqrt{3}+\sqrt{5})$, $(\sqrt{2}-\sqrt[3]{4})$, $(3\sqrt[5]{8}-7\sqrt[4]{15})$ etc.

(v) Like surds : Two or more surds are called like surds if order and radicand of their irrational factor are same.

ADDITION AND SUBTRACTION OF SURDS: Addition and subtraction of surds are possible only when the surds are like surds.

LAWS OF RATIONAL EXPONENTS

If a & b are positive real numbers and m & n are rational numbers, then

(i) $a^m \times a^n = a^{m+n}$

(ii) $a^m \div a^n = a^{m-n}$

(iii) $(a^m)^n = a^{mn}$

(iv) $a^{-n} = \frac{1}{a^n}$

(v) $a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = \left(a^{\frac{1}{n}}\right)^m$ i.e. $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

(vi) $(ab)^m = a^m b^m$

(vii) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

(viii) $(\sqrt[n]{a})^n = \sqrt[n]{a^n} = a$

(ix) $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$

(x) $\sqrt[n]{a} \div \sqrt[n]{b} = \sqrt[n]{\frac{a}{b}}$

(xi) $\sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a} = \sqrt[m]{\sqrt[n]{a}}$

Illustration 9 : Simplify : $13^{\frac{1}{5}}.17^{\frac{1}{5}}$

SOLUTION : $13^{\frac{1}{5}}.17^{\frac{1}{5}} = (13\times17)^{\frac{1}{5}} = 221^{\frac{1}{5}} = \sqrt[5]{221}$

Illustration 10 : Simplify $\pi^{3/4}\pi^{1/2}$

SOLUTION : $\pi^{3/4}.\pi^{1/2} = \pi^{\frac{3}{4}+\frac{1}{2}} = \pi^{\frac{5}{4}}$

Illustration 11 : Simplify: $\left(\dfrac{2^a}{2^b}\right)^{a+b}\left(\dfrac{2^b}{2^a}\right)^{b+c}\left(\dfrac{2^c}{2^a}\right)^{c+a}$

SOLUTION : $\left(\dfrac{2^a}{2^b}\right)^{a+b}\left(\dfrac{2^b}{2^c}\right)^{b+c}\left(\dfrac{2^c}{2^a}\right)^{c+a} = (2^{a-b})^{a+b}\cdot(2^{b-c})^{b+c}\cdot(2^{c-a})^{c+a}$

$$= 2^{(a^2-b^2)+(b^2-c^2)+(c^2-a^2)} = 2^0 = 1$$

Illustration 12 : $\left(\dfrac{x^l}{x^{-m}}\right)^{l^2+m^2-lm}\times\left(\dfrac{x^m}{x^{-n}}\right)^{m^2+n^2-mn}\times\left(\dfrac{x^n}{x^{-l}}\right)^{n^2+l^2-nl}$

SOLUTION : $\left(\dfrac{x^l}{x^{-m}}\right)^{l^2+m^2-lm}\times\left(\dfrac{x^m}{x^{-n}}\right)^{m^2+n^2-mn}\times\left(\dfrac{x^n}{x^{-l}}\right)^{n^2+l^2-nl}$

$$= (x^{l+m})^{l^2+m^2-lm}\times(x^{m+n})^{m^2+n^2-mn}\times(x^{n+l})^{n^2+l^2-nl}$$

$$= x^{(l+m)(l^2+m^2-lm)}\cdot x^{(m+n)(m^2+n^2-mn)}\cdot x^{(n+l)(n^2+l^2-nl)}$$

$$= x^{(l+m)(l^2+m^2-lm)+(m+n)(m^2+n^2-mn)+(n+l)(n^2+l^2-nl)}$$

$$= x^{l^3+m^2l-l^2m+ml^2+m^3-lm^2+m^3+mn^2-m^2n+nm^2+n^3-mn^2+n^3+nl^2-n^2l+ln^2+l^3-nl^2}$$

$$= x^{2(l^3+m^3+n^3)}$$

Illustration 13 : Assuming that x, y are positive real numbers, simplify each of the following:

(i) $\sqrt{x^{-2}y^3}$

(ii) $(x^{-2/3}\cdot y^{-1/2})^2$

(iii) $(\sqrt{x^{-3}})^5$

(iv) $(\sqrt{x})^{-2/3}\sqrt{y^4}\div\sqrt{xy^{-1/2}}$

(v) $\sqrt[3]{xy^{-2}}\div x^2y$

(vi) $\sqrt[4]{\sqrt[3]{x^2}}$

SOLUTION :

(i) We have,

$$\sqrt{x^{-2}y^3} = \sqrt{\frac{y^3}{x^2}} = \left(\frac{y^3}{x^2}\right)^{\frac{1}{2}} = \frac{(y^3)^{\frac{1}{2}}}{(x^2)^{\frac{1}{2}}} = \frac{y^{3/2}}{x^{2/2}} = \frac{y^{3/2}}{x}$$

(ii) We have,

$$(x^{-2/3}\times y^{-1/2})^2 = (x^{-2/3})^2(y^{-1/2})^2 = x^{-2/3\times2}y^{-1/2\times2} = x^{-4/3}y^{-1} = \frac{1}{x^{4/3}y}$$

(iii) We have,

$$(\sqrt{x^{-3}})^5 = \left[(x^{-3})^{\frac{1}{2}}\right]^5 = (x^{-3/2})^5 = x^{3/2\times5} = x^{-15/2} = \frac{1}{x^{15/2}}$$

(iv) We have,

$$(\sqrt{x})^{-2/3}\sqrt{y^4} \div \sqrt{xy^{-1/2}} = \frac{(x^{1/2})^{-2/3}(y^4)^{1/2}}{(xy^{-1/2})^{1/2}} = \frac{x^{1/2x-2/3}y^{4\times1/2}}{x^{1/2}(y^{-1/2})^{1/2}}$$

$$\Rightarrow (\sqrt{x})^{-2/3}\sqrt{y^4} \div \sqrt{xy^{-1/2}} = \frac{x^{-1/3}y^2}{x^{1/2}y^{-1/2\times1/2}} = \frac{x^{-1/3}y^2}{x^{1/2}y^{-1/4}}$$

$$\Rightarrow (\sqrt{x})^{-2/3}\sqrt{y^4} \div \sqrt{xy^{-1/2}} = \frac{1}{x^{1/3+1/2}} \times y^{2+1/4} = \frac{y^{9/8}}{x^{5/6}}$$

(v) We have,

$$\sqrt[3]{xy^2} + x^2y = \frac{\sqrt[3]{xy^2}}{x^2y} = \frac{(xy^2)^{\frac{1}{3}}}{x^2y}\ \frac{x^{1/3}y^{2/3}}{x^2y}$$

$$= x^{1/3}\,y^{2/3-1} = x^{-5/3}\,y^{-1/3}$$

(vi) We have,

$$\sqrt[4]{\sqrt[3]{x^2}} = \left[(x^2)^{1/3}\right]^{1/4} = (x^{2/3})^{\frac{1}{4}} = x^{2/3\times1/4} = x^{1/6}$$

Illustration 14 : Show that : $\dfrac{x^{a(b-c)}}{x^{b(a-c)}} \div \left(\dfrac{x^b}{x^a}\right)^c = 1$

SOLUTION :

We have,

$$\frac{x^{a(b-c)}}{x^{b(a-c)}} \div \left(\frac{x^b}{x^a}\right)^c$$

$$= \frac{x^{ab-ac}}{x^{ba-bc}} \div (x^{b-a})^c$$

$$= x^{(ab-ac)-(ba-bc)} \times \frac{1}{x^{(b-a)c}}$$

$$= x^{-ab-ac-ba+bc} \times \frac{1}{x^{bc-ac}} = x^{-ac+bc}\,x^{ac-bc}$$

$$= x^{-ac+bc+ac-bc} = x^0 = 1$$

RATIONALISING FACTOR

If the product of two surds is a rational number, then each of them is a rationalising factor of each other. Rationalising factor of a surd is not unique.

For examples:

(i) $\sqrt[n]{a^{n-m}}$ is the rationalising factor of $\sqrt[n]{a^m}$ and vice-versa.

(ii) $(\sqrt{a}+\sqrt{b})$ is the rationalising factor of $(\sqrt{a}-\sqrt{b})$ and vice-versa.

(iii) $(a+\sqrt{b})$ is the rationalising factor of $(a-\sqrt{b})$ and vice-versa.

(iv) $(\sqrt[3]{a}+\sqrt[3]{b})$ is a rationalising factor of $(a^{2/3}-a^{1/3}.b^{1/3}+b^{2/3})$ and vice-versa.

(v) $(\sqrt[3]{a}-\sqrt[3]{b})$ is a rationalising factor of $\left(a^{2/3}+a^{\frac{1}{3}}b^{1/3}+b^{\frac{2}{3}}\right)$ and vice-versa.

Illustration 15 : Rationalise the denominator of $\frac{1}{7+3\sqrt{2}}$

SOLUTION : We have,

$$\frac{1}{7+3\sqrt{2}}=\frac{1}{7+3\sqrt{2}}\times\frac{7-3\sqrt{2}}{7-3\sqrt{2}}=\frac{7-3\sqrt{2}}{(7)^2-(3\sqrt{2})^2}$$

$$=\frac{7-3\sqrt{2}}{49-18}=\frac{7-3\sqrt{2}}{31}$$

Illustraton 16 : Simplify each of the following by rationalising the denominator $\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}$

SOLUTION : Multiplying both numerator and denominator by the rationalisation factor of the denominator, we have

$$\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}=\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}\times\frac{2\sqrt{2}-3\sqrt{3}}{2\sqrt{2}-3\sqrt{3}}=\frac{(2\sqrt{3}-\sqrt{5})(2\sqrt{2}-3\sqrt{3})}{(2\sqrt{2}+3\sqrt{3})(2\sqrt{2}-3\sqrt{3})}$$

$$\Rightarrow\quad \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}=\frac{2\sqrt{3}\times2\sqrt{2}-2\sqrt{3}\times3\sqrt{3}-\sqrt{5}\times2\sqrt{2}+\sqrt{5}\times3\sqrt{3}}{(2\sqrt{2})^2-(3\sqrt{3})^2}$$

$$\Rightarrow\quad \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}=\frac{4\sqrt{3\times2}-6\sqrt{3\times3}-2\sqrt{5\times2}+3\sqrt{5\times3}}{4\times2-9\times3}$$

$$\Rightarrow\quad \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}=\frac{4\sqrt{6}-6\times30-2\sqrt{10}+3\sqrt{15}}{8-27}$$

$$\Rightarrow\quad \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}=\frac{4\sqrt{6}-18-2\sqrt{10}+3\sqrt{15}}{-19}$$

$$\Rightarrow\quad \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}=\frac{18+2\sqrt{10}-4\sqrt{6}-3\sqrt{15}}{19}$$

Illustration 17 : Simplify each of the following : $\frac{4+\sqrt{5}}{4-\sqrt{5}}+\frac{4-\sqrt{5}}{4+\sqrt{5}}$

SOLUTION : We have,

$$\frac{4+\sqrt{5}}{4-5}+\frac{4-\sqrt{5}}{4+\sqrt{5}}$$

$$=\frac{4+\sqrt{5}}{4-\sqrt{5}}\times\frac{4+\sqrt{5}}{4+\sqrt{5}}+\frac{4-\sqrt{5}}{4+\sqrt{5}}\times\frac{4-\sqrt{5}}{4-\sqrt{5}}$$

$$=\frac{(4+\sqrt{5})^2}{(4-\sqrt{5})(4+\sqrt{5})}+\frac{(4-\sqrt{5})^2}{(4+\sqrt{5})(4-\sqrt{5})}\ \frac{(4+\sqrt{5})^2}{(4-\sqrt{5})(4+\sqrt{5})}+\frac{(4-\sqrt{5})^2}{(4+\sqrt{5})(4-\sqrt{5})}$$

$$=\frac{4^2+(\sqrt{5})^2+2\times4\times\sqrt{5}}{4^2-(\sqrt{5})^2}+\frac{4^2+(\sqrt{5})^2-2\times4\times\sqrt{5}}{4^2-(\sqrt{5})^2}$$

$$=\frac{16+5+8\sqrt{5}}{16-5}+\frac{16+5-8\sqrt{5}}{16-5}=\frac{21+8\sqrt{5}+21-8\sqrt{5}}{16-5}=\frac{42}{11}$$

MISCELLANEOUS

Solved Examples

Example 1 : **Find the value of $2.\overline{6} - 0.\overline{9}$**

SOLUTION : Let $x = 2.\overline{6}$ ...(1)

$\therefore\ 10x = 26.\overline{6}$ ...(2)

Subtracting (1) from (2)

$9x = 24$

$\therefore\quad x = \frac{24}{9} = \frac{8}{3}$

Also suppose $y = 0.\overline{9}$ ...(3)

$\Rightarrow\ y = 0.9\overline{9}$

$\therefore\ 10y = 9.\overline{9}$ ...(4)

Subtracting equation (3) from (4), we get

$9y = 9$

$\therefore\quad y = \frac{9}{9} = 1$

$\therefore\quad 2.\overline{6} - 0.\overline{9} = x - y = \frac{8}{3} - 1 = \frac{8-3}{3} = \frac{5}{3}$

Example 2 : **Rationalise the denominator and simplify : $\frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$**

SOLUTION : $\frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$

$= \frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}} \times \frac{\sqrt{3}-\sqrt{6}}{\sqrt{3}-\sqrt{6}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} \times \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$

$= \frac{3\sqrt{2}(\sqrt{3}-\sqrt{6})}{3-6} - \frac{4\sqrt{3}(\sqrt{6}-\sqrt{2})}{6-2} + \frac{\sqrt{6}(\sqrt{2}-\sqrt{3})}{2-3}$

$= -\sqrt{2}(\sqrt{3}-\sqrt{6}) - \sqrt{3}(\sqrt{6}-\sqrt{2}) - \sqrt{6}(\sqrt{2}-\sqrt{3}) = -\sqrt{6} + \sqrt{12} - \sqrt{18} + \sqrt{6} - \sqrt{12} + \sqrt{18} = 0$

Example 3 : **Simplify : $5\sqrt[3]{250} + 7\sqrt[3]{16} - 14\sqrt[3]{54}$**

SOLUTION : $= 5\sqrt[3]{250} + 7\sqrt[3]{16} - 14\sqrt[3]{54}$

$= 5\sqrt[3]{125\times 2} + 7\sqrt[3]{8\times 2} - 14\sqrt[3]{27\times 2}$

$= 5\times 5\sqrt[3]{2} + 7\times 2\sqrt[3]{2} - 14\times 3\times \sqrt[3]{2} = (25+14-42)\sqrt[3]{2} = -3\sqrt[3]{2}$

Example 4 : Simplify : $\sqrt[6]{8a^5b} \times \sqrt[3]{4a^2b^2}$

SOLUTION : $= \sqrt[6]{8^3a^{15}b^3} \times \sqrt[6]{4^2a^4b^4} = 4a^3b\sqrt[6]{2ab}$

Example 5 : Divide $\sqrt{24}$ by $\sqrt[3]{200}$

SOLUTION : $\dfrac{\sqrt{24}}{\sqrt[3]{200}} = \dfrac{\sqrt[6]{(24)^3}}{\sqrt[6]{(200)^2}} = \sqrt[6]{\dfrac{216}{625}}$

Example 6: If $x = \dfrac{1}{2+\sqrt{3}}$, find the value of $x^3 - x^2 - 11x + 3$

SOLUTION : as $x = \dfrac{1}{2+\sqrt{3}} = 2 - \sqrt{3}$

$$x - 2 = -\sqrt{3}$$

Squaring both sides, we get

$$(x-2)^2 = (-\sqrt{3})^2 \Rightarrow x^2 + 4 - 4x = 3 \Rightarrow x^2 - 4x + 1 = 0$$

Now, $x^3 - x^2 - 11x + 3 = x^3 - 4x^2 + x - 3x^2 - 12x + 3$

$$x(x^2 - 4x + 1) + 3(x^2 - 4x + 1)$$

$$x \times 0 + 3(0)$$

$$0 + 0 = 0$$

Example 7 : If $x = \dfrac{\sqrt{a+2b} + \sqrt{a-2b}}{\sqrt{a+2b} - \sqrt{a-2b}}$, prove that $b^2x^2 - abx + b^2 = 0$

SOLUTION : $x = \dfrac{(\sqrt{a+2b} + \sqrt{a-2b})}{(\sqrt{a+2b} - \sqrt{a-2b})} \times \dfrac{(\sqrt{a+2b} + \sqrt{a-2b})}{(\sqrt{a+2b} + \sqrt{a-2b})}$

$$\Rightarrow \quad x = \frac{(\sqrt{a+2b} + \sqrt{a-2b})^2}{(a+2b) - (a-2b)} = \frac{a + 2b + a - 2b + 2\sqrt{(a+2b)(a-2b)}}{4b}$$

$$\Rightarrow \quad x = \frac{2\left[a + \sqrt{a^2 - 4b^2}\right]}{2b}$$

$$\Rightarrow \quad 2bx = a + \sqrt{a^2 - 4b^2}$$

$$\Rightarrow \quad 2bx - a = \sqrt{a^2 - 4b^2}$$

Squaring both sides, we get

$$(2bx - a)^2 = \left(\sqrt{a^2 - 4b^2}\right)^2$$

$$\Rightarrow \quad 4b^2x^2 + a^2 - 4abx = a^2 - 4b^2$$

$$\Rightarrow \quad 4b^2x^2 - 4abx + 4b^2 = 0$$

$$\Rightarrow \quad b^2x^2 - abx + b^2 = 0$$

Example 8 : Show that $\dfrac{(x^{a+b})^2(x^{b+c})^2(x^{c+a})^2}{(x^a x^b x^c)^4}=1$

SOLUTION : We have $=\dfrac{(x^{a+b})^2(x^{b+c})^2(x^{c+a})^2}{(x^a x^b x^c)^4}$

$$=\frac{x^{2(a+b)}x^{2(b+c)}x^{2(c+a)}}{(x^a)^4.(x^b)^4(x^c)^4}=\frac{x^{2a+2b}.x^{2b+2c}.x^{2c+2a}}{x^{4a}.x^{4b}.x^{4c}}=\frac{x^{2a+2b+2b+2c+2c+2a}}{x^{4a+4b+4c}}=\frac{x^{4a+4b+4c}}{x^{4a+4b+4c}}=1$$

Example 9 : If $\dfrac{9^n\times 3^2(3^{-n/2})-(27)^n}{3^{3m}\times 2^3}=\dfrac{1}{27}$, prove that $m-n=1$.

SOLUTION : We have, $\dfrac{9^n\times 3^2(3^{-n/2})^{-2}-(27)^n}{3^{3m}\times 2^3}=\dfrac{1}{27}$

$$\Rightarrow \frac{(3^2)^n\times 3^2\times 3^{-n/2\times -2}-(3^3)^n}{3^{3m}\times 2^3}=\frac{1}{27}$$

$$\Rightarrow \frac{3^{2n}\times 3^2\times 3^n-3^{3n}}{3^{3m}\times 2^3}=\frac{1}{27}$$

$$\Rightarrow \frac{3^{2n+2+n}-3^{3n}}{3^{3m}\times 2^3}=\frac{1}{27}.$$

$$\Rightarrow \frac{3^{3n}(3^2-1)}{3^{3m}\times 2^3}=\frac{1}{27}\Rightarrow \frac{3^{3n}.8}{3^{3m}.8}=\frac{1}{27}$$

$$\Rightarrow 3^{3n-3m}=\frac{1}{3^3}$$

$$\Rightarrow 3^{3n-3m}=3^{-3}$$

On equating the exponent, we get

$$\Rightarrow 3n-3m=-3\Rightarrow n-m=-1\Rightarrow m-n=1.$$

Example 10: Simplify each of the following by rationalising the denominator $\dfrac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}$

SOLUTION : Multiplying both numerator and denominator by the rationalisation factor of the denominator, we have

$$\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}=\frac{2\sqrt{3}-2\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}\times\frac{2\sqrt{2}-3\sqrt{3}}{2\sqrt{2}-3\sqrt{3}}=\frac{(2\sqrt{3}-\sqrt{5})(2\sqrt{2}-3\sqrt{3})}{(2\sqrt{2}+3\sqrt{3})(2\sqrt{2}-3\sqrt{3})}$$

$$\Rightarrow \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}=\frac{2\sqrt{3}\times 2\sqrt{2}-2\sqrt{3}\times 3\sqrt{3}-\sqrt{5}\times 2\sqrt{2}+\sqrt{5}\times 3\sqrt{3}}{(2\sqrt{2})^2-(3\sqrt{3})^2}$$

$$\Rightarrow \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}=\frac{4\sqrt{3\times 2}-6\sqrt{3\times 3}-2\sqrt{5\times 2}+3\sqrt{5}\times 3}{4\times 2-9\times 3}$$

$$\Rightarrow \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}=\frac{4\sqrt{6}-6\times 3-2\sqrt{10}+3\sqrt{15}}{8-27}$$

$$\Rightarrow \quad \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}} = \frac{4\sqrt{6}-18-2\sqrt{10}+3\sqrt{15}}{-19}$$

$$\Rightarrow \quad \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}} = \frac{18+2\sqrt{10}-4\sqrt{6}-3\sqrt{15}}{19}$$

Example 11: If both a and b are rational numbers, find the values of a and b in each of the following equalities:

$$\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = a - b\sqrt{6}$$

SOLUTION : Multiplying the numerator and denominator by rationalisation factor of the denominator, we get

$$\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = \frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} \times \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} = \frac{(\sqrt{2}+\sqrt{3})(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2}-2\sqrt{3})(3\sqrt{2}+2\sqrt{3})}$$

$$\Rightarrow \quad \frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = \frac{\sqrt{2}\times 3\sqrt{2}+\sqrt{2}+2\sqrt{3}+\sqrt{3}\times 3\sqrt{2}+\sqrt{3}\times 2\sqrt{3}}{(3\sqrt{2})^2-(2\sqrt{3})^2}$$

$$\Rightarrow \quad \frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = \frac{3\sqrt{2\times 2}+2\sqrt{3\times 2}+3\sqrt{3\times 2}+2\sqrt{3\times 3}}{9(\sqrt{2})^2-4(\sqrt{3})^2}$$

$$\Rightarrow \quad \frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = \frac{3\times 2+2\sqrt{6}+3\sqrt{6}+2\times 3}{9\times 2-4\times 3}$$

$$\Rightarrow \quad \frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = \frac{6+(2+3)\sqrt{6}+6}{18-12} = \frac{12+5\sqrt{6}}{6} = 2+\frac{5}{6}\sqrt{6}$$

$$\therefore \quad \frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = a-b\sqrt{6}$$

$$\Rightarrow \quad 2+\frac{5}{6}\sqrt{6} = a-b$$

$$\Rightarrow \quad a-b\sqrt{6} = 2-(-5/6)\sqrt{6}$$

$$\Rightarrow \quad a = 2 \text{ and } b = -5/6$$

Example 12 : If $x = 3\sqrt{3}+\sqrt{26}$ find the value of $\frac{1}{2}\left(x+\frac{1}{x}\right)$

SOLUTION : $x = 3\sqrt{3}+\sqrt{26}$

$$\frac{1}{x} = \frac{1}{3\sqrt{3}+\sqrt{26}} \times \frac{3\sqrt{3}-\sqrt{26}}{3\sqrt{3}-\sqrt{26}}$$

$$= \frac{3\sqrt{3}-\sqrt{26}}{(27)-(26)} = 3\sqrt{3}-\sqrt{26}$$

$$\therefore \frac{1}{2}\left(x+\frac{1}{x}\right) = \frac{1}{2}\left[(3\sqrt{3}+\sqrt{26})+(3\sqrt{3}-\sqrt{26})\right]$$

$$= \frac{1}{2}\times 6\sqrt{3} = 3\sqrt{3}$$

Example 13: If $x = 2 + 2^{1/3} + 2^{2/3}$ find $x^3 - 6x^2 + 6x - 2$.

SOLUTION : $x = 2 + 2^{1/3} + 2^{2/3}$

$x - 2 = 2^{1/3} + 2^{2/3} = 2^{1/3}(1 + 2^{1/3})$

$\Rightarrow (x-2)^3 = [2^{1/3}(1 + 2^{1/3})]^3$

$\Rightarrow x^3 - 8 - 3.x^2.2 + 3.x.2^2 = 2(1+2^{1/3})^3$

$\Rightarrow x^3 - 8 - 6x^2 + 12x = 2(1 + 2 + 3.1^2.2^{1/3} + 3.1.2^{2/3})$

$\Rightarrow x^3 - 6x^2 + 12x - 8 = 2[3 + 3.2^{1/3} + 3.2^{2/3}]$

$= 6(1 + 2^{1/3} + 2^{2/3})$

$= 6(x-1)$ ...(i) $\left[\begin{array}{l} \because\ x = 2 + 2^{\frac{1}{3}} + 2^{\frac{2}{3}} \\ \therefore\ x - 1 = 1 + 2^{\frac{1}{3}} + 2^{\frac{2}{3}} \end{array}\right]$

$\Rightarrow x^3 - 6x^2 + 12x - 8 = 6x - 6$

$\Rightarrow x^3 - 6x^2 + 12x - 6x - 8 + 6 = 0$

$\Rightarrow x^3 - 6x^2 + 6x - 2 = 0$

Example 14: Find $\sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots\ldots\infty}}}$

SOLUTION : Let $y = \sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots\ldots\infty}}}$

$= \sqrt{6 + y}$ [$\because$ the term are repeating]

$y^2 = 6 + y$ [on squaring]

$y^2 - y - 6 = 0$

$(y-3)(y+2) = 0$

$y = 3, -2$

But $y \neq -2$

$\therefore y = 3$

$\therefore \sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots\ldots\infty}}} = 3$

Example 15: If $a^p = b^q = c^r = abc$ then find the value of pqr.

SOLUTION : Let $a^p = b^q = c^r = k$ (ray)

$\therefore a^p = k \Rightarrow a = k^{1/p}$

$b^r = k \Rightarrow b = k^{1/r}$

$c^r = k \Rightarrow c = k^{1/r}$

$abc = k^{1/p} . k^{1/q} . k^{1/r}$ (On multiplying)

$abc = k^{\frac{1}{p}+\frac{1}{q}+\frac{1}{r}} = k$ [Given]

$\therefore \frac{1}{p} + \frac{1}{q} + \frac{1}{r} = 1$

$\frac{qr + pr + pq}{pqr} = 1$

$pq + qr + pr = pqr$

Example 16: If $\left[\left\{\left(\frac{1}{7^2}\right)^{-2}\right\}^{-1/3}\right]^{\frac{1}{4}} = 7^m$, then find the value of m.

SOLUTION :

$$\left[\left\{\left(\frac{1}{7^2}\right)^{-2}\right\}^{-\frac{1}{3}}\right]^{\frac{1}{4}} = 7^m$$

$$\Rightarrow \quad \left[\{(7^{-2})^{-2}\}^{-1/3}\right]^{\frac{1}{4}} = 7^m$$

$$\Rightarrow \quad \left[(7^4)^{-1/3}\right]^{\frac{1}{4}} = 7^m$$

$$\Rightarrow \quad (7^{-4/3})^{1/4} = 7^m$$

$$\Rightarrow \quad 7^{-1/3} = 7^m$$

$$\therefore \quad m = -1/3$$

Example 17 : Show that between any two distinct rational numbers a and b, there exists another rational number.

SOLUTION : Since $a \neq b$, without any loss of generality we may assume that $a < b$.

Now $a < b$, $\quad \therefore\ a + a < a + b$

$$\Rightarrow 2a < a + b$$

$$\Rightarrow a < \frac{a+b}{2}$$

Also, $a < b$, $\quad \therefore\ a + b < b + b$

$$\Rightarrow a + b < 2b$$

$$\Rightarrow \frac{a+b}{2} < b$$

Thus, $a < \frac{a+b}{2} < b$

Clearly, $\frac{a+b}{2}$ is a rational number lying between a and b.

Example 18 : Find two irrational numbers between 0.12 and 0.13.

SOLUTION : The two numbers 0.1201001000100001....... and 0.12101001000100001........ are the irrational numbers between 0.12 and 0.13.

Example 19 : Find a rational number between $\frac{1}{5}$ and $\frac{7}{10}$.

SOLUTION : One rational number between $\frac{1}{5}$ and $\frac{7}{10} = \frac{1}{2}\left(\frac{1}{5}+\frac{7}{10}\right) = \frac{1}{2}\left[\frac{2+7}{10}\right] = \frac{9}{20}$

Example 20 : Which of the following rational numbers can be represented as terminating decimals ?

(i) $\frac{7}{20}$ **(ii)** $\frac{-27}{49}$

SOLUTION :

(i) In $\frac{7}{20}$, $20 = 2^2 \times 5$, i.e., it has factors in 2's and 5's.

$\therefore$ $\frac{7}{20}$ has terminating decimal representation.

(ii) In $\frac{-27}{40}$, $40 = 2^3 \times 5$ i.e., it has factors in 2's and 5's.

$\therefore$ $\frac{27}{40}$ has terminating decimal representation.

Example 21 : Express $1.272727..... = 1.\overline{27}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

SOLUTION : Let $x = 1.272727$.......... Since two digits are repeating, we multiply x by 100 to get

$100x = 127.2727$ ....

So, $100x = 126 + 1.272727......... = 126 + x$

Therefore, $100x - x = 126, \Rightarrow 99x = 126 \Rightarrow x = \frac{126}{99} = \frac{14}{11}$

Example 22 : Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is $\pi = \frac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction ?

SOLUTION : There is no contradiction. Remember that when you measure a length with a scale or any other device, you only get anapproximate rational value. So, you may not realise that either c or d is irrational.

Example 23 : Express $0.12\overline{3}$ in $\frac{p}{q}$ form.

SOLUTION : Let $x = 012\overline{3}$ i.e. $x = 0.12333$ ... (1)

Multiply both sides of (1) by 100

$100x = 12.333$ ... (2)

Multiply both sides of (2) by 10

$1000x = 123.333$ ... (3)

Subtract (2) from (3)

$$\begin{aligned} 1000x &= 123.333 \\ 100x &= 12.333 \\ \hline 900x &= 111.0 \end{aligned}$$

$$\Rightarrow \quad x = \frac{111}{900} = \frac{3 \times 37}{900} = \frac{37}{300}$$

Example 24 : Prove that $2 + \sqrt{3}$ is irrational.

SOLUTION : Let $2 + \sqrt{3}$ be a rational number equals to r

$2 + \sqrt{3} = r$

$\Rightarrow \quad \sqrt{3} = r - 2$

Here, L.H.S. is an irrational no. while R.H.S. $(r - 2)$ is rational.

$\therefore$ L.H.S. $\neq$ R.H.S.

Hence is contradicts our assumption

$\therefore$ $(2 + \sqrt{3})$ is irrational.

EXERCISE 1

FIB Fill in the Blanks

DIRECTIONS: *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

1. The sum/difference of a rational and an irrational number is ______.
2. 0.578 is ______ number. (rational/irrational)
3. 0.72737475______ number. (rational/irrational)
4. Two mixed quadratic surds, $a+\sqrt{b}$ and $a-\sqrt{b}$, whose sum and product are rational, are called______.
5. If $x+\sqrt{5}=4+\sqrt{y}$, then $x + y =$ ______.(where x and y are rational)
6. Irrational number between $\frac{2}{5}$ and $\frac{3}{7}$ is
7. Value of a is .... if $\frac{\sqrt{3}-1}{\sqrt{3}+1}=a+b\sqrt{3}$
8. If the product of two surds is a rational number, then each of the two is a ______ of the other.
9. $\frac{\sqrt{7}}{3\sqrt{3}}$, in rational denominator is ______.
10. 125–1/3 = ______.

T/F True /False

DIRECTIONS: *Read the following statements and write your answer as true or false.*

1. Every whole number is a natural number.
2. Every rational number is an integer.
3. Every natural number is a whole number.
4. $\sqrt[3]{3+\sqrt{2}}$ is a surd.
5. If $p=2+\sqrt{3}$ and pq is a rational number, then q is a unique surd.
6. Every integer is a whole number.
7. Every rational number is a whole number.
8. All rational numbers when expressed in decimal form are either terminating decimals or repeating decimals.
9. All rational numbers can be represented by some point on the number line.
10. Square root of a prime number is rational.

MTC Match the Columns

DIRECTIONS: *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column I have to be matched with statements (p, q, r, s, t) in column II.*

1.

Column I	Column II
(A) 12 is a	(p) prime number
(B) 2,7 are	(q) not a rational number
(C) 2 is a	(r) composite number
(D) $\sqrt{2}$	(s) co-prime numbers

VSAQ Very Short Answer Questions

DIRECTIONS: *Give answer in one word or one sentence.*

1. Write the smallest whole number.
2. Write the integer other than 1, which is a reciprocal of itself.
3. Write Additive inverse of $\frac{1}{5}$.
4. Write multiplicative inverse of $\frac{3}{5}$.
5. Suppose a is a rational number. What is the reciprocal of the reciprocal of a ?
6. Write the repeating decimal for each of the following, and use a bar to show the repetend.

   (i) $-\frac{4}{3}$ (ii) $\frac{11}{12}$ (iii) $\frac{7}{13}$
7. Classify the following numbers as rational or irrational.

   (i) $\sqrt{225}$ (ii) 7.478478......
8. Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.
9. Rationalise the denominator of $\frac{1}{2+\sqrt{3}}$
10. Simplify :

    (i) $\left(\frac{1}{3^3}\right)^7$ (ii) $7^{\frac{1}{2}}.8^{\frac{1}{2}}$

11. Simplify :

(i) $(\sqrt{4})^{3/4}$ (ii) $\left(\frac{5}{8}\right)^3\left(\frac{4}{3}\right)^3$

(iii) $\left[\frac{\frac{\sqrt{5}}{3}}{\frac{5}{9}}\right]^6$ (iv) $\left(\frac{5}{3}\right)^3\cdot\left(\frac{9}{2}\right)^4$

(v) $\frac{2^6\times 8^2}{4^4}$

12. Which whole number is not a natural number ?

13. Find two irrational numbers between 0.1 and 0.2.

14. Determine, without actually dividing, which of the following rational numbers can be named, (a) by a terminating decimal, (b) by a repeating decimal.

(i) $\frac{7}{20}$ (ii) $\frac{1}{6}$

(iii) $\frac{1}{12}$ (iv) $3\frac{47}{160}$

15. Write down a fraction which is equivalent to 0.033636363..........

16. Find two rational numbers between 0.222332333233332.... and 0.252552555255552....

17. Which is greatest : $\sqrt[3]{4}$, $\sqrt[4]{5}$ or $\sqrt[4]{3}$?

18. Find four rational numbers between $\frac{1}{4}$ and $\frac{1}{3}$.

SAQ Short Answer Questions

DIRECTIONS: *Give answer in 2-3 sentences.*

1. If $\frac{3+\sqrt{5}}{4-2\sqrt{5}}=p+q\sqrt{5}$, where p and q are rational numbers, find the values of p and q.

2. Simplify :

$$\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$$

3. Examine whether the following numbers are rational or irrational :

(i) $(2-\sqrt{3})^2$ (ii) $(3+\sqrt{2})(3-\sqrt{2})$

4. The product of two given real numbers is a non-zero rational number. Show that the numbers are either both rational or both irrational. If the product of the numbers is zero, how does this result change?

5. Find the value of 'a' in the following:

$$\frac{6}{3\sqrt{2}-2\sqrt{3}}=3\sqrt{2}-a\sqrt{3}$$

6. Simplify : $\left[5\left(8^{\frac{1}{3}}+27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}}$

7. Examine whether the number is rational or irrational $\frac{(2+\sqrt{2})(3-\sqrt{5})}{(3+\sqrt{5})(2-\sqrt{2})}$.

8. Given that $\sqrt{3}=1.732$ find the value of

$$\sqrt{75}+\frac{1}{2}\sqrt{48}-\sqrt{192}$$

9. Simplify : $3\sqrt{2}+\sqrt[4]{64}+\sqrt[4]{64}+\sqrt[4]{2500}+\sqrt[6]{8}$

10. Simplify and express the results in simplest form

$$\frac{\sqrt{x^2-y^2}+x}{\sqrt{x^2+y^2}+y}+\frac{\sqrt{x^2+y^2}-y}{x-\sqrt{x^2-y^2}}$$

11. If $x=9-4\sqrt{5}$ find the value of $\sqrt{x}-\frac{1}{\sqrt{x}}$

12. Find x^2, if $x=\frac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}}$

LAQ Long Answer Questions

DIRECTIONS: *Give answer in four to five sentences.*

1. Express $0.6+0.\bar{7}+0.4\bar{7}$ in the form $\frac{p}{q}$, where p and q are integers and $q\neq 0$

2. If $x=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $y=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$.

Then find the value of x^2+y^2.

3. Express with a rational denominator :

$$\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$$

4. Simplify : $\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}-\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$

5. If $\frac{\sqrt{7}-1}{\sqrt{7}+1}-\frac{\sqrt{7}+1}{\sqrt{7}-1}=a+b\sqrt{7}$, find the values of a and b.

6. Simplify : $\frac{4+\sqrt{5}}{4-\sqrt{5}}+\frac{4-\sqrt{5}}{4+\sqrt{5}}$

7. If $\sqrt{5}=2.236$ and $\sqrt{6}=2.449$ find the value of $\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$.

8. If $x=\frac{\sqrt{p+2q}+\sqrt{p-2q}}{\sqrt{p+2q}-\sqrt{p-2q}}$ and $q\neq 0$, then prove that $qx^2-px+q=0$.

EXERCISE 2

MCQ Multiple Choice Questions

DIRECTIONS: *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. The value of x, when $2^{x+4}.3^{x+1} = 288$.

(a) 1 (b) -1
(c) 0 (d) None

2. When simplified the product $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)........\left(1+\frac{1}{n}\right)$ becomes

(a) n (b) $\frac{n-1}{2}$
(c) $\frac{n+1}{2}$ (d) $\frac{n}{2}$

3. The value of 0.423 is

(a) $\frac{423}{1000}$ (b) $\frac{423}{100}$
(c) $\frac{423}{990}$ (d) $\frac{419}{990}$

4. If $a = 2+\sqrt{3}$ and $b = 2-\sqrt{3}$ then $\frac{1}{a^2}-\frac{1}{b^2}$ is equal to

(a) 14 (b) -14
(c) $8\sqrt{3}$ (d) $-8\sqrt{3}$

5. Value of x satisfying $\sqrt{x+3}+\sqrt{x-2} = 5$, is

(a) 6 (b) 7
(c) 8 (d) 9

6. Rationalizing factor of $\sqrt{162}$ is

(a) $\sqrt{3}$ (b) $\sqrt{2}$
(c) $\sqrt{5}$ (d) $\sqrt{8}$

7. Rationalizing factor of $(2+\sqrt{3})=$

(a) $2-\sqrt{3}$ (b) $\sqrt{3}$
(c) $2+\sqrt{3}$ (d) $3+\sqrt{3}$

8. Rationalizing factor of $1+\sqrt{2}+\sqrt{3}$

(a) $1+\sqrt{2}-\sqrt{3}$ (b) 2
(c) 4 (d) $1+\sqrt{2}+\sqrt{3}$

9. Evaluate $\sqrt[3]{\left(\frac{1}{64}\right)^2}$

(a) 4 (b) 16
(c) $\frac{1}{4}$ (d) $\frac{1}{6}$

10. $\frac{2^{n+2}-2(2^n)}{2^{(2n+2)}}$ when simplified is

(a) $1-2(2^n)$ (b) $2^{n+3}-\frac{1}{4}$
(c) $\frac{1}{2^{n+1}}$ (d) $\frac{1}{2^{n-1}}$

11. Which of the following statement is not true?

(a) Between two integers, there exist infinite number of rational numbers
(b) Between two rational numbers, there exist infinite number of integers
(c) Between two rational numbers, there exist infinite number of rational numbers
(d) Between two real numbers, there exists infinite number of real numbers

MTOC More than One Correct

DIRECTIONS: *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

1. Which of the following is irrational?

(a) $\sqrt{17}$ (b) $\frac{\sqrt{12}}{\sqrt{3}}$
(c) $\sqrt{7}$ (d) $\sqrt{81}$

2. Value of $\sqrt[4]{(81)^{-2}}$ is

(a) $\frac{1}{9}$ (b) $\frac{1}{3}$
(c) 9 (d) 3

3. Which of the following is equal to x ?

(a) $x^{\frac{12}{7}} - x^{\frac{5}{7}}$ (b) $\sqrt[12]{(x^4)^{\frac{1}{3}}}$
(c) $(\sqrt{x^3})^{\frac{2}{3}}$ (d) $x^{12/19} \times x^{7/19}$

4. Decimal representation of a rational number can not be

(a) terminating
(b) non-terminating
(c) non-terminating repeating
(d) non-terminating non-repeating

5. Which of the following is/are not correct ?
   (a) Every whole number is a natural number.
   (b) Every integer is a rational number
   (c) Every rational number is an integer
   (d) Every rational number is a whole number
6. Which of the following is/are correct ?
   (a) There are infinitely many rational numbers between any two given rational numbers.
   (b) Every point on the number line represents a unique real number.
   (c) The decimal expansion of an irrational number is non-terminating non-recursing.
   (d) A number whose decimal expansion is non-terminating non-recurring is rational.

A&R Assertion & Reason

DIRECTIONS: *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.
(b) If both **Assertion** and **Reason** are correct, but Reason **is not the correct explanation** of Assertion.
(c) If **Assertion** is **correct** but **Reason** is **incorrect**.
(d) If **Assertion** is **incorrect** but **Reason** is **correct**.

1. **Assertion :** Every integer is a rational number
   Reason .: Every integer 'm' can be expressed in the form $\frac{m}{1}$.
2. **Assertion :** $\sqrt{2}$ is an irrational number.
   Reason : A number is called irrational, if it cannot be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
3. **Assertion :** $17^2 . 17^5 = 17^3$
   Reason : If $a > 0$ be a real number and p and q be rational numbers. Then $a^p . a^q = a^{p+q}$.
4. **Assertion :** $2+\sqrt{6}$ is an irrational number.
   Reason : Sum of a rational number and an irrational number is always an irrational number.
5. **Assertion :** A rational number between $\frac{1}{3}$ and $\frac{1}{2}$ is $\frac{5}{12}$.
   Reason : Rational number between two numbers a and b is $\sqrt{ab}$.

MMQ Multiple Matching Question

DIRECTIONS : *Following question has four statements (A. B. C and D) given in Column I and four statements (p. q. r. s) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in column I with entries in column II.*

1.

	Column I		Column II
(A)	$\frac{1}{2+\sqrt{3}}$	(p)	Irrational
(B)	$64^{1/2}$	(q)	$\sqrt{56}$
(C)	$16^{3/4}$	(r)	8
(D)	$7^{1/2}\, 8^{1/2}$	(s)	$2-\sqrt{3}$

HOTS Hots Subjective Questions

DIRECTIONS: *Answer the following questions.*

1 Prove that between two distinct rational numbers a and b, there exists another rational number.
2. Locate $\sqrt{2}$ on the number line
3. Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.
4. Express 2.5434343... in the form p/q where p and q are integers and $q \neq 0$
5. If $x = \frac{1}{7+4\sqrt{3}}, y = \frac{1}{7-4\sqrt{3}}$, find the value of $5x^2 - 7xy - 5y^2$.
6. Find the square root of $10+\sqrt{24}+\sqrt{60}+\sqrt{40}$.

SOLUTIONS

EXERCISE – I

FILL IN THE BLANKS

1. irrational 2. rational 3. irrational
4. conjugate 5. 9 6. $\sqrt{\frac{6}{35}}$
7. 2 8. rationalizing factor
9. $\frac{\sqrt{21}}{9}$ 10. 1/5

TRUE/FALSE

1. False 2. False 3. True 4. False
5. False 6. False 7. False 8. True
9. True 10. False

MATCH THE COLUMNS

1. (A) → r; (B) → s; (C) → p; (D) → q;
(A) $12 = 3 \times 4$ is a composite no.
(B) a.c.d (2,7) = 1
(C) $\sqrt{2}$ is not a rational no.

VERY SHORT ANSWER QUESTIONS

1. 0 2. –1 3. –1/5
4. 5/3 5. a
6. (i) $-1.\overline{3}$ (ii) $0.91\overline{6}$ (iii) $0.5\overline{38461}$
7. (i) rational (ii) rational
8. No, for example $\sqrt{4} = 2$ is a rational number.
9. $2 - \sqrt{3}$
10. (i) 3^{-21} (ii) $56^{1/2}$
11. (i) $\frac{3}{2^4}$ (ii) $\frac{125}{216}$
(iii) $\frac{729}{125}$ (iv) $\frac{30375}{16}$
(v) 2^4
12. Which whole number is not a natural number ?
12. 0
13. Find two irrational numbers between 0.1 and 0.2.
13. 0.10101001000100001 and 0.11001001000100001
14. (i) terminating (ii) repeating
(iii) repeating (iv) terminating
15. 37/1100 16. 0.25 and 0.2525
17. $\sqrt[3]{4}$ is the greatest 18. $\frac{7}{24}, \frac{13}{48}, \frac{15}{48}, \frac{31}{96}$

SHORT ANSWER QUESTIONS

1. $p = -11/2$, $q = -5/2$
2. 5
3. (i) irrational (ii) rational
4. At least one of the two given real numbers will be zero, the other may be rational or irrational.
5. $$\frac{6}{3\sqrt{2}-2\sqrt{3}} = \frac{6}{3\sqrt{2}-2\sqrt{3}} \times \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$$
$$= \frac{6(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2})^2-(2\sqrt{3})^2} = \frac{6(3\sqrt{2}+2\sqrt{3})}{18-12}$$
$$= \frac{6(3\sqrt{2}+2\sqrt{3})}{6} = 3\sqrt{2}+2\sqrt{3}$$
Therefore, $3\sqrt{2}+2\sqrt{3} = 3\sqrt{2} - a\sqrt{3}$
$\Rightarrow a = -2$
6. $$\left[5\left(8^{\frac{1}{3}}+27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}} = \left[5\left((2^3)^{\frac{1}{3}}+(3^3)^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}}$$
$$= \left[5(2+3)^3\right]^{\frac{1}{4}}$$
$$= \left[5(5)^3\right]^{\frac{1}{4}}$$
$$= \left[5^4\right]^{\frac{1}{4}} = 5$$
7. On Rationalizing the denominator, we get
$$\frac{(2+\sqrt{2})(3-\sqrt{5})}{(3+\sqrt{5})(2-\sqrt{2})} = \frac{(2+\sqrt{2})(3-\sqrt{5})}{(3+\sqrt{5})(2-\sqrt{2})} \times \frac{(3-\sqrt{5})(2+\sqrt{2})}{(3-\sqrt{5})(2+\sqrt{2})}$$
$$= \frac{(2+\sqrt{2})^2(3-\sqrt{5})^2}{(3^2-(\sqrt{5})^2)(2^2-(\sqrt{2})^2)} = \frac{(4+2+4\sqrt{2})(9+5-6\sqrt{5})}{(9-5)(4-2)}$$
$$= \frac{(6+4\sqrt{2})(14-6\sqrt{5})}{4\times 2} = \frac{84-36\sqrt{5}+56\sqrt{2}-24\sqrt{10}}{8}$$
Hence irrational.
8. $\sqrt{75}+\frac{1}{2}\sqrt{48}-\sqrt{192} = 5\sqrt{3}+\frac{4}{2}\sqrt{3}-8\sqrt{3} = \sqrt{3}(5+2-8)$
$= -1.732$

9. $3\sqrt{2}+\sqrt[4]{16\times 4}+\sqrt[4]{625\times 4}+\sqrt[6]{2^3}$

$=\sqrt{2}+\sqrt[4]{2^4\times 2^2}+\sqrt[4]{5^4\times 2^2}+\sqrt[6]{2^3}$

$=3\sqrt{2}+2\sqrt[4]{2^2}+5\sqrt[4]{2^2}+\sqrt[6]{2^3}$

$=3\sqrt{2}+2\sqrt{2}+5\sqrt{2}+\sqrt{2}$

$=(3+2+5+1)\sqrt{2}=11\sqrt{2}$

10. $\dfrac{\sqrt{x^2-y^2}+x}{\sqrt{x^2+y^2}+y}\times\dfrac{x-\sqrt{x^2-y^2}}{\sqrt{x^2+y^2}-y}=\dfrac{x^2-(\sqrt{x^2-y^2})^2}{(\sqrt{x^2+y^2})^2-y^2}$

$=\dfrac{x^2-(x^2-y^2)}{(x^2+y^2)-y^2}=\dfrac{y^2}{x^2}$

11. $x=9-4\sqrt{5}$ then

$\dfrac{1}{x}=\dfrac{9+4\sqrt{5}}{(9-4\sqrt{5})(9+4\sqrt{5})}=\dfrac{9+4\sqrt{5}}{81-80}$

$\dfrac{1}{x}=9+4\sqrt{5}$

$\sqrt{x}-\dfrac{1}{\sqrt{x}}=\sqrt{9-4\sqrt{5}}-\sqrt{9+4\sqrt{5}}$

$=\sqrt{9-4\sqrt{5}}-\sqrt{4+5+2\times 2\sqrt{5}}$

$=\sqrt{(\sqrt{5}-2)^2}-\sqrt{(\sqrt{5}+2)^2}$

$=(\sqrt{5}-2)-(\sqrt{5}+2)=-4$

12. $x^2=\dfrac{\sqrt{5}+2+\sqrt{5}-2+2\sqrt{(\sqrt{5})^2-2^2}}{\sqrt{5}+1}$

$=\dfrac{2\sqrt{5}+2}{\sqrt{5}+1}=\dfrac{2(\sqrt{5}+1)}{\sqrt{5}+1}=2$

LONG ANSWER QUESTIONS

1. $0.6+0.\overline{7}+0.4\overline{7}$

Let $x=0.\overline{7}$; $y=0.4\overline{7}$

$\Rightarrow 9x=7.\overline{7}$; $10y=4.\overline{7}$

$\Rightarrow 9x=7$; $100y=47.\overline{7}$

$\Rightarrow x=\dfrac{7}{9}$; $90y=43\Rightarrow y=\dfrac{43}{90}$

then $\dfrac{6}{10}+\dfrac{7}{9}+\dfrac{43}{90}$

2. 98

3. $\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}$

$=\sqrt{10}+\sqrt{4\times 5}+\sqrt{4\times 10}-\sqrt{5}-\sqrt{16\times 5}$

$=\sqrt{10}+2\sqrt{5}+2\sqrt{10}-\sqrt{5}-4\sqrt{5}$

$=3\sqrt{10}-3\sqrt{5}=3(\sqrt{10}-\sqrt{5})$

$\therefore$ Given expression

$=\dfrac{15}{3(\sqrt{10}-\sqrt{5})}=\dfrac{5}{\sqrt{10}-\sqrt{5}}=\dfrac{5}{\sqrt{10}-\sqrt{5}}\times\dfrac{\sqrt{10}+\sqrt{5}}{\sqrt{10}+\sqrt{5}}$

$=\dfrac{5(\sqrt{10}+\sqrt{5})}{10-5}=\sqrt{10}+\sqrt{5}$

4. Let $I=\dfrac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}-\dfrac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}-\dfrac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}=A-B-C$

Where $A=\dfrac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}\times\dfrac{\sqrt{10}-\sqrt{3}}{\sqrt{10}-\sqrt{3}}=\dfrac{7\sqrt{3}(\sqrt{10}-\sqrt{3})}{10-3}$

$=\dfrac{7\sqrt{30}-7\times 3}{7}=\dfrac{7(\sqrt{30}-3)}{7}=\sqrt{30}-3$

$B=\dfrac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}\times\dfrac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}}=\dfrac{2\sqrt{30}-2\times 5}{6-5}=2\sqrt{30}-10$

$C=\dfrac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}\times\dfrac{\sqrt{15}-3\sqrt{2}}{\sqrt{15}-3\sqrt{2}}=\dfrac{3\sqrt{30}-18}{15-18}=\dfrac{3\sqrt{30}-18}{-3}$

$=-\sqrt{30}+6$

Now $I=A-B-C$

$=(\sqrt{30}-3)-(2\sqrt{30}-10)-(-\sqrt{30}+6)$

$=\sqrt{30}-3-2\sqrt{30}+10+\sqrt{30}-6$

$=2\sqrt{30}-2\sqrt{30}-3+10-6=1$

5. L.H.S. $=\dfrac{\sqrt{7}-1}{\sqrt{7}+1}-\dfrac{\sqrt{7}+1}{\sqrt{7}-1}=\dfrac{(\sqrt{7}-1)^2-(\sqrt{7}+1)^2}{(\sqrt{7}+1)(\sqrt{7}-1)}$

$=\dfrac{(7+1-2\sqrt{7})-(7+1+2\sqrt{7})}{(\sqrt{7})^2-(1)^2}$

$=\dfrac{8-2\sqrt{7}-8-2\sqrt{7}}{7-1}$

$=-\dfrac{4\sqrt{7}}{6}=-\dfrac{2}{3}\sqrt{7}$

$\therefore -\dfrac{2}{3}\sqrt{7}=a+b\sqrt{7}$

$\Rightarrow a=0$ and $b=-\dfrac{2}{3}$

6. $\frac{4+\sqrt{5}}{4-\sqrt{5}}+\frac{4-\sqrt{5}}{4+\sqrt{5}}=\frac{(4+\sqrt{5})(4+\sqrt{5})}{(4-\sqrt{5})(4+\sqrt{5})}=\frac{(4-\sqrt{5})(4-\sqrt{5})}{(4+\sqrt{5})(4-\sqrt{5})}$

$= \frac{4^2+(\sqrt{5})^2+2\times4\sqrt{5}}{4^2-(\sqrt{5})^2}+\frac{4^2+(\sqrt{5})^2-2\times4\sqrt{5}}{4^2-(\sqrt{5})^2}$

$= \frac{16+5+8\sqrt{5}}{16-5}+\frac{16+5-8\sqrt{5}}{16-5}$

$= \frac{21+8\sqrt{5}}{11}+\frac{21-8\sqrt{5}}{11}$

$= \frac{21+8\sqrt{5}+21-8\sqrt{5}}{11}=\frac{42}{11}$

7. $\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

$= \frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$= \frac{\sqrt{5}-\sqrt{3}+\sqrt{2}\times\sqrt{5}-\sqrt{2}\times\sqrt{3}}{(\sqrt{5})^2-(\sqrt{3})^2}+\frac{\sqrt{5}+\sqrt{3}-\sqrt{2}\times\sqrt{5}-\sqrt{2}\times\sqrt{3}}{(\sqrt{5})^2-(\sqrt{3})^2}$

$= \frac{\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}}{2}+\frac{\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6}}{2}$

$= \frac{2\sqrt{5}-2\sqrt{6}}{2}=\sqrt{5}-\sqrt{6}=2.236-2.449=-0.213$

8. $\frac{x}{1}=\frac{\sqrt{p+2q}+\sqrt{p-2q}}{\sqrt{p+2q}-\sqrt{p-2q}}$

$\frac{x+1}{x-1}=\frac{2\sqrt{p+2q}}{2\sqrt{p-2q}}$ (By componendo and dividendo)

On squaring both sides, we get

$\frac{(x^2+1)+2x}{(x^2+1)-2x}=\frac{p+2q}{p-2q}$

On again applying componendo and dividendo.
we have,

$\frac{2(x^2+1)}{2.2x}=\frac{2.p}{2.2q}$

$qx^2+q=px$

Then, $qx^2-px+q=0$.

EXERCISE – 2

MULTIPLE CHOICE QUESTIONS

1. (a) $2^{x+4}.3^{x+1}=2^5.3^2 \Rightarrow x=1$
2. (c) 3. (a) 4. (d)
5. (a) $x=6$ satisfies the given equation.
6. (b) 7. (a) 8. (a)
9. (a) 10. (c) 11. (b)

MORE THAN ONE CORRECT

1. (a) and (c) 2. (a) and (d)
3. (c) and (d) 4. (a),(b) and (c)
5. (a),(c) and (d) 6. (a), (b) and (c)

ASSERTION AND REASON

1. (a) 2. (a)
3. (d) $17^2.17^5=17^{2+5}=17^7$
4. (a)
5. (c) $\frac{1}{2}\left(\frac{1}{3}+\frac{1}{2}\right)=\frac{5}{12}$

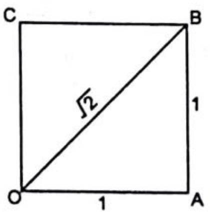
MULTIPLE MATCHING QUESTIONS

1. (A) $\to$ p, s; (B) $\to$ r; (C) $\to$ r; (D) $\to$ q

HOTS SUBJECTIVE QUESTIONS

1. Since $a \neq b$, therefore without any loss of generality, we may assume that $a < b$.
Now, $a < b$
By adding 'a' on both side, we get $a+a<b+a$
$\Rightarrow$ $2a < a+b$
$\Rightarrow$ $a < \frac{a+b}{2}$ ...(1)
Again from $a < b$
We have,
$a+b<b+b$ (By adding 'b' on both side)
$\Rightarrow$ $\frac{a+b}{2}<b$ ...(2)
Combining (1) and (2), we get $a<\frac{a+b}{2}<b$
Since, a, b and 2 ($\neq 0$) are rational numbers.
$\therefore$ $\frac{a+b}{2}$ is also a rational number.
Thus, there exists another rational number between two distinct rational numbers a and b.

2. Consider a unit square $OABC$ (a square with each side 1 unit length).



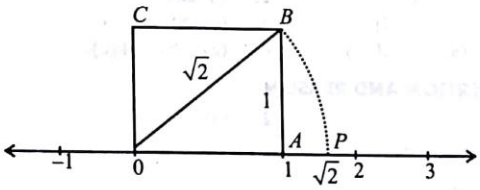
Now using Pythagoras Theorem, we have

$OB^2 = OA^2+AB^2$

$\Rightarrow$ $OB^2 = (1)^2+(1)^2$

$\Rightarrow$ $OB = \sqrt{2}$

Now we plot $\sqrt{2}$ on the number line.



When O corresponds to zero and A corresponds to 1 then clearly as above $OB = \sqrt{2}$.

Now using a compass with as O centre and radius OB, we get the point P on the number line.

P is the required point.

3. Consider the rational number $\frac{5}{7}$. On dividing by 7, we get,

$$
\begin{array}{l}
7\,)\,5.000000\,(0.714285........\\
\underline{49}\\
\;10\\
\;\;\underline{7}\\
\;\;30\\
\;\;\underline{28}\\
\;\;\;20\\
\;\;\;\underline{14}\\
\;\;\;\;60\\
\;\;\;\;\underline{56}\\
\;\;\;\;\;40\\
\;\;\;\;\;\underline{35}\\
\;\;\;\;\;\;5
\end{array}
$$

Thus, $\frac{5}{7} = 0.714285 \ldots\ldots = 0.\overline{714285}$

Now, consider $\frac{9}{11}$.

On dividing by 11, we get

$$
\begin{array}{l}
11)\,9.0000\,(0.8181......\\
\;\;\underline{88}\\
\;\;\;20\\
\;\;\;\underline{11}\\
\;\;\;\;90\\
\;\;\;\;\underline{88}\\
\;\;\;\;\;20\\
\;\;\;\;\;\underline{11}\\
\;\;\;\;\;\;9
\end{array}
$$

Thus, $\frac{9}{11} = 0.8181 \ldots\ldots = 0.\overline{81}$

Three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$ can be

0.75 075007500075000075......,

0.7670767000767........... and

0.808008000800008..........

4. Let $x = 2.5434343$

$\Rightarrow x = 2.5\overline{43}$

Multiplying both sides by 10 we get

$10x = 25.\overline{43}$ ... (i)

Again multiplying equation (i) by 100

$1000x = 2543.\overline{43}$ on subtracting

$1000x - 10x = 2543.\overline{43} - 25.\overline{43}$

$990x = 2518$

$x = \frac{2518}{990}$

$x = \frac{1259}{495}$

Hence, $2543.\overline{43} - 25.\overline{43} = \frac{1259}{495}$

5. $-7\left[1 + 80\sqrt{3}\right]$

6. Let the given expression be equal to $a + \sqrt{x} + \sqrt{y} + \sqrt{z}$

$a = 10$, $b = 24$, $c = 60$ and $d = 40$

$$x = \frac{1}{2}\sqrt{\frac{bd}{c}} = \frac{1}{2}\sqrt{\frac{24 \times 40}{60}} = 2$$

$$y = \frac{1}{2}\sqrt{\frac{bc}{d}} = \frac{1}{2}\sqrt{\frac{24 \times 60}{40}} = 3$$

$$z = \frac{1}{2}\sqrt{\frac{cd}{b}} = \frac{1}{2}\sqrt{\frac{60 \times 40}{24}} = 5$$

$\therefore \quad \sqrt{x} + \sqrt{y} + \sqrt{z} = \sqrt{2} + \sqrt{3} + \sqrt{5}$

$$
\begin{array}{r|l|l}
2x-1 & 2x^3-3x^2+7x-3 & x^2-x+3 \\
 & 2x^3-x^2 & \\
 & -\quad + & \\
\hline
 & -2x^2+7x-3 & \\
 & -2x^2+x & \\
 & +\quad - & \\
\hline
 & 6x-3 & \\
 & 6x-3 & \\
 & -\quad + & \\
\hline
 & 0 &
\end{array}
$$

2

CHAPTER

Polynomials

INTRODUCTION

In earlier classes, you have studied constants, variables, algebraic expressions, terms and some basic algebraic operations (addition, subtraction and multiplication) of polynomials. You have also studied to factorise simple algebraic expressions and use of simple algebraic identities in the factorization of algebraic expressions.

In this chapter you will study the polynomial which is a particular type of algebraic expression and various terminology related to it. You will also study the Division of Polynomials, Remainder Theorem, Factor Theorem, some more Algebraic Identities and their uses.

POLYNOMIAL

An algebraic expression in which the variable involved has only non-negative integral power, when the variable in any term is in numerator, is called a polynomial. For example: $3x^4 - 5x^2 + x - 10$, $5x + 3$, $x^2 + 2x - 5$, $x^3 - \frac{5}{2}x + \sqrt{7}$, $2\sqrt{3}x^4 - \frac{5}{6}x$, etc.

But $3x^3 + 2\sqrt{x} - 7$, $5x^4 - \frac{2}{x^2} + 2\sqrt{3}$, $7x^3 - 5x^{\frac{3}{2}} + 3$ and $x^5 + 2x^{-4} + x - 1$ are not the polynomials, because

(a) in $3x^3 + 2\sqrt{x} - 7$, $2\sqrt{x}$ is a term. $2\sqrt{x}$ means $2x^{\frac{1}{2}}$, hence the power of x is not a non-negative integer.

(b) in $5x^4 - \frac{2}{x^2} + 2\sqrt{3}$, $-\frac{2}{x^2}$ is a term. $-\frac{2}{x^2}$ means $-2x^{-2}$, hence the power of x is not a non-negative integer.

(c) in $7x^3 - 5x^{\frac{3}{2}} + 3$, $-5x^{\frac{3}{2}}$ is a term. The power of x is $\frac{3}{2}$, which is not a non-negative integer.

(d) in $x^5 + 2x^{-4} + x - 1$, $2x^{-4}$ is a term. The power of x is -4, which is not a non-negative integer.

The standard form of the polynomial in any variable x is $a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots a_3x^3 + a_2x^2 + a_1x + a_0$. Here $a_0, a_1, a_2, \dots a_n$ are real numbers and all the index (or power) of x are non-negative integers $a_n, a_{n-1}, a_{n-2}, \dots, a_3, a_2, a_1$ are called coefficients of $x^n, x^{n-1}, x^{n-2}, \dots, x^3, x^2$ and x. a_0 is called constant term. In the standard form of the polynomial in any one variable, terms are written in decreasing order of indices of x. The highest index n if $a_n \neq 0$, is called the degree of the polynomial. In particular, if $a_0 = a_1 = a_2 = a_3 = \dots, = a_n = 0$ (all the constants). We get the zero polynomial, which is denoted by O. The degree of zero polynomial is not defined. A polynomial may has one or more terms. If a polynomial has only one term, which is a non-zero constant, then the degree of this non-zero constant polynomial is zero. For example 5, $\sqrt{3}$, $-\frac{7}{2}$ are constant polynomial of degree zero, because 5, $\sqrt{3}$ and $-\frac{7}{2}$ can be written as $5x^\circ$, $\sqrt{3}\,x^\circ$ and $-\frac{7}{2}x^\circ$ respectively.

If a polynomial involves two or more variables, then in standard form terms are written in decreasing order of indices of the variable which is comes earlier in alphabatic order. To find the degree of such type of polynomial, sum of the power of all the variables in each term is taken up. The highest sum so obtained is the degree of the polynomial. For example $5x^4y^2 - 7x^3y^2 + \sqrt{5}y^4$ and $7x^5y - 3\sqrt{2}xy^6 + 8$ are polynomials in two variables x and y in standard form. The degree of the first polynomial is 6 and the degree of the second polynomial is 7.

Note : Variables are generally represented by small English alphabets which comes generally after the middle alphabet in the alphabetic order.

If there are two or more small English alphabets are involved in a polynomial, out of which some alphabet(s) come(s) earlier and other(s) come(s) after long gap of in alphabetic order, then the earlier alphabet(s) repersent(s) the constant and the other(s) repersent(s) the variable(s). Generally all capital English Alphabets repersent the constants.

A variable raised to any non-zero real number is also a variable. For example $x^{\frac{7}{3}}$, y^5, $x^{4.5}$, etc.

A number which is the product of a constant and a variable is also a variable. For example $\sqrt{5}x^3$, $-4x^{\frac{4}{5}}$, $2x^7$ etc.

A combination of two or more variables separated by a (+ or –) sign is also a variable. For example $x^3 - y^2$, $x + 2xy$, $\sqrt{5}x^2 + y$, etc.

TYPES OF POLYNOMIALS ON THE BASIS OF THEIR DEGREE

(I) LINEAR POLYNOMIAL

A polynomial of degree one is called a linear polynomial. The general form of a linear polynomial is $ax + b$, where a and b are any real constants and $a \neq 0$.

Example: $3x + 5$ is a linear polynomial.

(II) QUADRATIC POLYNOMIAL

A polynomial of degree two is called a quadratic polynomial. The general form of quadratic polynomial is $ax^2 + bx + c$, where $a \neq 0$.

Example: $2y^2 + 3y - 1$ is a quadratic polynomial.

(III) CUBIC POLYNOMIAL

A polynomial of degree three is called a cubic polynomial. The general form of a cubic polynomial is $ax^3 + bx^2 + cx + d$, where $a \neq 0$.

Example: $6x^3 - 5x^2 + 2x + 1$ is a cubic polynomial.

(IV) BIQUADRATIC POLYNOMIAL

A polynomial of degree four is called a biquadratic polynomial. The general form of a biquadratic polynomial is $ax^4 + bx^3 + cx^2 + dx + e$ where $a \neq 0$.

Example: $x^4 - 2x^2 + \sqrt{3}x + 2$ is a biquadratic polynomial.

A polynomial of degree five or more than five does not have any particular name. Such a polynomial usually called a polynomial of degree five or six or .... etc.

TYPES OF POLYNOMIALS ON THE BASIS OF NUMBER OF TERMS OF THE POLYNOMIAL

On the basis of the number of terms of the polynomial, the polynomial is named as follows:

(A) MONOMIAL

A polynomial is said to be a monomial if it has only one term.

For Example: $-3x^2, 5x^3, 10x$ are monomials.

(B) BINOMIAL

A polynomial is said to be a binomial if it contains two terms.

For Example: $2x^2 + 5, 3x^3 - 7, 6x^2 + 8x$ are binomials.

(C) TRINOMIALS

A polynomial is said to be a trinomial if it contains three terms.

For Example: $3x^3 - 8x + \frac{5}{2}$, $\sqrt{7}x^{10} + 8x^4 - 3x^2$, $5 - 7x + 8x^9$, etc, are trinomials.

There is no specific name of the polynomial which has more than 3 terms

Illustration 1 : Which of the following expressions is/are polynomial(s)?

(a) $5x^2 - 3x + 9$ (b) $\frac{4}{3}x^7 - 5x^4 + 3x^2 - 1$ (c) $\frac{5}{4}x^{-3} + 2x - 1$ (d) $\sqrt{2}x^4 + x^{\frac{3}{4}} - 7$

SOLUTION

(*a*) and (*b*) are polynomials.

Illustration 2 : Write down the degree of the following polynomials.

(a) $3x + 5$ (b) $3t^2 - 5t + 9t^4$ (c) $2 - y^2 - y^3 + 2y^8$

SOLUTION

(a) The highest power term is $3x$ and its exponent is 1.
$\therefore$ degree of $3x + 5 = 1$

(b) The highest power term is $9t^4$ and exponent of t is 4. So, degree of the given polynomial is 4.

(c) The highest power of the variable is 8. So, the degree of the polynomial is 8.

Illustration 3 : Write the following polynomials in standard form:

(a) $x^6 - 3x^4 + \sqrt{2}x + \frac{5}{2}x^2 + 7x^5 + 4$ (b) $5x^4 - 2x + 5 + \sqrt{3}x$

SOLUTION

The given polynomials in standard form are:

(a) $x^6+7x^5-3x^4+\frac{5}{2}x^2+\sqrt{2}x+4$

(b) $5x^4+(\sqrt{3}-2)\,x+5$

VALUE OF A POLYNOMIAL

A polynomial in any variable x is symbolically may be repersented by $P(x), f(x), \ldots$, (etc.)

The value of a polynomial $P(x)$ at $x=\alpha$ is obtained by substituting $x=\alpha$ in the given polynomial and is denoted by $P(\alpha)$.

Consider the polynomial $P(x)=5x^3-2x^2+3x-2$

If we replace x by 1 everywhere in $P(x)$, we get

$$P(1)=5\times(1)^3-2\times(1)^2+3\times(1)-2$$
$$=5-2+3-2=-4$$

So, we say that the value of $P(x)$ at $x=1$ is 4.

ZEROE(S) OR ROOT(S) OF A POLYNOMIAL

If value of a polynomial $P(x)$ at $x=\alpha$ is 0 i.e., $P(\alpha)=0$, then α is called the zero of the polynomial $P(x)$

Let $f(x)=2x^2-x-1$ is a polynomial

Now, $f(1)=2\times(1)^2-(1)-1=0$

Hence, $x=1$ is a zero or Root of the polynomial $f(x)$.

(a) Every real number is a zero of the zero polynomial

(b) A non-zero constant polynomial has no zero.

(c) Every linear polynomial in one variable has a unique zero.

(d) Every quadratic polynomial in one variable has two zeroes.

(e) Every cubic polynomial in one variable has three zeroes.

Illustration 4 : If $x=\frac{4}{3}$ is a root of the polynomial $f(x)=6x^3-11x^2+kx-20$, then find the value of k.

SOLUTION

$f(x)=6x^3-11x^2+kx-20$

$$f\left(\frac{4}{3}\right)=6\left(\frac{4}{3}\right)^3-11\left(\frac{4}{3}\right)^2+k\left(\frac{4}{3}\right)-20=0$$

$\Rightarrow\ 6\left(\frac{64}{27}\right)-11\left(\frac{16}{9}\right)+\frac{4k}{3}-20=0$

$\Rightarrow\ 128-176+12k-180=0$

$\Rightarrow\ 12k+128-356=0$

$\Rightarrow\ 12k=228 \quad\Rightarrow\quad k=19$

Illustration 5 : If $x=2$ and $x=0$ are roots of the polynomials $f(x)=2x^3-5x^2+ax+b$. Find the values of a and b.

SOLUTION

$f(2)=2(2)^3-5(2)^2+a(2)+b=0$

$\Rightarrow\ 16-20+2a+b=0$

$\Rightarrow\ 2a+b=4$

$f(0)=2(0)^3-5(0)^2+a(0)+b=0$

$\Rightarrow\ b=0 \quad\Rightarrow\quad 2a=4 \quad\Rightarrow\quad a=2$

$\therefore\ a=2,\ b=0$

TO FIND THE ZERO(ES) OF A POLYNOMIAL

To find the zero(es) of any polynomial (except zero polynomial and constant polynomial) put the polynomial equal to zero and solve them.

FACTORIZATION OF A QUADRATIC POLYNOMIAL BY SPLITTING THE MIDDLE TERM

To Factorize Quadratic Polynomial: $ax^2 + bx + c$

(i) Take the product of the constant term and the coefficient of x^2 i.e., ac.

(ii) Now this product ac is to split into two factors m and n such that $m + n$ is equal to the coefficient of x i.e., b.

(iii) Then we pair one of them, say mx, with x^2 and the other nx, with c and factorize.

Illustration 6 : Factorize $2x^2 + 5x - 3$ by splitting the middle term.

SOLUTION

Here, $2 \times (-3) = -6 = 6 \times (-1)$

and $6 + (-1) = 5$

$$\therefore \quad 2x^2 + 5x - 3 = 2x^2 + 6x - x - 3$$
$$= 2x(x + 3) - 1(x + 3)$$
$$= (x + 3)(2x - 1)$$

REMAINDER THEOREM

Let $p(x)$ be any polynomial of degree greater than or equal to one and let a be any real number. If $p(x)$ is divided by the linear polynomial $x - a$, then the remainder is $p(a)$.

DEDUCTION

If a polynomial $p(x)$ is divided by $(x + a)$, $(ax - b)$, $(ax + b)$, $(b - ax)$ then the remainder is the value of $p(x)$ at

$x = -a, \frac{b}{a}, -\frac{b}{a}, \frac{b}{a}$ i.e., $p\left(\frac{b}{a}\right), p\left(-\frac{b}{a}\right), p\left(\frac{b}{a}\right)$ are remainders respectively

Illustration 7 : Find the values of a and b so that the polynomial $x^3 + 10x^2 + ax + b$ is exactly divisible by $(x - 1)$ as well as $(x + 2)$.

SOLUTION

Let $p(x) = x^3 + 10x^2 + ax + b$

Now, $(x - 1)$ is a factor of $p(x) \Rightarrow p(1) = 0$

And $(x + 2)$ is a factor of $p(x) \Rightarrow p(-2) = 0$

Now, $p(1) = 1^3 + 10 \times 1^2 + a \times 1 + b = 11 + a + b$

And $p(-2) = (-2)^3 + 10 \times (-2)^2 + a(-2) + b = 32 - 2a + b$

So, we must have

$$11 + a + b = 0 \quad \ldots (1)$$

and

$$32 - 2a + b = 0 \quad \ldots (2)$$

Subtracting (2) from (1), we get $3a - 21 = 0$ or $a = 7$

Putting $a = 7$ in (1), we get $b = -18$

$\therefore \quad a = 7$ and $b = -18$

FACTOR THEOREM

If $P(x)$ be a polynomial of degree greater than one and $f(a) = 0$, then $(x - a)$ is a factor of $f(x)$.

Also if $f\left(\frac{b}{a}\right) = 0$, then $\left(x - \frac{b}{a}\right)$ i.e. $\frac{1}{a}(ax - b)$ is a factor of $f(x)$.

Here a and b are constants.

USE OF FACTOR THEOREM IN FINDING A LINEAR FACTOR OF A POLYNOMIAL F(X) WHOSE ONE OF THE TERMS IS CONSTANT

Let $f(x)$ be the given polynomial whose one of the term is constant term

(i) Let the constant term in $f(x)$ be 3 or -3.

Then, its factors are $1, -1, 3$ and -3.

See which one of these factor is zero of the polynomial $f(x)$.

If $f(1) = 0$, then $(x-1)$ is a factor of $f(x)$

If $f(-1) = 0$, then $(x+1)$ is a factor of $f(x)$ and so on.

(ii) Let the constant term in $f(x)$ be 4 or -4

Then, its factors are $1, -1, 2, -2, 4$ and -4

Find $f(1), f(-1), f(2), f(-2), f(4)$ and $f(-4)$

See which one of the factor is zero and find the factor of $f(x)$ as mentioned above.

Similarly, by taking the factors of the constant term in any given polynomial, we can find a linear factor of $f(x)$ by hit and trial method.

Illustration 8 : For what value of 'a' is $(x-2)$ a factor of $f(x) = x^2 - ax + 6$.

SOLUTION

If $(x-2)$ is factor of $f(x)$, then $f(2) = 0$ [By factor theorem]

$\therefore \quad f(2) = (2)^2 - a(2) + 6 = 0$

$\therefore \quad 4 - 2a + 6 = 0$

$\therefore \quad a = 5$

Ilustration 9 : Use the factor theorem to factorize $x^3 + x^2 - 4x - 4$ completely

SOLUTION

Let $f(x) = x^3 + x^2 - 4x - 4$

The constant term in $f(x)$ is -4

Its factors are $1, -1, 2, -2, 4$ and -4

Now, $f(2) = 2^3 + 2^2 - 4 \times 2 - 4 = 0$

$\therefore \quad (x-2)$ is a factor of $f(x)$

On dividing $f(x)$ by $(x-2)$,

$$
\begin{array}{r|l|l}
x-2 & x^3 + x^2 - 4x - 4 & x^2 + 3x + 2 \\
 & x^3 - 2x^2 & \\
 & \underline{-\quad +\quad} & \\
 & 3x^2 - 4x - 4 & \\
 & 3x^2 - 6x & \\
 & \underline{-\quad +\quad} & \\
 & 2x - 4 & \\
 & 2x - 4 & \\
 & \underline{-\quad +} & \\
 & 0 &
\end{array}
$$

$\therefore \; f(x) = (x-2)(x^2 + 3x + 2)$

$= (x-2)[x^2 + x + 2x + 2]$

$= (x-2)[x(x+1) + 2(x+1)]$

$= (x-2)(x+2)(x+1)$

ALGEBRAIC IDENTITIES

Some important Algebraic Identities, which are used in factorization, simplification and to find the product of polynomials are given below:

(i) $(x+y)^2 = x^2 + 2xy + y^2$

(ii) $(x-y)^2 = x^2 - 2xy + y^2$

(iii) $x^2 - y^2 = (x+y)(x-y)$

(iv) $(x+a)(x+b) = x^2 + (a+b)x + ab$

(v) $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(vi) $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$
$= x^3 + y^3 + 3x^2y + 3xy^2$

(vii) $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$
$= x^3 - y^3 - 3x^2y + 3xy^2$

(viii) $x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$

Special Case

If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$

Note : In each of the identities, the right hand side expression is the expended form of the left hand side expression.

Illustration 10 : Use a suitable identity to factorise each of the following :

(a) $4x^2 + 4xy + y^2$ **(b)** $25p^2 - 36q^2$ **(c)** $x^2 - y^2 + 2x + 1$ **(d)** $4a^2 - 4b^2 + 4a + 1$

SOLUTION

(a) Consider $4x^2 + 4xy + y^2 = (2x)^2 + 2(2x)(y) + (y)^2$
$= (2x+y)^2 = (2x+y)(2x+y)$ (Using Identity (i))

(b) Consider $25p^2 - 36q^2 = (5p)^2 - (6q)^2$
$= (5p+6q)(5p-6q)$ (Using Identity (iii))

(c) $x^2 - y^2 + 2x + 1 = x^2 + 2x + 1 - y^2$
$= (x+1)^2 - (y)^2 = (x+1+y)(x+1-y)$ (Using Identities (i) and (iii))

(d) $4a^2 - 4b^2 + 4a + 1 = (4a^2 + 4a + 1) - 4b^2$
$= \{(2a)^2 + 2(2a)(1) + (1)^2\} - (2b)^2$
$= (2a+1)^2 - (2b)^2 = (2a+1+2b)(2a+1-2b)$. (Using Identities (i) and (iii))

Illustration 11 : Simplify : $(a+b)^3 + (a-b)^3 + 6a(a^2-b^2)$

SOLUTION

Consider $(a+b)^3 + (a-b)^3 + 6a(a^2-b^2)$
$= (a+b)^3 + (a-b)^3 + 3(2a)(a+b)(a-b)$. (Using Identity (iii))
$= (a+b)^3 + (a-b)^3 + 3(a+b)(a-b)\{(a+b)+(a-b)\}$
$= \{(a+b)+(a-b)\}^3$
$= (2a)^3 = 8a^3$ (Using Identity vi)

Illustration 12 : Factorise

(a) $\frac{4}{9}a^2 + b^2 + \frac{4}{3}ab$ **(b)** $a^2 - ab + \frac{1}{4}b^2$ **(c)** $p^3 - p^2q + \frac{1}{3}pq^2 - \frac{1}{27}q^3$

SOLUTION

(a) Consider $\frac{4}{9}a^2 + b^2 + \frac{4}{3}ab = \left(\frac{2}{3}a\right)^2 + (b)^2 + 2\left(\frac{2}{3}a\right)(b)$
$= \left(\frac{2}{3}a+b\right)^2 = \left(\frac{2}{3}a+b\right)\left(\frac{2}{3}a+b\right)$ (Using Identity (i))

(b) Consider $a^2 - ab + \frac{1}{4}b^2 = (a)^2 - 2(a)\left(\frac{1}{2}b\right) + \left(\frac{1}{2}b\right)^2$

$= \left(a - \frac{1}{2}b\right)^2 = \left(a - \frac{1}{2}b\right)\left(a - \frac{1}{2}b\right)$ (Using Identity (ii))

(c) Consider $p^3 - p^2q + \frac{1}{3}pq^2 - \frac{1}{27}q^3 = p^3 - \frac{1}{27}q^3 - p^2q + \frac{1}{3}pq^2$

$= (p)^3 - \left(\frac{1}{3}q\right)^3 - 3p\left(\frac{1}{3}q\right)\left(p - \frac{1}{3}q\right) = \left(p - \frac{1}{3}q\right)^3$ (Using Identity (vii))

$= \left(p - \frac{1}{3}q\right)\left(p - \frac{1}{3}q\right)\left(p - \frac{1}{3}q\right)$

Illustration 13 : Factorise : $(a + b - c)^3 + (a - b + c)^3 - 8a^3$

SOLUTION

$(a + b - c)^3 + (a - b + c)^3 - 8a^3 = (a + b - c)^3 + (a - b + c)^3 + (-2a)^3$

$= 3(a + b - c)(a - b + c)(-2a)$ ($\because$ $(a + b - c) + (a - b + c) + (-2a) = 0$)

$= -6a(a + b - c)(a - b + c)$

Illustration 14 : Evaluate each of the following using suitable identities :

(i) **$(104)^3$** ***(ii)*** **$(999)^3$**

SOLUTION

(i) We have

$(104)^3 = (100 + 4)^3 = (100)^3 + (4)^3 + 3(100)(4)(100 + 4)$ (Using Identity (vi))

$= 1000000 + 64 + 124800 = 1124864$

(ii) We have

$(999)^3 = (1000 - 1)^3 = (1000)^3 - (1)^3 - 3(1000)(1)(1000 - 1)$ (Using Identity (vii))

$= 1000000000 - 1 - 2997000 = 997002999$

DIVISION OF A POLYNOMIAL BY A POLYNOMIAL

To divide a polynomial by other polynomial, the degree of the dividend will be either equal or greater than the degree of the divisor.

(A) DIVISION OF A POLYNOMIAL BY A NON-ZERO, NON-CONSTANT MONOMIAL

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

Illustration 15 : Divide $12x^4 - 15x^3 + 6\sqrt{3x}$ by $3x$

SOLUTION

$$\frac{12x^4 - 15x^3 + 6\sqrt{3}x}{3x}$$

$$= \frac{12x^4}{3x} - \frac{15x^3}{3x} + \frac{6\sqrt{3}x}{3x}$$

$$= 4x^3 - 5x^2 + 2\sqrt{3}$$

(B) DIVISION OF A POLYNOMIAL BY A POLYNOMIAL (NON-MONOMIAL)

(i) Factorization Method

To divide one polynomial by other polynomial (non-monomial), factorize the polynomial to be divided so that one of the factors is equal the polynomial by which we want to divide.

Illustration 16 : Divide $6x^2 + x - 12$ by $2x + 3$

SOLUTION

$$\frac{6x^2+x-12}{2x+3} = \frac{6x^2+9x-8x-12}{2x+3}$$

$$= \frac{3x(2x+3)-4(2x+3)}{2x+3}$$

$$= \frac{(2x+3)(3x-4)}{2x+3} = 3x-4$$

(ii) Long Division Method

(*a*) First arrange the terms of the dividend and the divisor in the descending order of their degrees.

(*b*) Now the first term of the quotient is obtained by dividing the first term of the dividend by the first term of the divisor.

(*c*) Then multiply all the terms of the divisor by the first term of the quotients and subtract the result from the dividend.

(*d*) Consider the remainder as new dividend and proceed as before.

(*e*) Repeat this process till we obtained the remainder which is either 0 or a polynomial of degree less than that of the divisor.

Illustration 17 : Divide $2x^3 - 3x^2 + 7x - 3$ by $2x - 1$

SOLUTION

$$\begin{array}{r|l|l}
2x-1 & 2x^3-3x^2+7x-3 & x^2-x+3 \\
 & \underline{\overset{-}{2x^3}\ \overset{+}{-x^2}} & \\
 & -2x^2+7x-3 & \\
 & \underline{\overset{+}{-2x^2}\ \overset{-}{+x}} & \\
 & 6x-3 & \\
 & \underline{\overset{-}{6x}\ \overset{+}{-3}} & \\
 & 0 &
\end{array}$$

$\therefore\ (2x^3 - 3x^2 + 7x - 3) \div (2x - 1) = x^2 - x + 3$

Quotient $= x^2 - x + 3$

Remainder $= 0$

Illustration 18 : Divide $3x^3 - 4x^2 - 7x - 5$ by $3x + 2$

SOLUTION

$$\begin{array}{r|l|l}
3x+2 & 3x^3-4x^2-7x-5 & x^2-2x-1 \\
 & \underline{\overset{-}{3x^3}\ \overset{-}{+2x^2}} & \\
 & -6x^2-7x-5 & \\
 & \underline{\overset{+}{-6x^2}\ \overset{+}{-4x}} & \\
 & -3x-5 & \\
 & \underline{\overset{+}{-3x}\ \overset{+}{-2}} & \\
 & -3 &
\end{array}$$

Quotient $= x^2 + 2x - \frac{11}{3}$

Remainder $= -3$

MISCELLANEOUS

Solved Examples

Example 1 : **Find α and β if $x + 1$ and $x + 2$ are factors of $p(x) = x^3 + 3x^2 - 2\alpha x + \beta$.**

SOLUTION : Put $x + 1 = 0$ or $x = -1$ and $x + 2 = 0$ or $x = -2$ in $p(x)$

Then, $p(-1) = (-1)^3 + 3(-1)^2 - 2\alpha(-1) + \beta = 0$

$\Rightarrow \quad -1 + 3 + 2\alpha + \beta = 0$

$\Rightarrow \quad \beta = -2\alpha - 2 \qquad \ldots (1)$

$p(-2) = (-2)^3 + 3(-2)^2 - 2\alpha(-2) + \beta = 0$

$\Rightarrow \quad -8 + 12 + 4\alpha + \beta = 0$

$\Rightarrow \quad \beta = -4\alpha - 4 \qquad \ldots (2)$

By equalising both of the above equation

$-2\alpha - 2 = -4\alpha - 4$

$\Rightarrow \quad 2\alpha = -2 \quad \Rightarrow \quad \alpha = -1$

$\alpha = -1$ put in eq. (1) $\Rightarrow \beta = -2(-1) - 2 = 2 - 2 = 0.$

Hence, $\alpha = -1, \beta = 0$

Example 2 : **What must be subtracted from $x^3 - 6x^2 - 15x + 80$ so that the result is exactly divisible by $x^2 + x - 12$.**

SOLUTION : In $p(x) = x^3 - 6x^2 - 15x + 80$ we subtracted $ax + b$ so that it is exactly divisible by $x^2 + x - 12$.

$\therefore \quad s(x) = x^3 - 6x^2 - 15x + 80 - (ax + b)$

$= x^3 - 6x^2 - (15 + a)x + (80 - b)$

Dividend = Divisior × quotient + remainder

But remainder will be zero.

$\therefore$ Dividend = Divisor × quotient

$\Rightarrow \quad s(x) = (x^2 + x - 12) \times$ quotient

$\Rightarrow \quad s(x) = x^3 - 6x^2 - (15 + a)x + (80 - b)$

$= x(x^2 + x - 12) - 7(x^2 + x - 12)$

$= x^3 + x^2 - 7x^2 - 12x - 7x + 84$

$= x^3 - 6x^2 - 19x + 84$

Hence, $x^3 - 6x^2 - 19x + 84 = x^3 - 6x^2 - (15 + a)x + 80 - b$

$\Rightarrow \quad -15 - a = -19$

$\Rightarrow \quad a = +4$

and $\quad 80 - b = 84 \qquad \Rightarrow b = -4$

Hence, if in $p(x)$ we subtract $4x - 4$ ($= ax + b$) then it is exactly divisible by $x^2 + x - 12$

Alternative Method

$$
\begin{array}{r|l|l}
x^2 + x - 12 & x^3 - 6x^2 - 15x + 80 & x - 7 \\
 & \underline{\underset{-}{x^3} + \underset{-}{x^2} \underset{+}{-} 12x} & \\
 & -7x^2 - 3x + 80 & \\
 & \underline{\underset{+}{-7x^2} \underset{+}{-} 7x \underset{-}{+} 84} & \\
 & \qquad 4x - 4 &
\end{array}
$$

So, if we subtract $4x - 4$ from dividend $(x^3 - 6x^2 - 15x + 80)$, then dividend is exactly divisibly by $x^2 + x - 12$.

Example 3 : **Find the value of $(99)^2$.**

SOLUTION : $(99)^2 = (100-1)^2$

$= 100^2 - 2(100)(1) + (1)^2 = 10000 - 200 + 1 = 9801$

Example 4 : **Factorize $6x^2 - 5xy - 4y^2 + x + 17y - 15$.**

SOLUTION : $6x^2 - 5xy - 4y^2 + x + 17y - 15$

$= 6x^2 + x[1-5y] - [4y^2 - 12y - 5y + 15]$

$= 6x^2 + x[1-5y] - [4y(y-3) - 5(y-3)]$

$= 6x^2 + x[1-5y] - (4y-5)(y-3)$

$= 6x^2 + 3(y-3)x - 2(4y-5)x - (4y-5)(y-3)$

$= 3x(2x+y-3) - (4y-5)(2x+y-3)$

$= (2x+y-3)(3x-4y+5)$

Example 5 : **Use the factor theorem to factorize $x^3 + x^2 - 4x - 4$ completely.**

SOLUTION : Let $f(x) = x^3 + x^2 - 4x - 4$

The constant term in $f(x)$ is -4

Its factors are $1, -1, 2, -2, 4$ and -4

Now, $f(2) = 2^3 + 2^2 - 4 \times 2 - 4 = 0$

$\therefore$ $(x-2)$ is a factor of $f(x) = x^3 + x^2 - 4x - 4$

On dividing $f(x)$ by $(x-2)$,

$$
\begin{array}{r|l|l}
x-2 & x^3 + x^2 - 4x - 4 & x^2 + 3x + 2 \\
 & \underline{x^3 - 2x^2} & \\
 & \quad {}_{-\ \ +} & \\
 & 3x^2 - 4x - 4 & \\
 & \underline{3x^2 - 6x} & \\
 & \quad {}_{-\ \ +} & \\
 & 2x - 4 & \\
 & \underline{2x - 4} & \\
 & \quad {}_{-\ \ +} & \\
 & 0 &
\end{array}
$$

$\therefore$ $f(x) = (x-2)(x^2+3x+2)$

$= (x-2)[x^2 + x + 2x + 2]$

$= (x-2)[x(x+1) + 2(x+1)]$

$= (x-2)(x+2)(x+1)$

Example 6 : **Solve : $0.645 \times 0.645 + 2 \times 0.645 \times 0.355 + 0.355 \times 0.355$**

SOLUTION : $0.645 \times 0.645 + 2 \times 0.645 \times 0.355 + 0.355 \times 0.355$

$= (0.645)^2 + 2 \times (0.645)(0.355) + (0.355)^2$

$= (0.645 + 0.355)^2 = (1)^2 = 1$

Example 7 : **Factorize :** $x^2 - x\left(\frac{a^2-1}{a}\right) - 1$.

SOLUTION : $x^2 - x\left(\frac{a^2-1}{a}\right) - 1 = x^2 - x\left(a - \frac{1}{a}\right) - 1$

$$= x^2 - ax + \frac{x}{a} - 1 = x(x-a) + \frac{1}{a}(x-a) = (x-a)\left(x + \frac{1}{a}\right)$$

Example 8 : **Solve :** $\frac{0.73 \times 0.73 - 0.27 \times 0.27}{0.73 - 0.27}$

SOLUTION : $\frac{0.73 \times 0.73 - 0.27 \times 0.27}{0.73 - 0.27} = \frac{(0.73)^2 - (0.27)^2}{0.73 - 0.27}$

$$= \frac{(0.73 + 0.27)(0.73 - 0.27)}{(0.73 - 0.27)} = 0.73 + 0.27 = 1$$

Example 9 : **Factorize :** $x^6 - 7x^3 - 8$

SOLUTION : $x^6 - 7x^3 - 8 = y^2 - 7y - 8$, where $x^3 = y$

$= y^2 - 8y + y - 8$

$= y(y-8) + 1(y-8)$

$= (y-8)(y+1)$

$= (x^3 - 8)(x^3 + 1)$ $\quad [\because \; y = x^3]$

$= (x^3 - 2^3)(x^3 + 1^3)$

$= (x-2)(x^2 + 2x + 4)(x+1)(x^2 - x + 1)$

Example 10 : **Using remainder theorem, find the remainder when $x^3 + x^2 - 2x + 1$ is divided by $x - 3$.**

SOLUTION : Let $p(x) = x^3 + x^2 - 2x + 1$

By remainder theorem, we know that $p(x)$ when divided by $(x - 3)$ gives a remainder equal to $p(3)$

Now. $p(3) = [3^3 + 3^2 - 2 \times 3 + 1] = (27 + 9 - 6 + 1) = 31$

Hence, the remainder is 31.

Example 11 : **Divide : $3x^4 + 2x^3 - 5x^2 + 3x + 1$ by $2x$**

SOLUTION :

$$
\begin{array}{r|l|l}
2x & 3x^4 + 2x^3 - 5x^2 + 3x + 1 & (3/2)x^3 + x^2 - (5/2)x + (3/2) \\
 & 3x^4 & \\
\hline
 & 2x^3 - 5x^2 + 3x + 1 & \\
 & 2x^3 & \\
\hline
 & -5x^2 + 3x + 1 & \\
 & -5x^2 & \\
\hline
 & 3x + 1 & \\
 & 3x & \\
\hline
 & 1 & \\
\hline
\end{array}
$$

$\therefore$ Quotient $= \left(\frac{3}{2}x^3 + x^2 - \frac{5}{2}x + \frac{3}{2}\right)$ and remainder $= 1$

Example 12 : Factorize $8x^3 + 27y^3 + 36x^2y + 54xy^2$

SOLUTION : The given expression can be written as

$(2x)^3 + (3y)^3 + 3(4x^2)(3y) + 3(2x)(9y)^2$

$= (2x)^3 + (3y)^3 + 3(2x)^2(3y) + 3(2x)(3y)^2$

$= (2x + 3y)^3$

$= (2x + 3y)(2x + 3y)(2x + 3y)$

Example 13 : Factorize : $x^3 - 23x^2 + 142x - 120$

SOLUTION : Let $p(x) = x^3 - 23x^2 + 142x - 120$

We shall now look for all the factors of -120.

Some of these are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 8, \pm 10, \pm 12, \pm 15, \pm 20, \pm 24, \pm 30, \pm 60$

By trial, we find that $p(1) = 0$. So $x - 1$ is a factor of $p(x)$.

Now we see that $x^3 - 23x^2 + 142x - 120 = x^3 - x^2 - 22x^2 + 22x + 120x - 120$

$= x^2(x - 1) - 22x(x - 1) + 120(x - 1)$

$= (x - 1)(x^2 - 22x + 120)$ [Taking $(x - 1)$ common]

We could have also got this by dividing $p(x)$ by $x - 1$

Now, $x^2 - 22x + 120$ can be factorized either by splitting the middle term or by using the factor theorem. By splitting the middle term, we have :

$x^2 - 22x + 120 = x^2 - 12x - 10x + 120$

$= x(x - 12) - 10(x - 12)$

$= (x - 12)(x - 10)$

So, $x^3 - 23x^2 + 142x - 120 = (x - 1)(x - 10)(x - 12)$

Example 14 : Find the value of p and q, if $(x + 3)$ and $(x - 4)$ are factors of $x^3 - px^2 - qx + 24$.

SOLUTION:

Let $f(x) = x^3 - px^2 - qx + 24$.

Since, $(x + 3)$ is a factor of $f(x)$, so by factor theorem, $f(-3) = 0$

$\therefore \quad f(-3) = (-3)^3 - p(-3)^2 - q(-3) + 24 = 0$

$\therefore \quad -27 - 9p + 3q + 24 = 0$

$\therefore \quad -3p + q - 1 = 0 \qquad \ldots (1)$

Similarly, if $(x - 4)$ is a factor of $f(x)$, then $f(4) = 0$

$\therefore \quad (4)^3 - p(4)^2 - q(4) + 24 = 0$

$\therefore \quad 64 - 16p - 4q + 24 = 0$

$\therefore \quad -4p - q + 22 = 0 \qquad \ldots (2)$

Solving eq. (1) and (2)

$-3p + q - 1 = 0$

$-4p - q + 22 = 0$

Adding, $-7p + 21 = 0, \quad \therefore\ p = 3$

Substituting, $p = 3$ in eq. (1), we get $-3(3) + q - 1 = 0$

$\therefore \quad q = 10$

$\therefore \quad p = 3$ and $q = 10$

EXERCISE 1

FIB Fill in the Blanks

DIRECTIONS: *Complete the following statements with an appropriate word/term to be filled in the blank space(s).*

1. A polynomial of one term is called a ....
2. A polynomial of three terms is called a ....
3. $11x^2 - 88x^3 + 14x^4$ is called a .... polynomial.
4. The degree of the polynomial $7x^3 y^{10} z^2$ is ....
5. A polynomial of degree one is called a .... polynomial
6. A polynomial of degree three is called a .... polynomial.
7. If $a + b + c = 0$, then $a^3 + b^3 + c^3 =$ ....
8. Factors of $x^6 - y^6$ is ....
9. $x - a$ is a factor of the polynomial $p(x)$, if $p(a) =$ ....
10. The remainder obtained when $80x^3 + 55x^2 + 20x + 172$ is divided by $x + 2$ is ....
11. The quotient of $8x^3 - 7x^2 + 5x$ when divided by $2x$ is ....
12. The square root of $a^{m^2}.b^{n^2}$ is ....
13. The factors of $a^3 + b^3 + c^3 - 3abc$ are ....

T/F True / False

DIRECTIONS: *Read the following statements and write your answer as true or false.*

1. $5x^3 - 4x^2 + 2x + 3$ is a polynomial over integers.
2. $3x^2 - 5x + 6$ is a polynomial of degree 2.
3. $7x^3 - 3x^2 + \sqrt{2}x + 5$ is a polynomial.
4. $2y^2 - \sqrt{2y} + 1 - 6y^3$ is a polynomial.
5. When $x^3 - 6x^2 + 9x + 7$ is divided by $(x - 1)$ remainder is 11.
6. The degree of a constant polynomial is 1.
7. Every binomial is a polynomial.
8. Every binomial is a polynomial of degree 2.
9. Every polynomial is a binomial.
10. $\sqrt{3}x^2 + 11x + 6\sqrt{3} = (x + 3\sqrt{3})(\sqrt{3}x + 2)$

MTC Match the Columns

DIRECTIONS: *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column I have to be matched with statements (p, q, r, s) in column II.*

1. Column II gives remainder when $x^3 + 3x^2 + 3x + 1$ is divided by expression given in column I, match them correctly.

Column I	Column II
(A) $x + 1$	(p) 27/8
(B) x	(q) –27/8
(C) $x - \frac{1}{2}$	(r) 1
(D) $5 + 2x$	(s) 0

2. Column II gives value of k for polynomials given in column I when it is divided by $x - 1$ match them correctly.

Column I	Column II
(A) $kx^2 - 3x + k$	(p) -2
(B) $x^2 + x + k$	(q) 3/2
(C) $2x^2 + kx + \sqrt{2}$	(r) $\sqrt{2} - 1$
(D) $kx^2 - \sqrt{2}x + 1$	(s) $-(2 + \sqrt{2})$

3. Column II gives factors for expression given in column I match them correctly.

Column I	Column II
(A) $9x^2 + 6xy + y^2$	(p) $(2x + 3y - 4z)(2x + 3y - 4z)$
(B) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$	(q) $\left(x + \frac{y}{10}\right)\left(x - \frac{y}{10}\right)$
(C) $27x^3 + y^3 + z^3 - 9xyz$	(r) $(3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3zx)$
(D) $x^2 - \frac{y^2}{100}$	(s) $(3x + y)(3x + y)$

VSAQ Very Short Answer Questions

DIRECTIONS: *Give answer in one word or one sentence.*

1. Check whether –2 and 2 are zeroes of the polynomial $x + 2$.
2. Write the following polynomial in standard form: $m^7 + 8m^5 + 4m^6 + 6m - 3m^2 - 11$
3. Evaluate $(104)^3$ using suitable identities
4. Factorize : $49a^2 + 70ab + 25b^2$
5. Factorize : $25x^2 + 60xy + 36y^2$
6. Factorize : $x^5 - x$
7. Factorize : $xy - 3z + xz - 3y$
8. Factorize : $axb + axc + 3aby + 3acy - 5b - 5c$
9. Find the value of 49^2
10. Factorize : $(a - 2b)^3 - a + 2b$
11. Factorize : $x^2 - y^2 - 4xz + 4z^2$

12. Factorise the following using appropriate identities : $9x^2 + 6xy + y^2$
13. Expand the following using suitable identities : $(-2x + 3y + 2z)^2$
14. If $A = x^3$, $B = 4x^2 + x - 1$, then find AB.
15. Find the remainder when x^{15} is divided by $x - 2$.
16. Factorize $a^5b - ab^5$.
17. The degree of a polynomial A is 7 and that of polynomial AB is 56, then find the degree of polynomial B.
18. Use the Factor Theorem to determine whether $q(x)$ is a factor of $p(x)$: $p(x) = 2\sqrt{2}x^2 + 5x + \sqrt{2}, q(x) = x + \sqrt{2}$

SAQ Short Answer Questions

DIRECTIONS: *Give answer in 2-3 sentences.*

1. Evaluate the following products without multiplying directly:
   (i) 103×107 (ii) 104×96
2. Factorize : $2x(a - b) + 3y(5a - 5b) + 4z(2b - 2a)$
3. Factorize: $x^2 - \left(a + \frac{1}{a}\right)x + 1$
4. Factorize: $6a^2b^2x - 48abx + 96x$
5. Factorize: $\frac{a^2}{4b^2} - \frac{1}{3} + \frac{b^2}{9a^2}$, $a \neq 0, b \neq 0$
6. Factorize: $4x^2 - 4\sqrt{3}x + 3$
7. Factorize : $3.7 \times 3.7 + 3.7 \times 2.6 + 1.3 \times 1.3$
8. Factorize : $\frac{0.564 \times 0.564 - 0.436 \times 0.436}{0.564 - 0.436}$
9. Factorize : $9x^3y + 11x^2y^2 + 20xy^3$
10. Factorize : $\frac{a}{b}x^2 + \left(\frac{a}{b} + \frac{c}{d}\right)x + \frac{c}{d}$; where $b \neq 0$, $d \neq 0$
11. Factorize : $12(a + 1)^2 - 25(a + 1)(b + 2) + 12(b + 2)^2$
12. Find the value of $a^3 + b^3 + c^3 - 3abc$, given that $a + b + c = 10$ and $a^2 + b^2 + c^2 = 83$.
13. Factorize : $x^4 + x^3 - 7x^2 - x + 6$
14. If $A = x^3$, $B = 4x^2 + x - 1$, $C = x + 1$, then find $(A - B)(A - C)$.
15. Factorize : $(a - b)^3 + (b - c)^3 + (c - a)^3$
16. If $x = \frac{4}{3}$ is a root of the polynomial $f(x) = 6x^3 - 11x^2 + kx - 20$ then find the value of k.
17. Find the remainder when $5x^2 + 3x + 1$ is divided by $2x$ using long division method.
18. Check whether $7 + 3x$ is a factor of $3x^3 + 7x$.

LAQ Long Answer Questions

DIRECTIONS: *Give answer in four to five sentences.*

1. Solve $2(x + 3)^2 + 9(x + 3) + 9$
2. Given $f(x) = x^3 + 5x^2 + 2x - 8$.
   Find (i) $f(1)$ (ii) $f(-2)$ (iii) $f(-4)$.
   Hence find all the factors of $f(x)$.
3. Show that $(3x - 2)$ is a factor of $(3x^3 + x^2 - 20x + 12)$

EXERCISE 2

Multiple Choice Questions

DIRECTIONS: *This section contains multiple choice questions. Each questions has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.*

1. In method of factorization of an algebraic expression. Which of the following statement is false ?
(a) Taking out a common factor from two or more terms
(b) Taking out a common factor from a group of terms.
(c) By using remainder theorem
(d) By using standard indentities

2. Factors of polynomial $x^3 - 3x^2 - 10x + 24$ are
(a) $(x-2)(x+3)(x-4)$ (b) $(x+2)(x+3)(x+4)$
(c) $(x+2)(x-3)(x-4)$ (d) $(x-2)(x-3)(x-4)$

3. The values of a and b so that the polynomial $x^3 - ax^2 - 13x + b$ is divisible by $(x+1)$ and $(x-3)$ are
(a) $a = 15, b = 3$ (b) $a = 0, b = -12$
(c) $a = -3, b = 15$ (d) $a = -12, b = 0$

4. Factors of $a^2 - b + ab - a$ are
(a) $(a-b)(a+1)$ (b) $(a+b)(a-1)$
(c) $(a-b)(a-1)$ (d) $(a+b)(a+1)$

5. Expansion of $\left(x+\frac{1}{x}\right)^2$ is
(a) $x^2+2x+\frac{1}{x^2}$ (b) $x^2-2x+\frac{1}{x^2}$
(c) $x^2+2+\frac{1}{x^2}$ (d) $x^2-2+\frac{1}{x^2}$

6. If $x^2 - x - 42 = (x+k)(x+6)$ then the value of k is
(a) 6 (b) -6
(c) 7 (d) -7

7. If $\left(x-\frac{1}{x}\right)^2 = x^2+x+\frac{1}{x^2}$ then the value of x is
(a) -2 (b) 2
(c) $2x$ (d) $-2x$

8. If one factor of $5 + 8x - 4x^2$ is $(2x + 1)$ then the second factor is
(a) $(5+2x)$ (b) $(2x-5)$
(c) $(5-2x)$ (d) $-(5+2x)$

9. Expansion of $(a-b)^2$ is
(a) $a^2+2ab+b^2$ (b) $a^2+2ab-b^2$
(c) $a^2-2ab-b^2$ (d) $a^2-2ab+b^2$

10. Factors of $x^2 - 7x + 12$ are
(a) $(x-3)(x+4)$ (b) $(x-3)(x-4)$
(c) $(x+3)(x-4)$ (d) $(x+3)(x+4)$

11. If $x = 2, y = -1$ then the value of $x^2 + 4xy + 4y^2$ is
(a) 0 (b) 1
(c) -1 (d) 2

12. If one factor of $a(x+y+z) + bx + by + bz$ is $(x+y+z)$ then the second factor is
(a) $ax + ay + az$ (b) $bx + by + bz$
(c) $bx + by - bz$ (d) $a + b$

13. Factors of : $36+11\left(z-\frac{y}{3}+x\right)-12\left(z-\frac{y}{3}+x\right)^2$
$+\left(4z-\frac{4}{3}y+4x-9\right)(5+3z-y+2x)$ are
(a) $(1-x)\left(4z-\frac{4y}{3}+4x-9\right)$
(b) $(1+x)\left(4z-\frac{4y}{3}+4x-9\right)$
(c) $(1-x)\left(4z+\frac{4y}{3}+4x-9\right)$
(d) $(1+x)\left(4z+\frac{4y}{3}+4x+9\right)$

14. The polynomials $ax^2 + 3x^2 - 3$ and $2x^3 - 5x + a$ when divided by $(x-4)$ leaves remainders R_1 and R_2 respectively then value of a if $2R_1 - R_2 = 0$, is
(a) $-\frac{18}{127}$ (b) $\frac{18}{32}$
(c) $\frac{17}{127}$ (d) $-\frac{17}{31}$

15. The value of n for which the expressions $9x^4 - 12x^3 - nx^2 - 8x + 4$ becomes a perfect square is
(a) 12 (b) 16
(c) 18 (d) 24

16. If $2x^2 + xy - 3y^2 + x + ay - 10 = (2x + 3y + b)(x - y - 2)$, then the values of a and b are
(a) 11 and 5 (b) 1 and -5
(c) -1 and -5 (d) -11 and 5

17. The expression $x^2 + px + q$ with p and q greater than zero has its minimum value when
(a) $x = -p$ (b) $x = p$
(c) $x = p/2$ (d) $x = -p/2$

MTOC More than One Correct

DIRECTIONS: *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONE or MORE may be correct.*

1. Factors of polynomial $12x^2 - 7x + 1$ are
   (a) $(3x-1)(4x-1)$ (b) $(4x+1)(3x-1)$
   (c) $12\left(x-\frac{1}{3}\right)\left(x-\frac{1}{4}\right)$ (d) $12\left(x+\frac{1}{4}\right)\left(x-\frac{1}{3}\right)$
2. Which of the following polynomials has a degree 3 ?
   (a) $2x^3 - 5x^2 + ax + b = 0$ (b) $6x^3 - 11x^2 + kx = 20$
   (c) $3x + 8 = 4$ (d) $\sqrt[3]{x^2} = x$
3. Which of the following is/are not false?
   (a) Highest power of the variable in a polynomial is the degree of polynomial.
   (b) Degree of zero polynomial is always defined.
   (c) A polynomial of degree one is called a linear polynomial.
   (d) A polynomial of degree two is called a constant polynomial.
4. Zeroes of the polynomial $x(x + 7)$ is /are
   (a) -7 (b) 7
   (c) 0 (d) none of these
5. Which of the following is/are cubic polynomials?
   (a) $x - x^3$ (b) $2x^2 + x^{3/2}$
   (c) $x^3 - x^{1/2}$ (d) $8x^2 + \frac{5}{2}x - 6x^3$
6. Expansion of $(x - y)^3$ is
   (a) $x^3 + y^3 - 3xy(x - y)$ (b) $x^3 - y^3 - 3x^2y - 3xy^2$
   (c) $x^3 - y^3 - 3x^2y + 3xy^2$ (d) $x^3 - y^3 - 3xy(x - y)$
7. If $a + b + c = 0$, then $a^3 + b^3 + c^3$ is equal to
   (a) $5abc$ (b) $2abc$
   (c) $abc + (abc)^0$ (d) $3abc$
8. Which of the following is/are a polynomial?
   (a) $x^3 - 4x^2 + 5\sqrt{x} + 1$ (b) $x^3 + 5x^2 + 2$
   (c) $x^{-2} + 4$ (d) x

PBQ Passage Based Questions

DIRECTIONS: *Study the given paragraph(s) and answer the following questions.*

$f(x) = 2x^3 - 9x^2 + x + 12$

1. The degree of the given polynomial is
   (a) 2 (b) 3
   (c) 0 (d) 1
2. Zeroes of the given polynomial is
   (a) (1, 3/2) (b) (–1, –3/2)
   (c) (–1, 3/2) (d) (1, 3/2)
3. If $f(x)$ is divided by $\left(x-\frac{3}{2}\right)$, then the remainder is
   (a) 1 (b) $\frac{3}{2}$
   (c) 0 (d) none of these

A&R Assertion and Reason

DIRECTIONS: *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.
(b) If both **Assertion** and **Reason** are correct, but Reason **is not the correct explanation** of Assertion.
(c) If **Assertion** is **correct** but **Reason** is **incorrect.**
(d) If **Assertion** is **incorrect** but **Reason** is **correct.**

1. **Assertion :** If $f(x) = 3x^7 - 4x^6 + x + 9$ is a polynomial, then its degree is 7.
   Reason : Degree of a polynomial is the highest power of the variable in it.
2. **Assertion :** If $p(x) = ax + b$, $a \neq 0$ is a linear polynomial, then $x = \frac{-b}{a}$ is the only zero of $p(x)$.
   Reason : A linear polynomial has one and only one zero.
3. **Assertion :** If $f(x) = x^4 + x^3 - 2x^2 + x + 1$ is divided by $(x - 1)$, then its remainder is 2.
   Reason : If $p(x)$ be a polynomial of degree greater than or equal to one, divided by the linear polynomial $x - a$, then the remainder is $p(-a)$.
4. **Assertion :** $(x + 2)$ is a factor of $x^3 + 3x^2 + 5x + 6$ and of $2x + 4$.
   Reason : If $p(x)$ be a polynomial of degree greater than or equal to one, then $(x - a)$ is a factor of $p(x)$, if $p(a) = 0$
5. **Assertion :** If $(x + 1)$ is a factor of $f(x) = x^2 + ax + 2$, then $a = -3$.
   Reason : If $(x - a)$ is a factor of $p(x)$, then $p(a) = 0$.

MMQ Multiple Matching Questions

DIRECTIONS : *Following question has four statements (A, B, C and D) given in Column I and statements (p, q, r, s and t) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in column I with entries in column II.*

1.

Column I (Degree)	Column II (Polynomial)
(A) 1	(p) $6 - y^3 + 4y^6 + y^2$
(B) 6	(q) $7x - \sqrt{3}$
(C) 3	(r) $2 + x^2$
(D) 2	(s) $x^2 + 5x^3 + 3$
	(t) $8x^2 - 4x + 6$

HOTS Hots Subjective Questions

DIRECTIONS: *Answer the following questions.*

1. Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x-y)^2 + (y-z)^2 + (z-x)^2]$.
2. Factorise $x^3 - 23x^2 + 142x - 120$.
3. If $x + y = 12$ and $xy = 27$, find the value of $x^3 + y^3$.
4. If the polynomials $(2x^3 + ax^2 + 3x - 5)$ and $(x^3 + x^2 - 2x + a)$ leave the same remainder when divided by $(x - 2)$, find the value of a. Also, find the remainder in each case.
5. Without actual division, prove that $(2x^4 - 6x^3 + 3x^2 + 3x - 2)$ is exactly divisible by $(x^2 - 3x + 2)$.

SOLUTIONS

EXERCISE – I

FILL IN THE BLANKS

1. monomial 2. trinomial 3. biquadratic
4. 15 5. linear 6. cubic
7. $a^3 + b^3 + c^3 = 3abc$
8. $(x-y)(x+y)(x^2+y^2-xy)(x^2+y^2+xy)$
9. 0 10. -288
11. $4x^2 - \frac{7}{2}x + \frac{5}{2}$ 12. $a^m \cdot b^n$
13. $(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

TRUE/FALSE

1. True 2. True 3. True 4. False
5. True 6. False 7. True 8. False
9. False 10. True

MATCH THE COLUMNS

1. (A) → (s); (B) → (r); (C) → (p); (D) → (q)
2. (A) → (q); (B) → (p); (C) → (s); (D) → (r)
3. (A) → (s); (B) → (p); (C) → (r); (D) → (q)

VERY SHORT ANSWER QUESTIONS

1. Let $r(x) = x + 2$.
   Then $r(2) = 2 + 2 = 4$, $r(-2) = -2 + 2 = 0$
   Therefore, -2 is a zero of the polynomial $x + 2$, but 2 is not.
2. $m^7 + 4m^6 + 8m^5 - 3m^2 + 6m - 11$
3. 1124864 4. $(7a+5b)(7a+5b)$
5. $(5x+6y)^2$ 6. $x(x^2+1)(x+1)(x-1)$
7. $(y+z)(x-3)$ 8. $(b+c)(ax+3ay-5)$
9. 2401 10. $(a-2b)(a-2b+1)(a-2b-1)$
11. $(x+y-2z)(x-y-2z)$
12. $9x^2 + 6xy + y^2 = (3x)^2 + 2(3x)(y) + (y)^2$
   $= (3x+y)^2 = (3x+y)(3x+y)$
13. $(-2x+3y+2z)^2 = (-2x)^2 + (3y)^2 + (2z)^2$
   $+ 2(-2x)(3y) + 2(3y)(2z) + 2(2z)(-2x)$
   $= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8zx$.
14. $4x^5 + x^4 - x^3$ 15. $f(2) = 2^{15}$
16. $ab(a^2+b^2)(a+b)(a-b)$ 17. 49
18. The zero of $q(x)$ is $-\sqrt{2}$
   Now, $p(-\sqrt{2}) = 2\sqrt{2}(-\sqrt{2})^2 + 5(-\sqrt{2}) + \sqrt{2}$
   $= 4\sqrt{2} - 5\sqrt{2} + \sqrt{2} = 0$
   ∴ By factor then, $q(x)$ is a factor of $p(x)$.

SHORT ANSWER QUESTIONS

1. (i) 11021 (ii) 9984
2. $(a-b)(2x+15y+8z)$
3. $(x-a)(x-1/a)$ 4. $6x(ab-4)^2$
5. $\left(\frac{a}{2b} - \frac{b}{3a}\right)^2$ 6. $(2x-\sqrt{3})^2$ 7. 25
8. 1 9. $xy(x+4y)(9x+5y)$
10. $(x+1)\left(\frac{a}{b}x + \frac{c}{d}\right)$ 11. $(4a-3b-2)(3a-4b-5)$
12. 745 13. $(x+1)(x-1)(x+3)(x-2)$
14. $x^6 - 4x^5 - 2x^4 + 4x^3 + 5x^2 - 1$
15. $3(a-b)(b-c)(c-a)$
16. $f(x) = 6x^3 - 11x^2 + kx - 20$
   $f\left(\frac{4}{3}\right) = 6\left(\frac{4}{3}\right)^3 - 11\left(\frac{4}{3}\right)^2 + k\left(\frac{4}{3}\right) - 20 = 0$
   $\Rightarrow 6.\frac{64}{9.3} - 11.\frac{16}{9} + \frac{4k}{3} - 20 = 0$
   $\Rightarrow 128 - 176 + 12k - 180 = 0$
   $\Rightarrow 12k + 128 - 356 = 0$
   $\Rightarrow 12k = 228 \Rightarrow k = 19$
17.
$$\begin{array}{r|l|l} 2x & 5x^2 + 3x + 1 & \frac{5}{2}x + \frac{3}{2} \\ & 5x^2 & \\ & - & \\ \hline & 3x + 1 & \\ & 3x & \\ & - & \\ \hline & 1 & \end{array}$$
   ∴ Quotient $= \frac{5}{2}x + \frac{3}{2}$, Remainder = 1
18. $7 + 3x$ will be a factor of $3x^3 + 7x$ only if $7 + 3x$ divides $3x^3 + 7x$ leaving no remainder.
   Let $p(x) = 3x^3 + 7x$
   put $7 + 3x = 0$
   $\Rightarrow x = -\frac{7}{3}$
   ∴ Remainder $= p\left(-\frac{7}{3}\right) = 3\left(-\frac{7}{3}\right)^3 + 7\left(-\frac{7}{3}\right)$
   $= -\frac{343}{9} - \frac{49}{3} = -\frac{490}{9} \neq 0$
   ∴ $7 + 3x$ is not a factor of $3x^3 + 7x$.

LONG ANSWER QUESTIONS

1. $2(x+3)^2 + 9(x+3) + 9$
   $= 2a^2 + 9a + 9$ (where $(x+3) = a$)

$= 2a^2 + 6a + 3a + 9 = (2a^2 + 6a) + (3a + 9)$
$= 2a(a + 3) + 3(a + 3) = (a + 3)(2a + 3)$
$= (x + 3 + 3)\{2(x + 3) + 3\}$, (substituting the value of a)
$= (x + 6)(2x + 9)$

2. (i) $f(1) = (1)^3 + 5(1)^2 + 2(1) - 8 = 1 + 5 + 2 - 8 = 0$
(ii) $f(-2) = (-2)^3 + 5(-2)^2 + 2(-2) - 8$
$= -8 + 20 - 4 - 8 = 0$
(iii) $f(-4) = (-4)^3 + 5(-4)^2 + 2(-4) - 8$
$= -64 + 80 - 8 - 8 = 0$
Since, $f(1) = 0 \therefore (x - 1)$ is a factor of $f(x)$
$f(-2) = 0 \quad \therefore (x + 2)$ is a factor of $f(x)$
$f(-4) = 0 \quad \therefore (x + 4)$ is a factor of $f(x)$
Hence all the factors of $f(x)$ are
$(x - 1)$, $(x + 2)$ and $(x + 4)$

3. Let $f(x) = 3x^3 + x^2 - 20x + 12$.
Now, $3x - 2 = 0 \Rightarrow x = \frac{2}{3}$.
$(3x - 2)$ is a factor of $f(x) = 3x^3 + x^2 - 20x + 12$
Iff $f\left(\frac{2}{3}\right) = 0$
Now, $f\left(\frac{2}{3}\right) = \left[3\times\left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^2 - 20\times\left(\frac{2}{3}\right) + 12\right]$
$= \left(\frac{8}{9} + \frac{4}{9} - \frac{40}{3} + 12\right) = 0$
Hence, $(3x - 2)$ is a factor of the given polynomial $f(x)$.

EXERCISE – 2

MULTIPLE CHOICE QUESTIONS

1. (c) 2. (a) 3. (b) 4. (b) 5. (c)
6. (d) 7. (a) 8. (c) 9. (d) 10. (b)
11. (a) 12. (d) 13. (a) 14. (b) 15. (b)
16. (d) 17. (d)

MORE THAN ONE CORRECT

1. (a, c) 2. (a, b, d) 3. (a, c) 4. (a, c)
5. (a, d) 6. (c, d) 7. (a, d) 8. (b, d)

PASSAGE BASED QUESTIONS

1. (b) 2. (c) 3. (c)

ASSERTION & REASON

1. (a) 2. (a) 3. (c) 4. (a) 5. (d)

MULTIPLE MATCHING QUESTIONS

1. (A) → (q); (B) → (p); (C) → (s); (D) → (r), (t)

HOTS SUBJECTIVE QUESTIONS

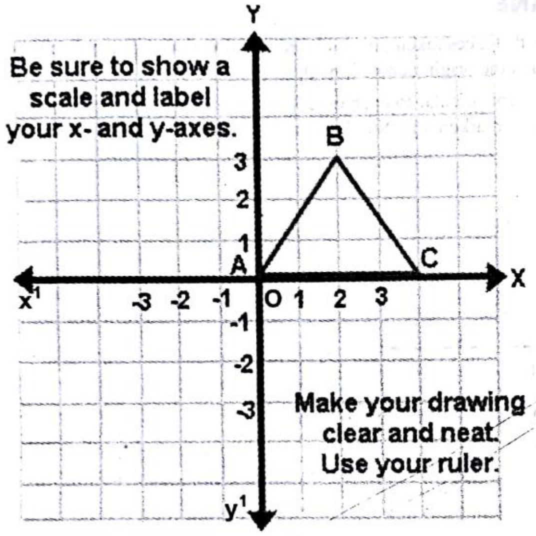
1. L.H.S. $= x^3 + y^3 + z^3 - 3xyz$
$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$
$= \frac{1}{2}(x + y + z)\{2(x^2 + y^2 + z^2 - xy - yz - zx)\}$
$= \frac{1}{2}(x + y + z)(2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx)$
$= \frac{1}{2}(x + y + z)\{(x^2 - 2xy + y^2) + (y^2 - 2yz + z^2) + (z^2 - 2zx + x^2)\}$
$= \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2] =$ R.H.S.

2. Let $r(x) = x^3 - 23x^2 + 142x - 120$
Some of the factors of -120 are $\pm1, \pm2, \pm3, \pm4, \pm5, \pm6, \pm8, \pm10, \pm12, \pm15, \pm20, \pm24, \pm30, \pm60$.
By trial, we find that $r(1) = 0$. So $x - 1$ is a factor of $r(x)$.
Now, $x^3 - 23x^2 + 142x - 120 = x^3 - x^2 - 22x^2 + 22x + 120x - 120 = x^2(x - 1) - 22x(x - 1) + 120(x - 1)$
$= (x - 1)(x^2 - 22x + 120)$ [Taking $(x - 1)$ common]
We could have also got this by dividing $r(x)$ by $x - 1$.
Now, $x^2 - 22x + 120$ can be factorised either by splitting the middle term or by using the Factor theorem. By splitting the middle term, we have :
$x^2 - 22x + 120 = x^2 - 12x - 10x + 120$
$= x(x - 12) - 10(x - 12)$
$= (x - 12)(x - 10)$
So, $x^3 - 23x^2 - 142x - 120 = (x - 1)(x - 10)(x - 12)$.

3. $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
$= (x + y)[(x + y)^2 - 3xy] = 12[12^2 - 3 \times 27]$
$= 12 \times 63 = 756$

4. Let $f(x) = 2x^3 + ax^2 + 3x - 5$ and $g(x) = x^3 + x^2 - 2x + a$.
When $f(x)$ is divided by $(x - 2)$, remainder $= f(2)$.
When $g(x)$ is divided by $(x - 2)$, remainder $= g(2)$.
Now, $f(2) = (2 \times 2^3 + a \times 2^2 + 3 \times 2 - 5) = (17 + 4a)$.
And, $g(2) = (2^3 + 2^2 - 2 \times 2 + a) = (8 + a)$.
Since, both the polynomials leave the same remainder when divided by $(x - 2)$
$\therefore \ 17 + 4a = 8 + a \Rightarrow 3a = -9 \Rightarrow a = -3$.
Hence, $a = -3$.
Remainder in each case $= (8 - 3) = 5$

5. Let $f(x) = 2x^4 - 6x^3 + 3x^2 + 3x - 2$
Let $g(x) = (x^2 - 3x + 2) = x^2 - 2x - x + 2$.
$= x(x - 2) - (x - 2) = (x - 2)(x - 1)$
Now, $f(x)$ will be exactly divisible by $g(x)$ if it is exactly divisible by $(x - 2)$ as well as $(x - 1)$.
For this, we must have $f(2) = 0$ and $f(1) = 0$
Now, $f(2) = (2 \times 2^4 - 6 \times 2^3 + 3 \times 2^2 + 3 \times 2 - 2)$
$= (32 - 48 + 12 + 6 - 2) = 0$
And, $f(1) = (2 \times 1^4 - 6 \times 1^3 + 3 \times 1^2 + 3 \times 1 - 2)$
$= (2 - 6 + 3 + 3 - 2) = 0$
$\therefore f(x)$ is exactly divisible by $(x - 2)$ as well as $(x - 1)$.
So, $f(x)$ is exactly divisible by $(x - 2)(x - 1)$.
Hence, $f(x)$ is exactly divisible by $(x^2 - 3x + 2)$.



3

CHAPTER

Co-ordinate Geometry

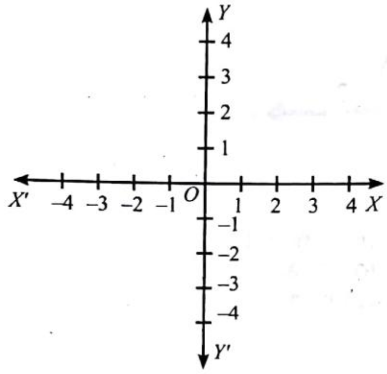
INTRODUCTION

You have already studied the number line and know how to locate the position of a number on the number line. In this chapter you will learn how to locate the position of a point or any thing in a plane which will be further used to draw various type of graph. You will also learn how to find the distance between two points and to find the ratio in which a point on it divides the line segment joining the two points.

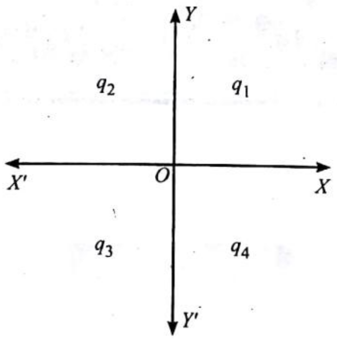
TO LOCATE THE POSITION OF A POINT IN A PLANE

On the number line, distances from a fixed point are marked at equal distances taken positively in one direction and negatively in other direction. The point from which the distances are marked is called the origin denoted by O.

To locate a point on a plane we require two such number lines, both perpendicular to each other and meeting each other at origin. One number line is kept horizontal marked XX' and other perpendicular marked YY'. Numbers are written on both of them just as written on number line. XX' is called X-axis and YY' is called Y-axis.



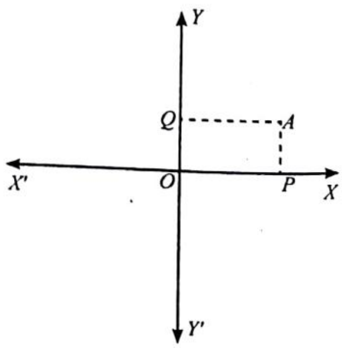
From the figure, it is clear that positive numbers lies on OX and OY. OX and OY are called positive direction of X-axis and Y-axis respectively. Similarly OX' and OY' are called negative direction of X-axis and Y-axis respectively because negative numbers are lies on OX' and OY'.



Here we see that the two axis divide the plane into four parts, these parts are called quadrants denoted by q_1, q_2, q_3 and q_4 as show in the figure.

The entire plane consists of the two axis and four quadrants. We call this plane as XY-plane or Cartesian plane or co-ordinate plane. The two axes (plural of axis) are called co-ordinate axes.

Let us take a point A in XY-plane as shown in figure and draw two perpendiculars AP and AQ on X and Y-axis respectively.

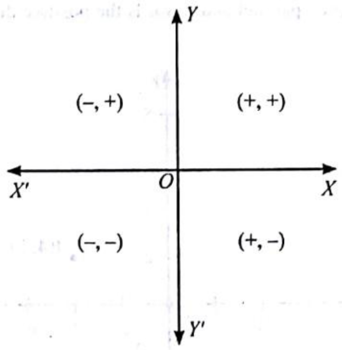


Distance $AQ = OP$, is called x-co-ordinate or abscissa of point A and distance $AP = OQ$, is called y-co-ordinate or ordinate of point P.

Actually in XOY plane, x-co-ordinate of a point is the distance of the point from Y-axis and y-co-ordinate of the point is the distance of the point from x-axis. The abscissa and ordinate of a point taken together is known as co-ordinate of a point. Thus if $OP = x$ and $OQ = y$ then (x, y) is co-ordinates of point A. Note that in the co-ordinate of a point, first entry is always x-co-ordinate and second entry is always y-co-ordinate. Hence $(x, y) \neq (y, x)$.

The order of x and y in (x, y) are important. So (x, y) is an ordered pair.

Now, we know about the co-ordinates of a point.



The figure shown the sign of the co-ordinates of a point in different quadrants.

We can clearly see from figure that

(a) All the points in first quadrant have both abscissa and ordinate positive.

(b) In second quadrant abscissa is negative and ordinate is positive.

(c) In third quadrant both abscissa and ordinate are negative

(d) In fourth quadrant abscissa is positive and ordinate is negative.

All the points which lie on x-axis have zero-ordinate as their distance from x-axis is zero.

Similarly all the points on y-axis have zero abscissa as their distance from y-axis is zero.

Note : That any part of x or y-axis does not lie in any quadrant.

CO-ORDINATES OF ORIGIN

Origin is the point of inter-section of x-axis and y-axis. Now the distance of origin from any of the axes is zero, so origin has zero abscissa and zero ordinate. Therefore the co-ordinates of origin is (0, 0).

PLOTING OF A POINT WHOSE CO-ORDINATES ARE KNOWN

A point can be plotted by measuring its proper distances from the axes. Thus any point (h, k) can be plotted as follows:

(i) Measure OM equal to h units along the x-axis.

(ii) Now measure PM perpendicular to OM at M and equal to K units.

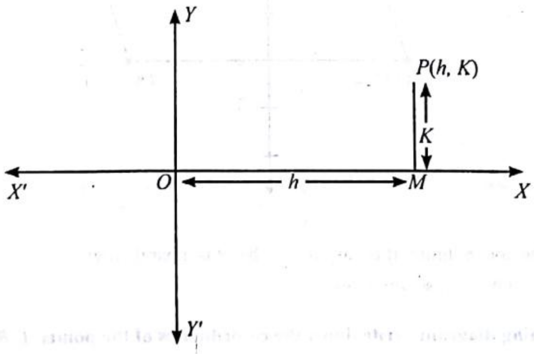


Illustration 1 : Plot the point $A(4, 2)$ on a graph paper.

SOLUTION : On a graph paper, draw the co-ordinate axes XOX' and YOY' intersecting at origin O. With proper scale, mark the numbers on the twc co-ordinate axes. For plotting any point, two steps are to be adopted. e.g. to plot point $A(4, 2)$.

Step 1 : Starting from the origin O, move 4 units along the positive direction of X-axis i.e. to the right of the origin O.

Step 2 : Now, from there, move 2 units up (i.e., parallel and towards the positive direction of y-axis) and place a dot at the point reached. Label this point as $A(4, 2)$

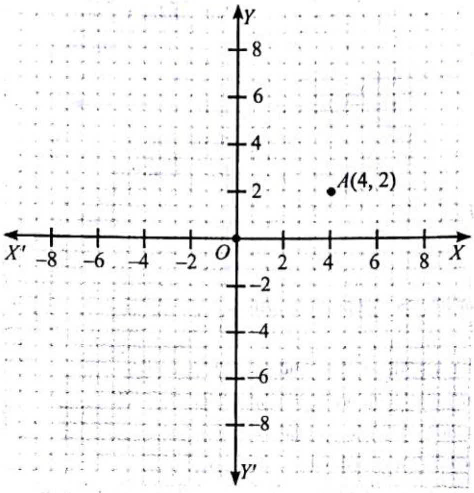
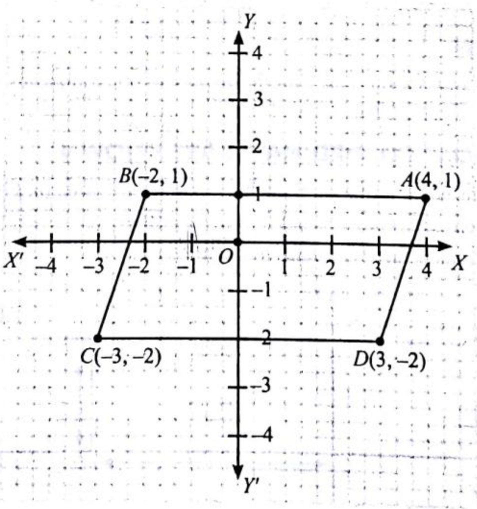


Illustration 2 : Plot the points $A(4, 1)$, $B(-2, 1)$, $C(-3, -2)$ and $D(3, -2)$. Name the figure $ABCD$. Find its area.

SOLUTION :



Points A, B, C, D are marked in the above figure. It is easy to see that it is a parallelogram.

Area = base × height = 6 units × 3 units = 18 square units

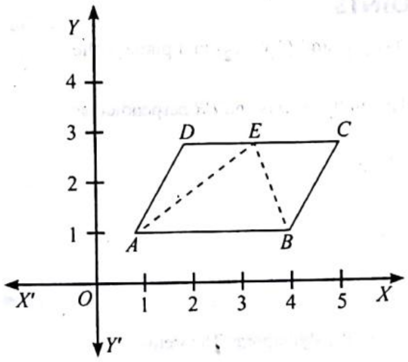
Illustration 3 : Using the adjoining diagram, write down the co-ordinates of the points A, B, C, D and E.

(i) Name the figure $ABCD$ and find its area.

(ii) Name the figure ABE, find its area.

(iii) Is area of the figure $ABCD$ twice the area of the figure ABE ?

What conclusion can you derive from the above result ?



SOLUTION : Clearly from the diagram, we have

$A(1, 1), B(4, 1), C(5, 3), D(2, 3), E(3, 3)$.

(i) $ABCD$ is a parallelogram, since $AB = DC = 3$ units, and AB is parallel to DC.

Area = base × height = 3 × 2 = 6 sq. units

(ii) ABE is a triangle, since it is a three-sided figure.

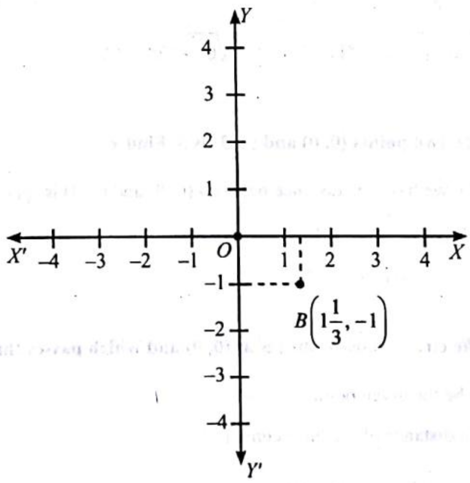
Area = $\frac{1}{2}$ × base × height = $\frac{1}{2} \times 3 \times 2 = 3$ sq. units

(iii) Yes, Area of parallelogram $ABCD$ = 2 (Area of triangle ABE)

Conclusion : The area of parallelogram is twice the area of a triangle on the same base and between the same parallels.

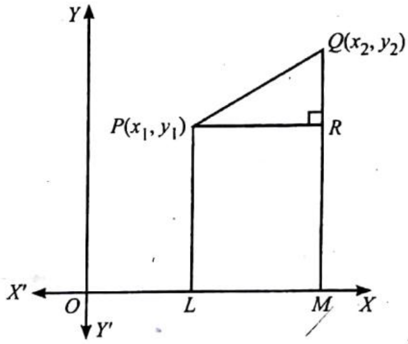
Illustration 4 : Plot the points : $B\left(1\frac{1}{3}, -1\right)$

SOLUTION : Plotting the points : $B\left(1\frac{1}{3}, -1\right)$



By taking suitable scale, mark the co-ordinate axes in such a way that the fraction $\frac{1}{3}$ can easily be read. For this, take three division equal to one unit.

DISTANCE BETWEEN TWO POINTS



The distance between any two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ in a plane is the length of the line segment PQ.

From P, Q draw PL and QM perpendiculars on the X-axis and PR perpendicular on QM.

Then, $OL = x_1$, $OM = x_2$, $PL = y_1$ and $QM = y_2$

$\therefore \quad PR = LM = OM - OL = x_2 - x_1$

$QR = QM - RM = QM - PL = y_2 - y_1$

Since PQR is a right angled triangle

$\therefore \quad PQ^2 = PR^2 + QR^2$

$= (x_2 - x_1)^2 + (y_2 - y_1)^2$ (By the Pythagoras Theorem)

$\therefore \quad PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance between two points $= \sqrt{(\text{difference of abscissae})^2 + (\text{difference of ordinates})^2}$

Corollary : The distance of the point (x_1, y_1) from the origin (0, 0) is

$$\sqrt{(x_1 - 0)^2 + (y_1 - 0)^2} = \sqrt{x_1^2 + y_1^2}$$

Illustration 5 : Find the distance between each of the following points

(a) $P(6, 8)$ and $Q(-9, -12)$ **(b) $A(-6, -1)$ and $B(-6, 11)$**

SOLUTION :

(a) Here the points are $P(6, 8)$ and $Q(-9, -12)$

By using distance formula, we have

$$PQ = \sqrt{(-9-6)^2 + \{(-12-8)\}^2} = \sqrt{15^2 + 20^2} = \sqrt{225 + 400} = \sqrt{625} = 25$$

Hence, $PQ = 25$ units.

(b) Here the points are $A(-6, -1)$ and $B(-6, 11)$

By using distance formula, we have

$$AB = \sqrt{\{-6-(-6)\}^2 + \{11-(-1)\}^2} = \sqrt{0^2 + 12^2} = 12$$

Hence, $AB = 12$ units.

Illustration 6 : The distance between two points (0, 0) and $(x, 3)$ is 5. Find x.

SOLUTION : By using distance formula, we have the distance between (0, 0) and $(x, 3)$ is $\sqrt{(x-0)^2 + (3-0)^2}$

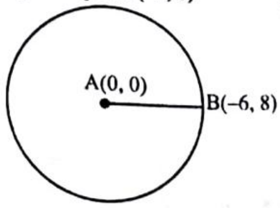
It is given that $\sqrt{(x-0)^2 + (3-0)^2} = 5$ or $\sqrt{x^2 + 3^2} = 5$

Squaring both sides, $x^2 + 9 = 25$ or $x^2 = 16$ or $x = \pm 4$

Hence, $x = +4$ or $x = -4$

Illustration 7 : Find the radius of the circle whose centre is at (0, 0) and which passes through the point (–6, 8)

SOLUTION : Let $A(0, 0)$ and $B(-6, 8)$ be the given points.



Now, radius of the cirlce is same as the distance of the line segment AB.

Now, $AB = \sqrt{(-6-0)^2 + (8-0)^2} = \sqrt{36 + 64} = \sqrt{100} = 10$

Hence, radius of the cirlce is 10 units.

APPLICATIONS OF DISTANCE FORMULA

(i) For given three points A, B, C to decide whether they are collinear or vertices of a particular triangle. After finding AB, BC, and CA we shall find that the point are
 (a) Collinear, if the sum of any two distances is equal to the third.
 (b) Vertices of an equilateral triangle if $AB = BC = CA$
 (c) Vertices of an isosceles triangle if $AB = BC$ or $BC = CA$ or $CA = AB$
 (d) Vertices of a right angled triangle if $AB^2 + BC^2 = CA^2$ or $AB^2 + CA^2 = BC^2$ or $BC^2 + CA^2 = AB^2$.

(ii) For given four points A, B, C, D
 (a) $AB = BC = CD = DA$; $AC = BD \Rightarrow ABCD$ is a square
 (b) $AB = BC = CD = DA$; $AC \neq BD \Rightarrow ABCD$ is a rhombus
 (c) $AB = CD$, $BC = DA$; $AC = BD \Rightarrow ABCD$ is a rectangle
 (d) $AB = CD$, $BC = DA$; $AC \neq BD \Rightarrow ABCD$ is a parallelogram
 (e) Diagonal of square, rhombus, rectangle and parallelogram always bisect each other

Illustration 8 : Show that the points (1, 1), (3, 0) and (–1, 2) are collinear

SOLUTION : Let $P(1, 1)$, $Q(3, 0)$ and $R(-1, 2)$ be the given points

$$PQ = \sqrt{(3-1)^2 + (0-1)^2} = \sqrt{4+1} = \sqrt{5} \text{ units}$$

$$QR = \sqrt{(-1-3)^2 + (2-0)^2} = \sqrt{16+4} = 2\sqrt{5} \text{ units}$$

$$RP = \sqrt{(-1-1)^2 + (2-1)^2} = \sqrt{4+1} = \sqrt{5} \text{ units}$$

Now, $PQ + RP = (\sqrt{5} + \sqrt{5})$ units $= 2\sqrt{5}$ units $= QR$

$\therefore$ P, Q and R are collinear points.

Illustration 9 : If the three vertices of a rectangle taken in order are the points (2, –2), (8, 4) and (5, 7). The co-ordinates of the fourth vertex is –

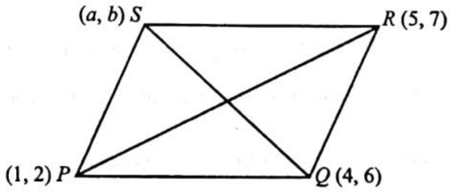
(A) (1, 1) **(B)** (1, –1) **(C)** (–1, 1) **(D)** None of these

SOLUTION : (C) Let fourth vertex be (x, y), then $\frac{x+8}{2} = \frac{2+5}{2}$ and $\frac{y+4}{2} = \frac{-2+7}{2} \Rightarrow x = -1, y = 1$

Illustration 10 : If $P(1, 2)$, $Q(4, 6)$, $R(5, 7)$ and $S(a, b)$ are the vertices of a parallelogram $PQRS$, then –

(A) $a = 2, b = 4$ **(B)** $a = 3, b = 4$ **(C)** $a = 2, b = 3$ **(D)** $a = 3, b = 5$

SOLUTION : (C) Diagonals cut each other at middle points.



Hence, $\frac{a+4}{2} = \frac{1+5}{2} \Rightarrow a = 2$

$\frac{b+6}{2} = \frac{2+7}{2} \Rightarrow b = 3$

Illustration 11 : If $A(3, 5)$, $B(-5, -4)$, $C(7, 10)$ are the vertices of a parallelogram taken in the order, then the co-ordinates of the fourth vertex are –

(A) (10, 19) **(B)** (15, 10) **(C)** (19, 10) **(D)** (15, 19)

SOLUTION : Mid point of $A(3, 5)$ and $C(7, 10) = M\left(5, \frac{15}{2}\right)$

$\therefore$ Mid points of $BD = M\left(5, \frac{15}{2}\right)$

$B(-5, -4)$ and $D(x, y)$

$\therefore \quad \dfrac{-5+x}{2} = 5, x = 10 + 5 = 15$

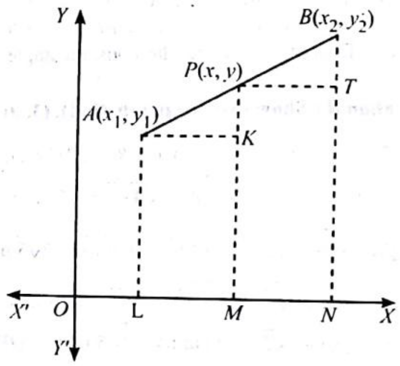
$\dfrac{-4+y}{2} = \dfrac{15}{2}, y = 15 + 4 = 19$

Co-ordinates of fourth vertex $D = (15, 19)$

SECTION FORMULA

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be the two given points and $P(x, y)$ be a point on AB which divides it in the ratio $m : n$ internally. We have to find the co-ordinates of P, draw the perpendiculars AL, PM, BN on OX, and, AK, PT on PM and BN respectively. Then, from similar triangles APK and PBT,

we have $\quad \dfrac{AP}{PB} = \dfrac{AK}{PT} = \dfrac{KP}{TB} \quad \ldots$ (i)



Now, $\quad AK = LM = OM - OL = x - x_1$

$PT = MN = ON - OM = x_2 - x$

$KP = MP - MK = MP - LA = y - y_1,$

$TB = NB - NT = NB - MP = y_2 - y$

From (i), we have, $\dfrac{m}{n} = \dfrac{x - x_1}{x_2 - x} = \dfrac{y - y_1}{y_2 - y}$

The first two relations give, $\dfrac{m}{n} = \dfrac{x - x_1}{x_2 - x}$

or $\quad mx_2 - mx = nx - nx_1$

or $\quad x(m+n) = mx_2 + nx_1$

or $\quad x = \dfrac{mx_2 + nx_1}{m+n}$

Similarly, from the relation $\dfrac{AP}{PB} = \dfrac{KP}{TB}$, we get $\dfrac{m}{n} = \dfrac{y - y_1}{y_2 - y}$ which gives on simplification.

$y = \dfrac{my_2 + ny_1}{m+n}$

Hence co-ordinates of a point which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) in the ratio $m : n$ internally is

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

Sometimes the word internally is not written but it is understood.

MID-POINT FORMULA

The co-ordinates of the mid-point of the line segment joining two points (x_1, y_1) and (x_2, y_2) can be obtained by taking $m = n$ in the section formula.

$\therefore \quad$ The co-ordinates of the mid-point joining two points (x_1, y_1) and $(x_2, y_2) = \left(\dfrac{x_2 + x_1}{2}, \dfrac{y_2 + y_1}{2}\right)$

Illustration 12 : Find the co-ordinates of a point which divides the line segment joining each of the following points in the given ratio :

(a) (2, 3) and (7, 8) in the ratio 2 : 3 internally **(b) (–1, 4) and (0, –3) in the ratio 1 : 4 internally.**

SOLUTION :

(*a*) Let $A(2, 3)$ and $B(7, 8)$ be the given points.

Let $P(x, y)$ divide AB in the ratio 2 : 3 internally.

Using section formula, we have,

$$x = \frac{2 \times 7 + 3 \times 2}{2+3} = \frac{20}{5} = 4 \text{ and } y = \frac{2 \times 8 + 3 \times 3}{2+3} = \frac{25}{5} = 5$$

$\therefore \quad P(4, 5)$ divides AB in the ratio 2 : 3 internally.

(b) Let $A(-1, 4)$ and $B(0, -3)$ be the given points.

Let $P(x, y)$ divide AB in the ratio 1 : 4 internally

Using section formula, we have

$$x = \frac{1\times 0 + 4\times(-1)}{1+4} = -\frac{4}{5} \text{ and } y = \frac{1\times(-3)+4\times 4}{1+4} = \frac{13}{5}$$

$\therefore$ $P\left(-\frac{4}{5}, \frac{13}{5}\right)$ divides AB in the ratio 1 : 4 internally.

Illustration 13 : Find the mid-point of the line-segment joining two points (3, 4) and (5, 12)

SOLUTION : Let $A(3, 4)$ and $B(5, 12)$ be the given points.

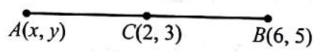
Let $C(x, y)$ be the mid-point of AB. Using mid-point formula, we have,

$$x = \frac{3+5}{2} = 4 \text{ and } y = \frac{4+12}{2} = 8$$

$\therefore$ (4, 8) are the co-ordinates of the mid-point of the line segment joining two points (3, 4) and (5, 12).

Illustration 14 : The co-ordinates of the mid-point of a line segment are (2, 3). If co-ordinates of one of the end points of the line segment are (6, 5), find the co-ordinats of the other end point.

SOLUTION : Let the other end point be $A(x, y)$



It is given that $C(2, 3)$ is the mid-point

$\therefore$ We can write, $2 = \frac{x+6}{2}$ and $3 = \frac{y+5}{2}$

or $4 = x + 6$ and $6 = y + 5$

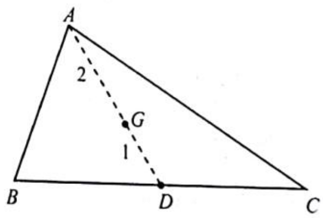
or $x = -2$ and $y = 1$

$\therefore$ $A(-2, 1)$ be the co-ordinates of the other end point.

CO-ORDINATE OF SOME PARTICULAR POINTS

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are vertices of any triangle ABC, then

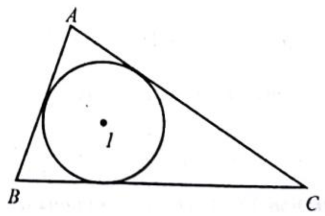
1. Centroid :



The centroid is the point of intersection of the medians (Line joining the mid-point of sides and opposite vertices). Centroid divides the median in the ratio of 2 : 1

Co-ordinates of centroid, $G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

2. Incentre :



The incentre is the point of intersection of internal bisector of the angles of the triangle. Also it is a centre of circle touching all the sides of a triangle.

Co-ordinates of incentre, $I = \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$

where a, b, c are length of the sides of triangle ABC.

Illustration 15 : The centroid of a triangle, whose vertices are (2, 1), (5, 2) and (3, 4) is –

(A) $\left(\frac{8}{3}, \frac{7}{3}\right)$ **(B)** $\left(\frac{10}{3}, \frac{7}{3}\right)$ **(C)** $\left(-\frac{10}{3}, \frac{7}{3}\right)$ **(D)** $\left(\frac{10}{3}, -\frac{7}{3}\right)$

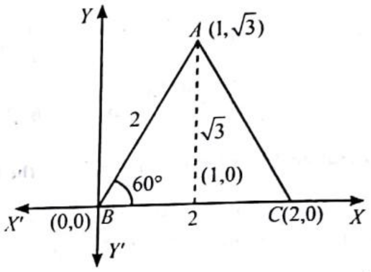
SOLUTION : (B) $x = \frac{2+5+3}{3} = \frac{10}{3}$ and $y = \frac{1+2+4}{3} = \frac{7}{3}$

Illustration 16 : The incentre of the equilateral triangle with vertices $(1,\sqrt{3}), (0,0)$ and $(2, 0)$ is –

(A) $\left(1,\frac{\sqrt{3}}{2}\right)$ (B) $\left(\frac{2}{3},\frac{1}{\sqrt{3}}\right)$ (C) $\left(\frac{2}{3},\frac{\sqrt{3}}{2}\right)$ (D) $\left(1,\frac{1}{\sqrt{3}}\right)$

SOLUTION : (D) Clearly, the triangle is equilateral.

So, the incentre is the same as the centroid.



$\therefore$ Incentre $=\left(\frac{1+0+2}{3},\frac{\sqrt{3}+0+0}{3}\right)=\left(1,\frac{1}{\sqrt{3}}\right)$

AREA OF A TRIANGLE

If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle then its area

$$=\frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$$

(i) Condition of Collinearity: Three points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are collinear if area of $\Delta ABC = 0$

i.e. $x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0$

(ii) Let the vertices of quadrilateral $ABCD$ are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ and $D(x_4, y_4)$.

So, Area of quadrilateral $ABCD$ = Area of ΔABC + Area of ΔACD

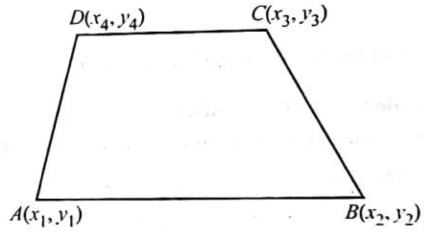


Illustration 17 : The area of a triangle is 5. Two of its vertices are (2, 1) and (3, –2). The third vertex lies on $y = x + 3$. Find the third vertex.

SOLUTION : Let the third vertex be (x_3, y_3), area of triangle $=\frac{1}{2}|[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]|$

As $x_1 = 2, y_1 = 1, x_2 = 3, y_2 = 2$, Area of $\Delta = 5$

$\Rightarrow \quad 5=\frac{1}{2}|2(-2-y_1)+3(y_3-1)+x_3(1+2)|$

$\Rightarrow \quad 10=|3x_3+y_3-7| \quad \Rightarrow \quad 3x_3+y_3=\pm 19$

Taking positive sign,

$3x_3+y_3-7=10 \quad \Rightarrow \quad 3x_3+y_3=17$ .... (i)

Taking negative sign

$3x_3+y_3-7=-10 \quad \Rightarrow \quad 3x_3+y_3=-3$ .... (ii)

Given that $(x_3, -y_3)$ lies on $y = x + 3$

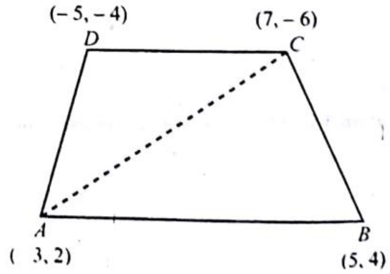
So, $-x_3+y_3=3$ .... (iii)

Solving equations (i) and (iii), $x_3=\frac{7}{2}$, $y_3=\frac{13}{2}$

Solving equations (ii) and (iii), $x_3=\frac{-3}{2}$, $y_3=\frac{3}{2}$.

So the third vertex are $\left(\frac{7}{2},\frac{13}{2}\right)$ or $\left(\frac{-3}{2},\frac{3}{2}\right)$

Illustration 18 : Find the area of quadrilateral whose vertices, taken in order, are $A(-3, 2)$, $B(5, 3)$, $C(7, -6)$ and $D(-5, -4)$.



SOLUTION : Area of quadrilateral = Area of $\Delta\, ABC$ + Area of $\Delta\, ACD$

So, Area of $\Delta\, ABC=\frac{1}{2}|(-3)(4+6)+5(-6-2)+7(2-4)|$

$=\frac{1}{2}|-30-40-14|=\frac{1}{2}|-84|=42$ sq. units

So, Area of $\Delta ACD = \frac{1}{2}|-3(-6+4)+7(-4-2)+(-5)(2+6)|$

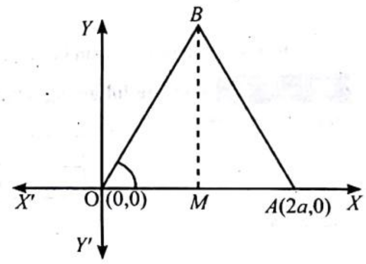
$= \frac{1}{2}|+6-42-40| = \frac{1}{2}|-76| = 38$ sq. units

So, Area of quadrilateral $ABCD = 42 + 38 = 80$ sq. units.

MISCELLANEOUS

Solved Examples

Example 1 : In the figure OAB is an equilateral triangle. Find the co-ordinate of vertex B.



SOLUTION : In figure,

OAB is an equilateral triangle of $2a$.

$\therefore \quad OA = AB = OB = 2a$

Now, from the point B, draw BM perpendicular on OA.

$\therefore \quad OM = MA = a$

Therefore from right triangle OMB,

$OB^2 = OM^2 + MB^2$

or, $(2a)^2 = (a)^2 + MB^2$

or, $MB^2 = 3a^2$

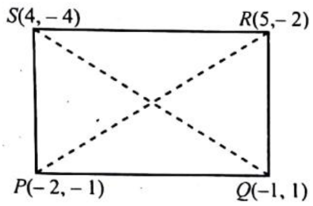
$\therefore \quad MB = \sqrt{3}a$

Since $OM = a$ and $MB = \sqrt{3}a$

Hence, co-ordinates of vertex B are $(a, \sqrt{3}a)$.

Example 2 : Prove that the points (–2, –1), (–1, 1), (5, –2) and (4, – 4) are the vertices of a rectangle.

SOLUTION : Let $P(-2, -1)$, $Q(-1, 1)$, $R(5, -2)$ and $S(4, -4)$ be the given points.



Now, $PQ = \sqrt{[-2-(-1)]^2 + [-1-1]^2} = \sqrt{(-1)^2 + (-2)^2} = \sqrt{5}$

$QR = \sqrt{[5-(-1)]^2 + [-2-1]^2} = \sqrt{(6)^2 + (-3)^2} = \sqrt{45}$

$RS = \sqrt{[4-5]^2 + [-4-(-2)]^2} = \sqrt{(-1)^2 + (-2)^2} = \sqrt{5}$

$SP = \sqrt{[4-(-2)]^2 + [-4-(-1)]^2} = \sqrt{(6)^2 + (-3)^2} = \sqrt{45}$

$\therefore \quad PQ = RS$ and $QR = SP$

Hence, the opposite sides are equal

Again, diagonal

$PR = \sqrt{[5-(-2)]^2 + [-2-(-1)]^2} = \sqrt{(7)^2 + (-1)^2} = \sqrt{50}$

$QS = \sqrt{[4-(-1)]^2 + [-4-1]^2} = \sqrt{(5)^2 + (-5)^2} = \sqrt{50}$

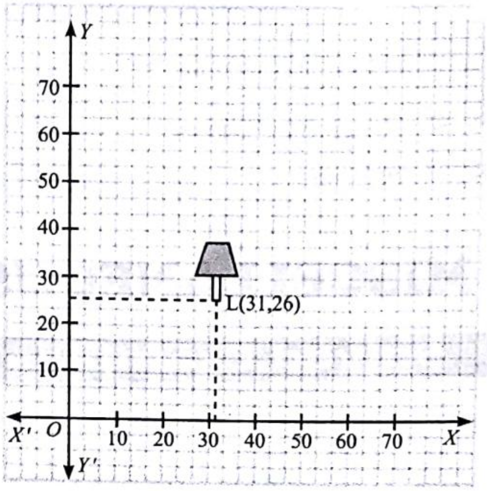
Hence the diagonals are equal.

Hence, the given point P, Q, R, S are the vertices of rectangle.

Example 3 : How will you describe the position of a table lamp on your rectangular top study table to another person?

SOLUTION : In order to locate the position of the lamp, we take a fixed point (i.e. corner of the table) for reference.

Let lamp be a point and table as plane. Draw perpendicular from the lamp point to two edges of the table and measure the distance of these perpendiculars from the fixed point (i.e., corner of the table). Let these distances are 31 cm and 26 cm as shown in the figure.

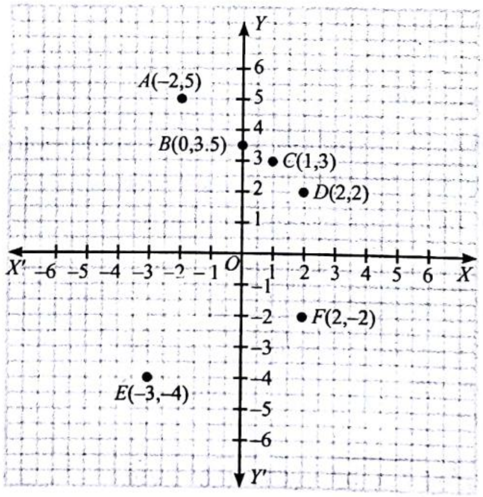


Then position of lamp can be written as L (31, 26) with respect to origin i.e., fixed point (or corner) of table.

Example 4 : **Plot the following ordered pairs of numbers (x, y) as points in the Cartesian plane. Use the scale 1 cm = 1 unit on the axes.**

x	−2	0	1	2	−3	+2
y	5	3.5	3	2	−4	−2

SOLUTION :



Example 5 : **Write the co-ordinates of a point**

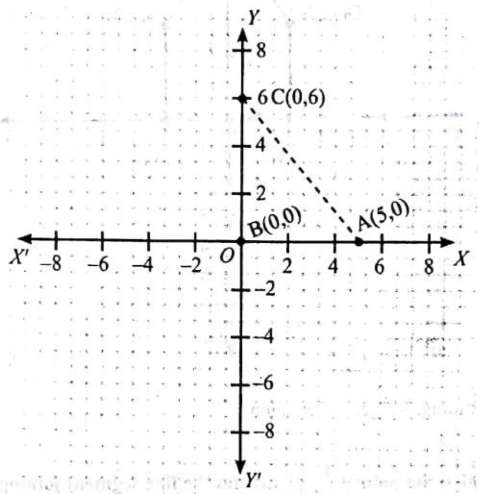
(i) **Above x-axis lying on y-axis and at a distance of 3 units.**
(ii) **Below x-axis and on y-axis at a distance of 8 units**
(iii) **Right of origin and on x-axis at a distance of 2 units.**
(iv) **Left of y-axis and on x-axis at a distance of 4 units.**

SOLUTION :

(i) The co-ordinates of the point are (0, 3)
(ii) The coordinates of the point are (0, −8)
(iii) (2, 0)
(iv) (− 4, 0)

Example 6 : **Find the area of the figure formed by joining the points (5, 0), (0, 0), (0, 6).**

SOLUTION : Let given points are $A(5, 0)$, $B(0, 0)$, and $C(0, 6)$. After plotting the points A, B, C we get (figure)



ΔABC is a right-triangle in which $\angle ABC = 90°$, $AB = 5$ units and $BC = 6$ units.

$\therefore$ Area of rt $\Delta ABC = \frac{1}{2}$ base × height $= \frac{1}{2} \times 5 \times 6$

$\therefore$ ar (ΔABC) = 15 sq. units.

Example 7 : **If the point (x, y) is equidistant from the points $(a + b, b - a)$ and $(a - b, a + b)$, prove that $bx = ay$.**

SOLUTION : Let $P(x, y)$, $Q(a + b, b - a)$ and $R(a - b, a + b)$ be the given points, therefore according to the question.

$PQ = PR$ or $PQ^2 = PR^2$

or $[x - (a + b)]^2 + [y - (b - a)]^2 = [x - (a - b)]^2 + [y - (a + b)]^2$

or $x^2 - 2(a + b)x + (a + b)^2 + y^2 - 2(b - a)y + (b - a)^2$

$= x^2 - 2(a - b)x + (a - b)^2 + y^2 - 2(a + b)y + (a + b)^2$

or $-2(a + b)x - 2(b - a)y = -2(a - b)x - 2(a + b)y$

or $ax + bx + by - ay = ax - bx + ay + by$ or $2bx = 2ay \Rightarrow bx = ay$

Example 8 : **If the distance between points $(x, 3)$ and $(5, 7)$ is 5, find the value of x.**

SOLUTION : Let $P(x, 3)$ and $Q(5, 7)$ be the given points then $PQ = 5$

$\therefore \sqrt{(x-5)^2 + (3-7)^2} = 5$

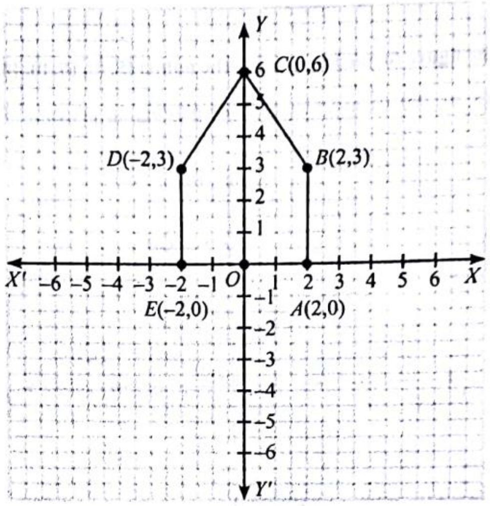
Squaring both sides

$(x - 5)^2 + (-4)^2 = 25$ or $x^2 - 10x + 25 + 16 = 25$

or $x^2 - 10x + 16 = 0$ or $(x - 2)(x - 8) = 0$ $\therefore$ $x = 2, 8$

Example 9 : **Plot the points (2, 0), (2, 3), (0, 6), (–2, 3) and (–2, 0) and join them in order. Find the type of figure thus formed**

SOLUTION : Let the given points are $A(2, 0)$, $B(2, 3)$, $C(0, 6)$, $D(-2, 3)$ and $E(-2, 0)$. After plotting and joining, we get the figure.



After plotting A, B, C, D and E and joining, we get a 'Pentagon.'

Example 10 : **Find the ratio in which the point $\left(\frac{1}{2}, 6\right)$ divides the line segment joining the points (3, 5) and (–7, 9)**

SOLUTION : Let $\left(\frac{1}{2}, 6\right)$ divide the line segment joining the points (3, 5) and (–7, 9) in the ratio of k : 1. Using section formula, the x-co-ordinater of the dividing point is given as

$$x = \frac{kx_2 + x_1}{k+1}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{k(-7)+3}{k+1} \Rightarrow \frac{1}{2} = \frac{-7k+3}{k+1} \Rightarrow k+1 = -14k+6 \Rightarrow 15k = 5 \Rightarrow k = \frac{1}{3}$$

$\therefore$ ratio is 1: 3

Example 11 : **Find the values of x for which the distance between the points $A(x, 5)$ and $B(0, -3)$ is $4\sqrt{5}$ units**

SOLUTION : Using distance formula,

$$AB = \sqrt{(0-x)^2 + (-3-5)^2} \Rightarrow 4\sqrt{5} = \sqrt{x^2 + 64}$$

Squaring both sides, we get

$80 = x^2 + 64 \Rightarrow x^2 = 16$ or $x = \pm 4$.

Example 12 : **Find the value of x such that $PQ = QR$ where co-ordinates of P, Q, R, are (6, –1), (1, 3), and (x, 8) respectively.**

SOLUTION : Since $PQ = QR \Rightarrow Q$ is mid-point of PR.

$\therefore$ using mid-point formula,

$$1 = \frac{6+x}{2} \Rightarrow 6 + x = 2 \Rightarrow x = -4.$$

EXERCISE 1

FIB Fill in the Blanks

DIRECTIONS: *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

1. Given a quadrilateral whose vertices are (1, 4), (–5, 4), (–5, –3) and (1, –3). The type of quadrilateral is ....
2. The points (–2, 0), (2, 0), (2, 2), (0, 4), (–2, 2) are joined in order. Obtained figure is ....
3. The opposite vertices of a square are (5, 4) and (–3, 2). The length of its diagonal is ....
4. If $x > 0$ and $y < 0$, then the point $(x, -y)$ lies in .... quadrant.
5. The points (0, 0), (0, 4) and (4, 0) form a/an .... triangle.
6. One end of a line is (4, 0) and the mid point is (4, 1). The coordinates of the other end is ....
7. If $P(2, -1)$, $Q(3, 4)$, $R(-2, 3)$ and $S(-3, -2)$ be four points in the plane then, $PQRS$ is a ....
8. A linear equation in two variables is always a ....
9. If (x, y) represents a point and $xy > 0$, then the point may lie in .... or .... quadrant.
10. If the point (x, y) lies in the second quadrant, then x is .... and y is ....
11. If $x - y = 2$ then point (x, y) is equidistant from (7, 1) and (..., ...)
12. Distance between (2, 3) and (4, 1) is ....
13. Point on the x-axis which is equidistant from (2, –5) and (–2, 9) is ....

T/F True / False

DIRECTIONS: *Read the following statements and write your answer as true or false.*

1. The ordinate of a point is its x-co-ordinate.
2. The point (a, b) lies on y-axis if $b = 0$.
3. The origin lies in the first quadrant.
4. If the ordinate of a point is equal to its abscissa, then the point lies either in the first quadrant or in the second quadrant.
5. The y-axis is the vertical number line.
6. Every point is located in one of the four quadrant.
7. The point (– 2, 0) lies on the y-axis.
8. The point (0, –4) lies on the y-axis.
9. The point (– 1, 2) lies below the x-axis.
10. The point (3, – 2) lies below the x-axis.
11. The point (–1, – 1) lies on the x-axis.
12. The point (4, – 5) lies in the third quadrant.
13. The mid point of a line joining points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $C\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

MTC Match the Columns

DIRECTIONS: *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column I have to be matched with statements (p, q, r, s) in column II.*

1. Column II give quadrant for points given in column I, match them correctly.

Column I	Column II
(A) 4, 4	(p) I quadrant
(B) –3, 7	(q) II quadrant
(C) 2, –3	(r) III quadrant
(D) –1, –3	(s) IV quadrant

VSAQ Very Short Answer Questions

DIRECTIONS: Give answer in one word or one sentence.

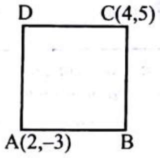
1. What are the coordinates of the origin?
2. Write the answer of each of the following questions:
   (i) What is the name of horizontal and the vertical lines drawn to determine the position of any point in the Cartesian plane?
   (ii) What is the name of each part of the plane formed by horizontal and vertical lines?
   (iii) Write the name of the point where horizontal and vertical lines intersect.
3. In which quadrant do the point (–2, –5) lies?
4. If $x > 0$ and $y < 0$, then $(x, -y)$ lies in which quadrant?
5. If $(x, -y)$ lies in 2nd quadrant then (x, y) belongs to which quadrant?
6. Find the distance between the points (–4, 5) and (2, –3).
7. Find the area of the circle whose centre is (–1, –2) and (3, 4) is a point on the circle.
8. Find the mid-point of the line segment joining the points (2, –6) and (6, –4).
9. Find the centroid of ΔABC whose vertices, are $A(2, -3)$, $B(4, 2)$ and $C(-3, -2)$.

10. Find the third vertex of ΔABC, if two of its vertices are $A(-2, 3)$, $B(4, 5)$ and its centroid is $G(1, 2)$
11. Find the point on the line joining $A(5, -4)$ $B(-3, 2)$ so that it is twice as far from A as from B.
12. Find the coordinate of the point whose abscissa is 5 and which lies on x-axis.

SAQ Short Answer Questions

DIRECTIONS: *Give answer in 2-3 sentences.*

1. Find a if the distance between the points $A(8, -7)$ and $B(-4, a)$ is 13 units.
2. Find the area of the square whose one pair of opposite vertices are $(2, -3)$ and $(4, 5)$.



3. In which quadrant or on which axis do each of the points $(-2, 4)$, $(3, -1)$, $(-1, 0)$ $(1, 2)$ and $(-3, -5)$ lie? Verify your answer by locating them on the Cartesian Plane.
4. A point lies on the x-axis at a distance of 7 units from the y-axis. What are its co-ordinates? What will be the coordinates if it lies on y-axis at a distance of –7 units from x-axis?
5. Find the co-ordinates of the mid-point of the join of points $P(2, -1)$ and $Q(-3, 4)$.
6. If $(-4, 0)$, $(0, 3)$, $(0, -3)$, are the vertices of a triangle, then find the shape of the triangle.
7. The two vertices of a triangle are $(6, 3)$ and $(-1, 7)$ and its centroid is $(1, 5)$. Find the third vertex.
8. Find the co-ordinates of the point which divides externally the line joining $(1, -3)$, and $(-3, 9)$ in the ratio $1 : 3$.
9. Find the area of the triangle whose vertices are $A(1, -2)$, $B(3, 4)$ and $C(2, 3)$.
10. Find the area of the triangle formed by the mid-points of the sides of ΔABC where $A(3, 2)$, $B(-5, 6)$ and $C = (8, 3)$.

LAQ Long Answer Questions

DIRECTIONS: *Give answer in four to five sentences.*

1. In what ratio does the point $(6, -6)$ divide the join of $(1, 4)$ and $(9, -12)$?
2. The points $(0, 0)$, $(0, 10)$, $(8, 16)$ and $(8, 6)$ are joined to form a quadrilateral. Find the type of the quadrilateral.
3. Find a point equidistant from the points $(6, 2)$, $(-1, 3)$ and $(-3, -1)$.
4. In what ratio does the point $(-3, y)$ divide the join of $(-5, 11)$ and $(4, -7)$ and hence find the value of y.
5. Find the centroid of the triangle formed by the lines $x = 0$, $y = 0$ and $x + y = 10$ as sides.
6. If $A(-5, 7)$, $B(-4, -5)$, $C(-1, -6)$ and $D(4, 5)$ are the vertices of a quadrilateral, find the area of the quadrilateral $ABCD$.
7. The area of a triangle is 5. Two of its vertices are $(2, 1)$ and $(3, -2)$. The third vertex lies on $y = x + 3$. Find the third vertex.
8. Find the co-ordinates of the points of trisection of the line segment joining $(4, -1)$ and $(-2, -3)$.
9. Show that the points $A(2, -2)$, $B(14, 10)$, $C(11, 13)$ and $D(-1, 1)$ are the vertices of a rectangle.
10. The co-ordinates of the mid-point of the line joining the points $(3p, 4)$ and $(-2, 2q)$ are $(5, p)$. Find the values of p and q.

EXERCISE 2

Multiple Choice Questions

DIRECTIONS: *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. The quadrilateral, whose vertices are (–1, 1), (0, –3), (5, 2) and (4, 6) is
(a) a square (b) a rectangle
(c) a rhombus (d) a parallelogram

2. The distance between two points (0, 3) and (–2, 0) is
(a) $\sqrt{14}$ (b) $\sqrt{15}$
(c) $\sqrt{13}$ (d) $\sqrt{5}$

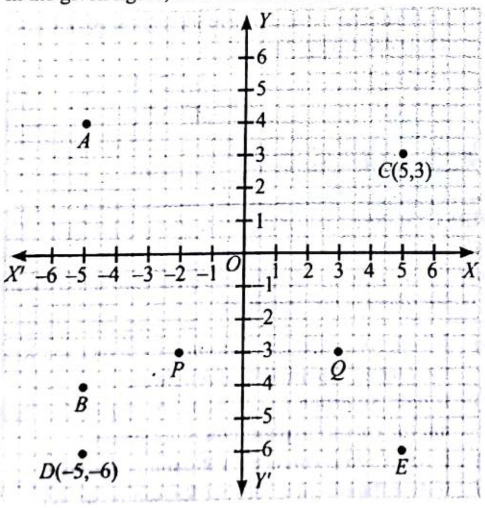
3. The distance of the point (3, 4) from y-axis is
(a) 1 (b) 4
(c) 2 (d) 3

4. The triangle, whose vertices are (2, 1), (2, –2) and (5, 2) is
(a) Right angled triangle (b) Equilateral triangle
(c) Isosceles (d) None of them

5. The distance of the point (5, –2) from x-axis is
(a) 5 (b) –2
(c) 3 (d) 4

See Graph for (Qs. 6 to 9)

6. In the given figure, find Coordinate of A



(a) (–5, 4) (b) (5, 4)
(c) (–5, – 4) (d) (4, 5)

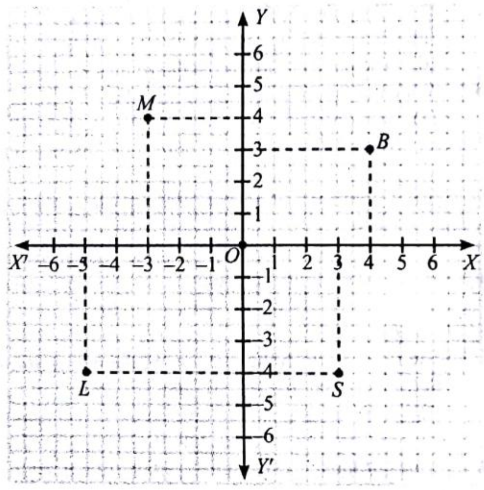
7. In the given figure, points identified by the coordinates (–2, –3)
(a) P(–2, –3) (b) Q(–2, –3)
(c) A(–2, –3) (d) C(–2, –3)

8. In the given figure, find Abscissa of C
(a) 4 (b) 5
(c) 6 (d) 7

9. In the given figure, find co-ordinate of the point E
(a) (1, 2) (b) (5, –6)
(c) (5, 6) (d) (–5, –6)

See Graph for (Qs. 10 to 13)

10. In the figure, the coordinates of B is



(a) (–3, 4) (b) (4, 3)
(c) (–5, –4) (d) (3, – 4)

11. In the figure, the co-ordinates of M are –
(a) (–3, 4) (b) (4, 3)
(c) (–5, –4) (d) (3, – 4)

12. In the figure, the co-ordinates of L are –
(a) (–3, 4) (b) (4, 3)
(c) (–5, –4) (d) (3, – 4)

13. In the figure, the co-ordinates of S are –
(a) (–3, 4) (b) (4, 3)
(c) (–5, –4) (d) (3, – 4)

14. The point (3, –4) lies in the quadrant
(a) 1st (b) II nd
(c) III rd (d) IV th

15. The abscissa of a point is –7 and the ordinate is 2, then the point is
(a) (2, –7) (b) (–7, 2)
(c) (–2, 7) (d) (7, –2)

16. The point (0, 5) lies
(a) on the x-axis (b) on the y-axis
(c) in the IInd quadrant (d) in the Ist quadrant

17. If we join the points (–2, 0), (0, 1), (2, 0) and (0, –1) then name the figure formed
(a) square (b) rectangle
(c) rhombus (d) parallelogram

MTOC More than One Correct

DIRECTIONS: *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE or MORE may be correct.*

1. Which of the following point does not lie in IIIrd quadrant?
(a) (–1, 2) (b) (6, 3)
(c) (–1, –2) (d) (–6, –3)
2. Which of the following is/are correct.
(a) The point (–2, 0) lies in the II quadrant.
(b) The point (5, 6) lies in the IV quadrant
(c) The point (–1, –4) lies in the I quadrant
(d) The point (3, –4) lies in the III quadrant
3. Which of the following is/are not false
(a) Mid point of $A(3, 4)$ and $B(5, 2)$ is $C(4, 3)$.
(b) Distance between two points T(4, 6) and Q(0, 3) is 5 units
(c) Coordinates of origin of a coordinate system is always (0, 0).
(d) The point of intersection of the axes is called the origin.
4. Which of the following point does not lie in IV quadrant.
(a) (–4, –1) (b) (–2, 6)
(c) (–1, 3) (d) (2, –4)
5. The ratio in which the line joining the points $A(4, 4)$ and $B(7, 7)$, divided by $P(-1, -1)$ is
(a) 5 : 8 (b) 3 : 8
(c) 10 : 16 (d) 9 : 24
6. Which of the following given vertices of a triangle has its centroid as O(2, 3)?
(a) $A(1, 3), B(2, 4), C(3, 2)$
(b) $P(0, 3), Q(2, 4), R(3, 2)$
(c) $X(0, 2), Y(2, 1), Z(4, 6)$
(d) none of these
7. The distance between which two points is 2 units?
(a) (–2, –3) and (–2, –4) (b) (4, –3) and (2, –3)
(c) (7, 5) and (6, 2) (d) (0, 4) and (6, 0)
8. Which of the following points is nearest to the origin?
(a) (0, 6) (b) (–6, 0)
(c) (–3, –4) (d) (4, 3)

PBQ Passage Based Questions

DIRECTIONS: *Study the given paragraph(s) and answer the following questions.*

PASSAGE-I

If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are vertices of a ΔABC, then its centroid is $G\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$ and in a parallelogram, diagonals bisect each other.

Read this passage carefully and mark the correct answer.

1. The two vertices of a triangle are (6, 3) and (–1, 7) and its centroid is (1, 5). Then the third vertex is
(a) (2, 5) (b) (–2, 5)
(c) (2, –5) (d) (–2, –5)
2. The centroid of a triangle formed by $(7, p)$, $(q, -6)$, (9, 10) is (6, 3). The $p + q$
(a) 6 (b) 5
(c) 7 (d) 8
3. If (3, 2), (6, 3), (x, y) and (6, 5) are the vertices of a parallelogram, then $x + y =$
(a) 13 (b) 14
(c) 16 (d) 15

PASSAGE-II

Let $A(x_1, y_1)$, $B(x_2, y_2)$, be two given points. If (x, y) divides AB internally in the ratio $K : 1$, then $x = \frac{Kx_2 + x_1}{K+1}$, $y = \frac{Ky_2 + y_1}{K+1}$

Read this passage carefully and mark the correct choice.

1. The ratio in which the point (6, –6) divide the join of (1, 4) and (9, –12) is
(a) 3 : 5 (b) 5 : 3
(c) 2 : 3 (d) 3 : 2
2. In what ratio is the segment joining the points (4, 6) and (–7, –1) divided by x-axis?
(a) 1 : 6 (b) 6 : 2
(c) 2 : 6 (d) 6 : 1
3. In what ratio is the segment joining the points (–3, 2) and (6, 1) divided by y-axis?
(a) 1 : 3 (b) 2 : 1
(c) 1 : 2 (d) 3 : 1

A&R Assertion & Reason

DIRECTIONS: *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.
(b) If both **Assertion** and **Reason** are correct, but Reason **is not the correct explanation** of Assertion.
(c) If **Assertion** is **correct** but **Reason** is **incorrect.**
(d) If **Assertion** is **incorrect** but **Reason** is **correct.**

1. **Assertion :** The ratio in which the segment joining the points (–3, 10) and (0, –8) is divided by (–1, 6) is 2 : 7.
Reason : If $A(x_1, y_1)$, $B(x_2, y_2)$, are two points. Then the point $C(x, y)$ such that C divides AB internally in the ratio $K : 1$ is given by $x = \frac{Kx_2 + x_1}{K+1}$, $y = \frac{Ky_2 + y_1}{K+1}$

2. **Assertion :** MN, is a straight line in which $M(3, 2)$ and $N(-3,-6)$. The distance between M and N is 10 units.

Reason : Distance between $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

3. **Assertion :** P is the point $(-5, 3)$ and Q is the point $(-5, m)$. If the length of the straight line PQ is 8 units, then the possible value of "m" is -5 or 11.

Reason : Mid-point of line joining $A(x_1, y_1)$, $B(x_2, y_2)$ is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

4. **Assertion :** If the co-ordinates of the vertices of a triangle is $A(-4, -2)$, $B(-3, -5)$ and $C(3, -2)$, then its area is 10.5 sq. units.

Reason : Area of a triangle having its vertices as $P(x_1, y_1)$, $Q(x_2, y_2)$, and $R(x_3, y_3)$ is given by

Area (ΔPQR)

$$= \frac{1}{2}|[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]|$$

5. **Assertion :** If the co-ordinates of the vertices of a triangle is $(1, 1)$, $(2, -3)$ and $(3, 4)$, then its centroid is $\left(\frac{2}{3}, 2\right)$.

Reason : Centroid of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$\left(\frac{x_1+x_2+x_3}{2}, \frac{y_1+y_2+y_3}{2}\right)$$

MMQ Multiple Matching Questions

DIRECTIONS : *Following question has four statements (A, B, C and D) given in Column I and four statements (p, q, r, ...) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in column I with entries in column II.*

1.

Column I	Column II
(A) Ist quadrant	(p) $(-1, -4)$
(B) IVth quadrant	(q) $(5, -1)$
(C) IInd quadrant	(r) $(0, 6)$
(D) IIIrd quadrant	(s) $(-3, 2)$
	(t) $(-3, -2)$
	(u) $(4, 1)$

HOTS Hots Subjective Questions

DIRECTIONS: *Answer the following questions.*

1. Determine the ratio in which the point $P(a, -2)$ divides the join of $A(-4, 3)$ and $B(2, -4)$. Also find the value of a.
2. Prove that the points (a, a), $(-a, -a)$ and $(-\sqrt{3}a, \sqrt{3}a)$ are the vertices of an equilateral triangle. Calculate the area of this triangle.
3. The point A divides the join of points $(-5, 1)$ and $(3, 5)$ in the ratio $k : 1$ and coordinates of points B and C are $(1, 5)$ and $(7, -2)$ respectively. If the area of ΔABC be 2 units, then find the value of k.
4. The line segment joining $A(2, 3)$ and $B(-3, 5)$ is extended through each end by a length equal to its original length. Find the coordinates of the new ends.
5. A and B are two points $(3, 4)$ and $(5, -2)$. Find the coordinates of a point P such that $PA = PB$ and the area of the ΔPAB is 10 sq. units
6. Find the area of triangle, given that the coordinates of the mid-points of sides are $(-1, -2)$, $(6, 1)$ and $(3, 5)$.
7. The mid-points of the sides of a triangle are $(1, 2)$, $(3, -1)$ and $(5, 0)$. Find the vertices of the triangle.

SOLUTIONS

EXERCISE – I

FILL IN THE BLANKS

1. Rectangle 2. Pentagon 3. 10
4. I 5. Right angled isosceles triangle
6. (4, 2) 7. rhombus 8. Straight line
9. The first or third quadrant 10. (– ve, + ve)
11. (3, 5) 12. $2\sqrt{2}$ 13. (–7, 0)

TRUE/FALSE

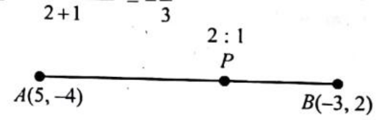
1. False 2. False 3. False 4. False
5. True 6. False 7. False 8. True
9. False 10. True 11. False 12. False
13. True

MATCH THE COLUMNS

1. (A) → (p); (B) → (q); (C) → (s); (D) → (r)

VERY SHORT ANSWER QUESTIONS

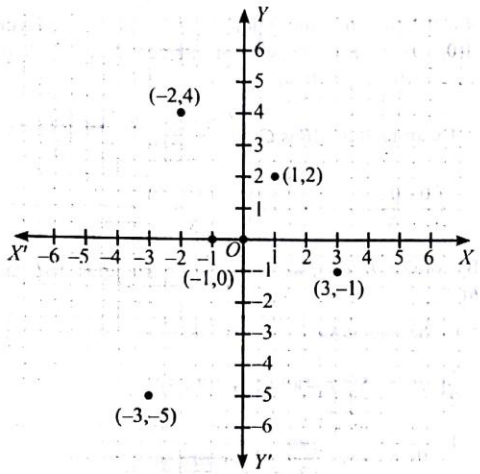
1. (0, 0)
2. (i) Horizontal line is the x-axis and vertical line is the y-axis.
   (ii) Part of the plane formed by these two lines is called quadrants.
   (iii) The origin.
3. The point (–2, –5) lies in the third quadrant.
4. $y < 0 \Rightarrow -y > 0$
   $\therefore$ The point $(x, -y)$ lies in first quadrant.
5. Given $(x, -y)$ lies in 2nd quadrant $\Rightarrow x < 0, -y < 0$.
   $\therefore$ (x, y) belongs to third quadrant.
6. Let the given points be $A(-4, 5)$ and $B(2, -3)$
   $AB = \sqrt{(2+4)^2 + (-3-5)^2}$
   $= \sqrt{36+64} = 10$ units
7. Let the centre of the circle be $A(-1, -2)$ and the point on the circumference be $B(3, 4)$
   Radius of circle $= AB$
   $= \sqrt{(3+1)^2 + (4+2)^2} = \sqrt{52}$ units
   $\therefore$ The area of the circle $= \pi r^2$
   $= \pi.(\sqrt{52})^2 = 52\pi$ sq.units.
8. Let $A(2, -6)$ and $B(6, -4)$ be the given points and M be the mid-point of AB.
   Then, $M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$
   $= \left(\dfrac{2+6}{2}, \dfrac{-6+(-4)}{2}\right) = (4, -5)$
   Hence, the mid-point of AB is (4, –5)
9. Given, $A(2, -3)$, $B(4, 2)$ and $C(-3, -2)$
   So, centroid of ΔABC
   $= \left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3}\right) = \left(\dfrac{2+4-3}{3}, \dfrac{-3+2-2}{3}\right)$
   $= (1, -1)$
   Hence, $(1, -1)$ is the centroid of ΔABC
10. Let $C(x, y)$ be the third vertex
    Given, centroid of $\Delta ABC = (1, 2)$
    $\Rightarrow \left(\dfrac{-2+4+x}{3}, \dfrac{3+5+y}{3}\right) = (1, 2)$
    $\Rightarrow \left(\dfrac{x+2}{3}, \dfrac{y+8}{3}\right) = (1, 2)$
    $\Rightarrow \dfrac{x+2}{3} = 1, \quad \dfrac{y+8}{3} = 2$
    $\Rightarrow x = 1, y = -2$
    $\therefore$ The third vertex is $(1, -2)$
11. $x = \dfrac{2(-3)+1\times 5}{2+1} = -\dfrac{1}{3}$



    $y = \dfrac{2\times 2+(1)(-4)}{2+1} = 0$
    $\therefore$ point is $\left(-\dfrac{1}{3}, 0\right)$
12. (5, 0)

SHORT ANSWER QUESTIONS

1. Given AB = 13
   $\Rightarrow \sqrt{(-4-8)^2 + (a+7)^2} = 13$
   Taking squares on both sides, we get
   $(a+7)^2 = 169 - 144 = 25$
   $a + 7 = \sqrt{25}$
   $a + 7 = \pm 5$
   $a = -2$ or -12
2. Let the given vertices be $A(2, -3)$ and $C(4, 5)$
   Length of $AC = \sqrt{(4-2)^2 + (5+3)^2} = \sqrt{68}$ units
   $\therefore$ Area of the square $= \dfrac{AC^2}{2} = \dfrac{(\sqrt{68})^2}{2} = 34$ sq.units
3. The point (– 2, 4) lies in the II quadrant.
   The point (3, – 1) lies in the IV quadrant.
   The point (– 1, 0) lies on the negative x-axis.
   The point (1, 2) lies in the I quadrant.
   The point (– 3, – 5) lies in the III quadrant.



4. (7, 0) (0, −7)

5. The coordinate of the mid-point is

$$x = \frac{2-3}{2} = -\frac{1}{2}$$

$$y = \frac{-1+4}{2} = \frac{3}{2}$$

$\therefore$ Coordinate of the mid-point is $\left(-\frac{1}{2}, \frac{3}{2}\right)$

6. If $A(-4, 0)$, $B(0, 3)$, and $C(0, -3)$, then

$$AB = \sqrt{(-4-0)^2 + (0-3)^2} = \sqrt{25} = 5 \text{ units}$$

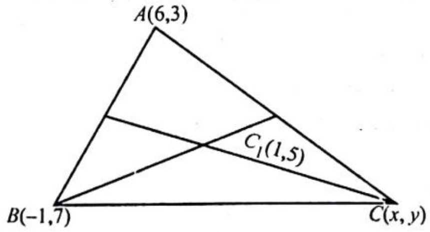
$$BC = \sqrt{(0-0)^2 + (3+3)^2} = \sqrt{36} = 6 \text{ units}$$

$$AC = \sqrt{(-4-0)^2 + (0+3)^2} = \sqrt{25} = 5 \text{ units}$$

Since, $AB = AC \neq BC$, therefore triangle is isosceles.

7. Let ABC be a triangle whose vertices are

$A = (6, 3)$; $B = (-1, 7)$; $C = (x, y)$ and centroid $C_1 = (1, 5)$



Then using the formula, for coordinates of centroid, we have

$$1 = \frac{6+(-1)+x}{3} \text{ and } 5 = \frac{3+7+y}{3}$$

$\Rightarrow x = -2$ and $y = 5$. Hence, the third vertex is $C = (-2, 5)$

8. Required point $= \left(\frac{1(-3)-3(1)}{1-3}, \frac{1(9)-3(-3)}{1-3}\right)$

$$= \left(\frac{-6}{-2}, \frac{18}{-2}\right) = (3, -9)$$

9. Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 1-3 & -2-4 \\ 3-2 & 4-3 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -2 & -6 \\ 1 & 1 \end{vmatrix}$

$$= \frac{1}{2}|-2-(-6)| = 2 \text{ sq. units}$$

10. Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 3-(-5) & 2-6 \\ -5-8 & 6-3 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 8 & -4 \\ -13 & 3 \end{vmatrix}$

$$= \frac{1}{2}|8(3)-4(13)| = \frac{1}{2}|24-52|$$

$$= \frac{1}{2}|-28| = 14 \text{ sq. units}$$

Hence, the area of triangle formed by the mid-points of the sides of $\Delta ABC = \frac{1}{4}$ (Area of ΔABC)

$$= \frac{1}{4}(14) = 3.5 \text{ sq. units}$$

LONG ANSWER QUESTIONS

1. Let the point $R(6, -6)$ divides the join of $P(1, 4)$ and $Q(9, -12)$ in the ratio $k : 1$.

By section formula, the coordinates of R are

$$P \xrightarrow{\quad k \quad} R \xrightarrow{\quad 1 \quad} Q$$

$$\left(\frac{k(9)+1(1)}{k+1}, \frac{k(-12)+1(4)}{k+1}\right), \text{ i.e., } \left(\frac{9k+1}{k+1}, \frac{-12k+4}{k+1}\right)$$

But the coordinates of R are given to be $(6, -6)$.

$$\therefore \frac{9k+1}{k+1} = 6 \text{ and } \frac{-12k+4}{k+1} = -6$$

$\Rightarrow 9k + 1 = 6k + 6$ and $-12k + 4 = -6k - 6$

$\Rightarrow 3k = 5$ and $-6k = -10$

In either case, $k = \frac{5}{3}(+ve)$

$\therefore$ R divides PQ internally in the ratio $\frac{5}{3} : 1$ i.e., $5 : 3$

2. Let the points $A(0, 0)$, $B(0, 10)$, $C(8, 16)$ and $D(8, 6)$

Here,

$$AB = \sqrt{0^2 + 10^2} = 10$$

$$BC = \sqrt{8^2 + (16-10)^2} = \sqrt{8^2 + 6^2} = 10$$

$$CD = \sqrt{0^2 + (16-6)^2} = \sqrt{10^2} = 10$$

$$DA = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$$

Again diagonal $AC = \sqrt{8^2 + 16^2} = \sqrt{320}$

and diagonal $BD = \sqrt{8^2 + (10-6)^2} = \sqrt{8^2 + 4^2} = \sqrt{80}$

So, $AB = BC = CD = DA$ (sides equal)

$AC \neq BD$ (diagonals not equal)

So, the quadrilateral is a rhombus.

3. Let $P(x, y)$ be the required point equidistant from the given points $A(6, 2)$, $B(-1, 3)$, $C(-3, -1)$

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (x-6)^2 + (y-2)^2 = (x+1)^2 + (y-3)^2$

$\Rightarrow x^2 - 12x + 36 + y^2 - 4y + 4 = x^2 + 2x + 1 + y^2 - 6y + 9$

$\Rightarrow 7x - y = 15$ ... (i)

and $PA^2 = PC^2$

$\Rightarrow (x-6)^2 + (y-2)^2 = (x+3)^2 + (y+1)^2$

$\Rightarrow x^2 - 12x + 36 + y^2 - 4y + 4 = x^2 + 6x + 9 + y^2 + 2y + 1$

$\Rightarrow 3x + y = -5$ ... (ii)

Solving (i) and (ii)

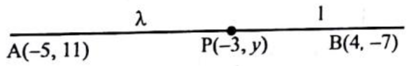
$x = 1, y = -8$

Hence, the required point is $P(1, -8)$

4. Let the required ratio be $\lambda : 1$

$$-3 = \frac{\lambda \times 4 + 1 \times -5}{\lambda + 1}$$

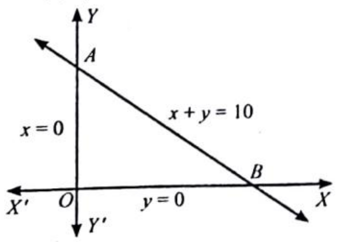
$$\Rightarrow -3\lambda - 3 = 4\lambda - 5 \Rightarrow \lambda = \frac{2}{7}$$



Hence, the required ratio $\frac{2}{7}:1$ i.e., $2:7$

$$y = \frac{ly_2 + my_1}{l+m} = \frac{2 \times -7 + 7 \times 11}{2+7} = 7$$

5. Let OAB be the triangle formed by the given lines. O is the point of intersections of $x = 0$ and $y = 0$ i.e., origin $= O(0, 0)$



A is the point of intersection of $x = 0$ and $x + y = 10$ i.e., $A(0, 10)$ and B is the point of intersection of $y = 0$ and $x + y = 10$, i.e., $B(10, 0)$

$\therefore$ Centroid of ΔOAB is $G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

$$= \left(\frac{0+0+10}{3}, \frac{0+10+0}{3}\right) = \left(\frac{10}{3}, \frac{10}{3}\right)$$

6. By joining B to D, we will get two triangles ABD and BCD.

Now the area of ΔABD

$$= \frac{1}{2}[-5(-5-5) + (-4)(5-7) + 4(7+5)]$$

$$= \frac{1}{2}(50 + 8 + 48) = \frac{106}{2} = 53 \text{ square units}$$

Also, the area of ΔBCD

$$= \frac{1}{2}[-4(-6-5) - 1(5+5) + 4(-5+6)]$$

$$= \frac{1}{2}(44 - 10 + 4) = 19 \text{ square units}$$

So, the area of quadrilateral $ABCD = 53 + 19 = 72$ square units.

7. Let the third vertex be (x_3, y_3), area of triangle

$$= \frac{1}{2}|[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]|$$

As $x_1 = 2, y_1 = 1, x_2 = 3, y_2 = -2$, Area of $\Delta = 5$

$$\Rightarrow 5 = \frac{1}{2}|2(-2-y_3) + 3(y_3 - 1) + x_3(1+2)|$$

$$\Rightarrow 10 = |3x_3 + y_3 - 7|$$

Taking positive sign,

$3x_3 + y_3 - 7 = 10 \Rightarrow 3x_3 + y_3 = 17$ .... (i)

Taking negative sign

$3x_3 + y_3 - 7 = -10 \Rightarrow 3x_3 + y_3 = -3$ .... (ii)

Given that (x_3, y_3) lies on $y = x + 3$

So, $-x_3 + y_3 = 3$ .... (iii)

Solving eq. (i) and (iii), $x_3 = \frac{7}{2}, y_3 = \frac{13}{2}$

Solving eq. (ii) and (iii), $x_3 = \frac{-3}{2}, y_3 = \frac{3}{2}$.

So, the third vertex are $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(\frac{-3}{2}, \frac{3}{2}\right)$

8. Let P and Q be points of trisection (points dividing in three equal parts) of AB, i.e. $AP = PQ = QB$.

The point P divides AB internally in the ratio 1 : 2.

Therefore, the coordinates of P will be given by :

$$x = \frac{m_2x_1 + m_1x_2}{m_1 + m_2} = \frac{2\times4 + 1(-2)}{1+2} = 2$$

$$y = \frac{m_2y_1 + m_1y_2}{m_1 + m_2} = \frac{2\times(-1) + 1(-3)}{1+2} = \frac{-5}{3}$$

Therefore, the co-ordinate of P are $\left(2, -\frac{5}{3}\right)$

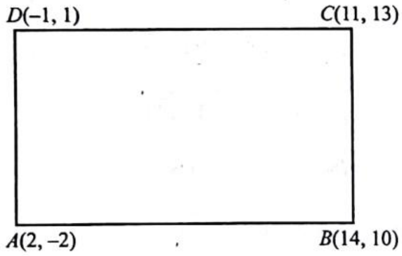
For the co-ordinate of Q, $m_2 = 2$ and $m_1 = 1$,

$$x = \frac{m_2x_1 + m_1x_2}{m_1 + m_2} = \frac{1(4) + 2(-2)}{1+2} = 0$$

$$y = \frac{m_2y_1 + m_1y_2}{m_1 + m_2} = \frac{1(-1) + 2(-3)}{1+2} = -\frac{7}{3}$$

$\therefore$ Co-ordinate of $Q = \left(0, -\frac{7}{3}\right)$

9.



$$AB = \sqrt{(14-2)^2 + (10+2)^2} = 12\sqrt{2} \text{ units}$$

$$BC = \sqrt{(11-14)^2 + (13-10)^2} = 3\sqrt{2} \text{ units}$$

$$CD = \sqrt{(-1-11)^2 + (1-13)^2} = 12\sqrt{2} \text{ units}$$

$$AD = \sqrt{(-1-2)^2 + (1+2)^2} = 3\sqrt{2} \text{ units}$$

$\Rightarrow$ $AB = CD$ and $BC = AD$

$\therefore$ $ABCD$ is a || gm

Now, $AC = \sqrt{(11-2)^2 + (13+2)^2} = \sqrt{306}$

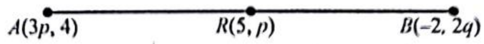
$\Rightarrow$ $AC^2 = 306$ units, $AB^2 = 288$ units, $BC^2 = 18$ units

$AB^2 + BC^2 = 306$ units

$\Rightarrow$ $AC^2 = AB^2 + BC^2$ $\therefore$ $\angle ABC = 90°$

$\Rightarrow$ $ABCD$ is a rectangle.

10. $R(5, p)$ is the mid-point of the line segment joining the points $A(3p, 4)$ and $B(-2, 2q)$



$\therefore$ $\left(\frac{3p-2}{2}, \frac{4+2q}{2}\right) = (5, p)$

$\Rightarrow$ $\frac{3p-2}{2} = 5$ and $\frac{4+2q}{2} = p$

$\Rightarrow$ $3p = 10 + 2$ and $4 + 2q = 2p$ .... (1)

$\Rightarrow$ $3p = 12$ $\Rightarrow$ $p = 4$

Substituting $p = 4$ in $4 + 2q = 2p$, we get

$4 + 2q = 8$ $\Rightarrow$ $2q = 4$ $\Rightarrow$ $q = 2$

$\therefore$ $p = 4$ and $q = 2$

EXERCISE – 2

MULTIPLE CHOICE QUESTIONS

1. (d) **2.** (c) **3.** (d) **4.** (d) **5.** (b)

6. (a) **7.** (a) **8.** (b) **9.** (b) **10.** (b)

11. (a) **12.** (c) **13.** (d)

14. (d) As in IVth quadrant x-coordinate is positive and y-coordinate is negative.

15. (b) As for point (x, y), x represents abscissa and y represents ordinate

16. (b) **17.** (c)

MORE THAN ONE CORRECT

1. (a, b) **2.** (a, d) **3.** (a, b, c, d)

4. (b, c, d) **5.** (a, c) **6.** (a, c)

7. (b, c) **8.** (c, d)

PASSAGE BASED QUESTIONS

PASSAGE-I

1. (b) Let the third vertex be (α, β)

$\therefore$ $\frac{\alpha+6-1}{3} = 1, \frac{\beta+3+7}{3} = 5$ $\Rightarrow$ $\alpha = -2, \beta = 5$

$\therefore$ required point is $(-2, 5)$

2. (c) By the given condition $\frac{7+q+9}{3} = 6$ and

$\frac{p-6+10}{3} = 3$

$\Rightarrow$ $q = 2$ and $p = 5$ $\therefore$ $p + q = 5 + 2 = 7$

3. (d) Since (3, 2), (6, 3), (x, y) and (6, 5) are the vertices of a parallelogram.

$\therefore$ $\frac{3+x}{2} = \frac{6+6}{2}$ and $\frac{2+y}{2} = \frac{3+5}{2}$

$\Rightarrow$ $x = 9$ and $y = 6$

$\therefore$ $x + y = 15$

PASSAGE-II

1. (b) 2. (d) 3. (c)

ASSERTION & REASON

1. (a) Reason (R) is true. (Standard Result)

For Assertion (A).

Let (–1, 6) divide the join of (–3, 10) and (6, –8) in the ratio $K : 1$

$\therefore$ $-1 = \frac{6K-3}{K+1}$ and $6 = \frac{-8K+10}{K+1}$

$\Rightarrow$ $-K - 1 = 6K - 3$ and $6K + 6 = -8K + 10$

$\Rightarrow$ $7K = 2$ and $14K = 4$ $\Rightarrow$ $K = \frac{2}{7}$ and $K = \frac{4}{14} = \frac{2}{7}$

Hence required ratio is 2 : 7 $\therefore$ (a) is true. Since (R) gives (A)

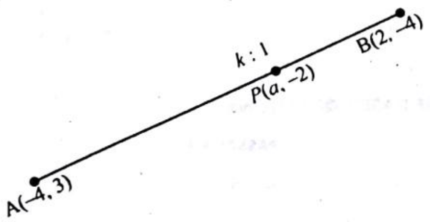
2. **(a)** 3. **(b)** 4. **(a)** 5. **(d)**

MULTIPLE MATCHING QUESTIONS

1. (A) $\to$ (r, u); (B) $\to$ (p, t); (C) $\to$ (s); (D) $\to$ (q)

HOTS SUBJECTIVE QUESTIONS

1. Let ratio be $k : 1$



$\therefore$ Co-ordinates of P are

$$\left(\frac{2k-4}{k+1}, \frac{-4k+3}{k+1}\right) = (a, -2)$$

$$\frac{2k-4}{k+1} = a \qquad \text{.....I}$$

and $$\frac{-4k+3}{k+1} = -2 \qquad \text{.....II}$$

$\Rightarrow$ $-4k + 3 = -2k - 2$ $\Rightarrow$ $2k = 5$ $\Rightarrow$ $k = \frac{5}{2}$

$\therefore$ ratio is $\frac{5}{2} : 1$ i.e., 5 : 2

By Putting value of k in I, we get $a = \frac{5-4}{\frac{5}{2}+1} = \frac{2}{7}$

2. Let A (a, a), B $(-a, -a)$ and C $(-\sqrt{3}a, \sqrt{3}a)$

Then,

$$AB = \sqrt{(-a-a)^2 + (-a-a)^2} = \sqrt{(-2a)^2 + (-2a)^2}$$

$$= \sqrt{8a^2} = 2\sqrt{2}\,a \text{ units}$$

$$BC = \sqrt{(-\sqrt{3}a + a)^2 + (\sqrt{3}a + a)^2}$$

$$= \sqrt{a^2(1-\sqrt{3})^2 + a^2(1+\sqrt{3})^2}$$

$$= \sqrt{a^2\{(1-\sqrt{3})^2 + (1+\sqrt{3})^2\}}$$

$$= \sqrt{8a^2} = 2\sqrt{2}\,a \text{ units}$$

$$AC = \sqrt{(-\sqrt{3}a - a)^2 + (\sqrt{3}a - a)^2}$$

$$= \sqrt{a^2\{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2\}}$$

$$= \sqrt{8a^2} = 2\sqrt{2}\,a \text{ units}$$

$\therefore$ $AB = BC = AC = 2\sqrt{2}\,a$

Hence, Δ ABC is an equilateral triangle whose side is $2\sqrt{2}\,a$ units.

$\therefore$ area of Δ $ABC = \left[\frac{\sqrt{3}}{4} \times (\text{side})^2\right] = \left[\frac{\sqrt{3}}{4} \times (2\sqrt{2}\,a)^2\right]$

$$= \frac{\sqrt{3}}{4} \times 8a^2 = 2\sqrt{3}\,a^2 \text{ sq. units.}$$

3. Co-ordinates of point A that divides the join of these two points is $A \equiv \left(\frac{3k-5}{k+1}, \frac{5k+1}{k+1}\right)$, Co-ordinates of B, and C are (1, 5) and (7, –2) respectively. Area of $\Delta\, ABC = 2$ units

$$\Rightarrow \quad \frac{1}{2}\left[\frac{3k-5}{k+1}(5+2)+1\left(-2-\frac{5k+1}{k+1}\right)+7\left(\frac{5k+1}{k+1}-5\right)\right]$$

$= \pm 2$

We know that the area of the triangle with vertices having co-ordinates (x_1, y_1) (x_2, y_2) and (x_3, y_3) is given by

$$A = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$$

$$\Rightarrow \frac{1}{2(k+1)}[21k-35-2k-2-5k-1+35k+7-35k-35]$$

$= \pm 2$

$\Rightarrow \quad 14k - 66 = \pm 4\,(k+1) \;\Rightarrow\; k = 7$ or 31/9.

4. Let the line segment joining $A(2, 3)$ and $B(-3, 5)$ is extended through each end by a length equal to its original length AB as shown in fig.

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be the new end points

Now $PA = AB = BQ$

$\therefore$ A is the mid-point of PB.

$\Rightarrow \quad 2 = \frac{x_1+(-3)}{2}$ and $3 = \frac{y_1+5}{2}$

$\Rightarrow \quad 4 = x_1 - 3$ and $6 = y_1 + 5$

Which gives $x_1 = 7$ and $y_1 = 1$

Thus, the coordinate of the point P are (7, 1),

Again B is the mid-point of AQ

$\therefore \frac{2+x_2}{2} = -3$ and $\frac{3+y_2}{2} = 5$

$\Rightarrow \quad 2 + x_2 = -6$ and $3 + y_2 = 10 \;\Rightarrow\; x_2 = -8$ and $y_2 = 7$

$\therefore$ the coordiantes of Q are (–8, 7).

Hence the coordinates of the new ends are (7, 1) and (–8, 7).

5. Co-ordinates of A and B are (3, 4) and (5, –2) respectively.

Let the co-ordinates of P be (x, y),

$PA = PB \Rightarrow PA^2 = PB^2$

$\therefore \quad (x-3)^2 + (y-4)^2 = (x-5)^2 + (y+2)^2$

$\Rightarrow \quad x^2 - 6x + 9 + y^2 - 8y + 16 = x^2 - 10x + 25 + y^2 + 4y + 4$

$\Rightarrow \quad 4x - 12y - 4 = 0 \;\Rightarrow\; x - 3y - 1 = 0$ ...(i)

Also area of $\Delta PAB = 10$

$\Rightarrow \quad \frac{1}{2}[x\{4-(-2)\}+3(-2-y)+5(y-4)] = \pm 10$

$\Rightarrow \quad [6x - 6 - 3y + 5y - 20] = \pm 20$

$\Rightarrow \quad 6x + 2y - 26 = \pm 20$

Taking +ve sign, we get $3x + y - 23 = 0$ ... (ii)

Taking –ve sign, we get $3x + y - 3 = 0$ ... (iii)

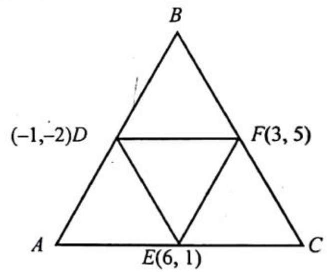
Solving (i) and (ii), we get $x = 7$ and $y = 2$

From (i) and (iii), we get $x = 1$ and $y = 0$

Hence, the coordinates of P are (7, 2) or (1, 0).

6. Let ABC be the given triangle and let $D(-1, -2)$, $E(6, 1)$ and $F(3, 5)$ be the mid-points of the sides AB, BC, and AC respectively.

Area of ΔDEF



$= \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

Where (x_1, y_1), (x_2, y_2) and (x_3, y_3) are coordinates of D, E and F respectively.

$\Rightarrow$ Area of ΔDEF

$= \frac{1}{2}[(-1)(1-5)+6\{5-(-2)\}+3(-2-1)]$

$= \frac{1}{2}(4+42-9) = \frac{1}{2} \times 37 = 18.5$ sq.units

$\therefore$ Area of $\Delta ABC = 4$(Area of ΔDEF)

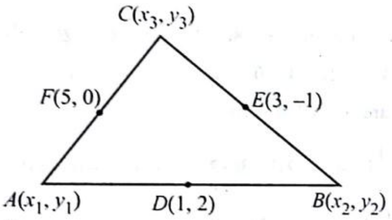
Hence area of $\Delta ABC = 4\,(18.5) = 74$ sq. units.

7. Let the vertices be (x_1, y_1), (x_2, y_2) and (x_3, y_3)

Let D be the mid point of AB

$\frac{x_1+x_2}{2} = 1 \Rightarrow x_1 + x_2 = 2$ .... (i)

$\frac{y_1+y_2}{2} = 2 \Rightarrow y_1 + y_2 = 4$ .... (ii)



Similarly, we get $x_2 + x_3 = 6$ .... (iii)

$y_2 + y_3 = -2$ .... (iv)

$x_1 + x_3 = 10$ .... (v)

$y_1 + y_3 = 0$ .... (vi)

Adding (i), (iii), (v) and (ii), (iv), (vi), we get

$2(x_1 + x_2 + x_3) = 2 + 6 + 10 \Rightarrow x_1 + x_2 + x_3 = 9$ .... (vii)

$2(y_1 + y_2 + y_3) = 4 - 2 + 0 = 2 \Rightarrow y_1 + y_2 + y_3 = 1$.... (viii)

From (i) and (vii), $x_3 = 9 - 2 = 7$

From (ii) and (viii), $y_3 = 1 - 4 = -3$

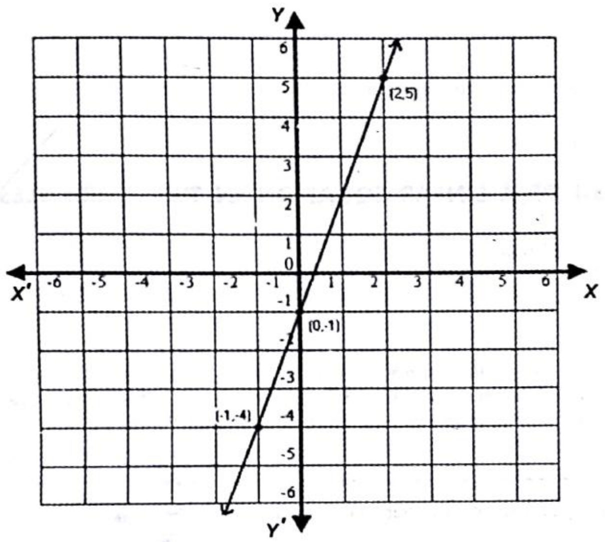
From (iii) and (vii), $x_1 = 9 - 6 = 3$

From (iv) and (viii), $y_1 = 1 + 2 = 3$

From (v) and (vii), $x_2 = 9 - 10 = -1$

From (vi) and (viii), $y_2 = 1 - 0 = 1$

The vertices of the triangle are $A(3, 3)$, $B(-1, 1)$ and $C(7, -3)$.



4

CHAPTER

Linear Equation In Two Variables

INTRODUCTION

In earlier classes, you have studied linear equation in one variable. You know that standard form of a linear equation in one variable is $ax + b = 0$, $(a \neq 0)$. Its solution $x = -\frac{b}{a}$, is unique. In this chapter, you will study the linear equation in two variables like $5x - y + \frac{7}{2} = 0$, $x - 2y + 1 = 0$, etc. You will see that a linear equation in two variables has infinite many solutions. You will also study, how to draw the graph of a linear equation in two variables in a graph paper.

LINEAR EQUATION IN TWO VARIABLES

An equation of the form $ax + by + c = 0$ where a, b, c are real numbers ($a, b \neq 0$) and x, y are variables, is called a linear equation in two variables.

Here a is called coefficient of x, b is called coefficient of y and c is called constant term.

eg. $6x + 2y + 5 = 0$, $5x - 2y + 3 = 0$ etc.

In the linear equation in two variables, index of each of the variable is 1.

TO FIND THE SOLUTION(S) [OR ROOT(S)] OF A LINEAR EQUATION IN TWO VARIABLES ALGEBRICALLY

Standard form of a linear equation in two variables : $ax + by + c = 0$ ($a, b \neq 0$)

Express any one variable y in terms of other variable say x as

$$y = -\frac{ax+c}{b} \qquad \ldots(i)$$

Now by putting any real value of x in above equation (i), you will get the corresponding real value of y. The value of x and its corresponding value of y is one set of solution of the given linear equation in two variables. In the same way by putting another real value of x in the above equation (i), you will get another value of y. Thus you will get another set of solution. In this way you will get infinite sets of solution.

Note : A solution of a linear equation in two variables can be find out only after taking any real value of any one of the two variables.

Illustration 1 : Find any three sets of solution of $-x + 2y = 4$

SOLUTION : $-x + 2y = 4$

$\Rightarrow \quad x = 2y - 4 \qquad \ldots(i)$

Put $y = 0$ in equation (i), then $x = 2 \times 0 - 4 = -4$

Hence, one set of solution : $x = -4, y = 0$

Put $y = 1$ in equation (i), then $x = 2 \times 1 - 4 = 2 - 4 = -2$

Hence, second set of solution : $x = -2, y = 1$

Put $y = 2$ in equation (i), then $x = 2 \times 2 - 4 = 0$

Hence, third set of solution : $x = 0, y = 2$

Thus, three sets of solution are $(-4, 0)$, $(-2, 1)$ and $(0, 2)$

GRAPH OF A LINEAR EQUATION IN TWO VARIABLES

To draw the graph of a linear equation . $ax + by + c = 0$; $a, b \neq 0$; we may follow the following steps:

(i) Obtain the linear equation : $ax + by + c = 0$

(ii) Express any one variable y in terms of other variable x as $y = -\frac{ax+c}{b}$

(iii) Put any three values x_1, x_2, x_3 for x one by one in the equation obtained in step (ii) and calculate the corresponding values y_1, y_2, y_3 of y. Thus you obtained three solution sets, (x_1, y_1) (x_2, y_2) and (x_3, y_3)

(iv) Plot points (x_1, y_1), (x_2, y_2) and (x_3, y_3) on a graph paper.

(v) Join the points marked in step (iv).

You will see that the three points lie on a straight line. The straight line passing through these three points is the graph of the linear equation in two variables, because coordinates of each point on the line satisfy the linear equation in two variables and hence coordinates of each point on the line is a set of solution of the equation.

The graph of a linear equation in two variable is always a straight line.

Note : (i) To draw the graph of a linear equation in two variables, you can take only two points also.
(ii) If possible take such integer as value of x that the corresponding values of y are also integer.

Illustration 2 : Draw the graph of the equation $y - x = 2$.

SOLUTION : We have,

$y - x = 2$

$\Rightarrow \quad y = x + 2$

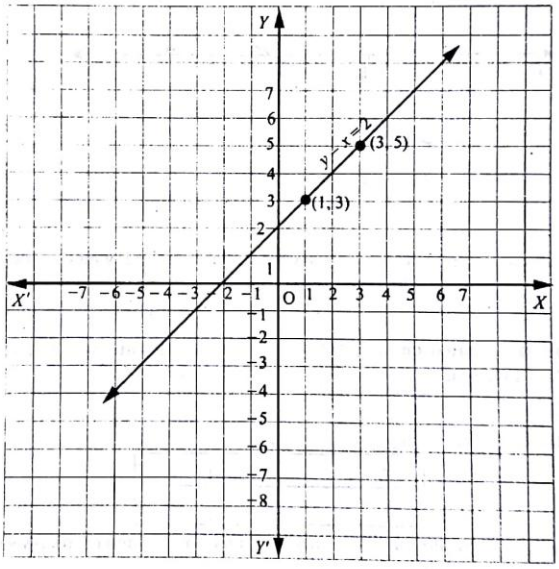
When $x = 1$, we have $y = 1 + 2 = 3$

When $x = 3$, we have $y = 3 + 2 = 5$

Thus, we have the following table exhibiting the abscissae and ordinates of points on the line represented by the given equation.

x	1	3
y	3	5

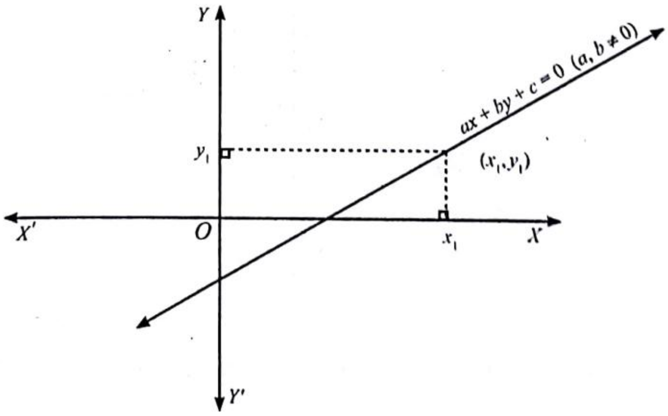
Plotting the points (1, 3) and (3, 5) on the graph paper and drawing a line joining them, we obtain the graph of the line represented by the given equation as shown in the Figure.



TO FIND THE SOLUTION OF A LINEAR EQUATION IN TWO VARIABLES GRAPHICALLY

The solution can be find out by the following two ways.

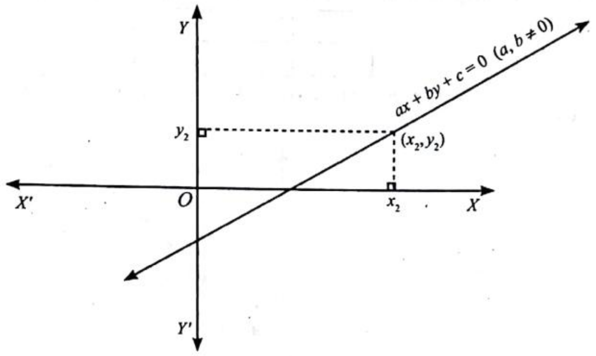
(I) BY TAKING ANY REAL VALUE 'x_1' OF ANY ONE VARIABLE, SAY x WHOSE VALUES ARE TAKEN ALONG x-AXIS:



To find the solution first draw the graph of the given equation, which will be a straight line. Then draw a line perpendicular to x-axis at ($x = x_1$), which intersects the line, which is the graph of the given equation at a point, from this point draw a line parallel to x-axis which intersects y-axis at $y = y_1$. Then a solution corresponding to the value x_1 of x is $x = x_1$, $y = y_1$. Similarly you can find any number of solutions by taking different values of x.

(II) BY TAKING ANY REAL VALUE y_2 OF ANY ONE VARIABLE, SAY y WHOSE VALUES ARE TAKEN ALONG y-AXIS:

To find the solution, first draw the graph of the given equation, which will be a straight line.



Then draw a line perpendicular to y-axis at $y = y_2$, which intersects the line, which is the graph of the given equation at a point. From this point draw a line parallel to y-axis which intersects the x-axis at a $x = x_2$. Then a solution corresponding to the value y_2 of y is $x = x_2$, $y = y_2$. Similarly you can find any number of solutions by taking different values of y.

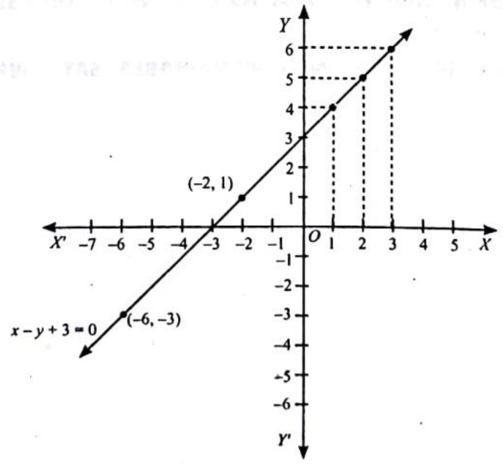
Illustration 3 : Draw the graph of the equation $x - y + 3 = 0$. Use it to find some solution of the equations and check from the graph that $x = 0$ and $y = 3$ is a solution.

SOLUTION: The given equation is $x - y + 3 = 0$

To draw the graph we use the table of corresponding values of x and y.

x	–2	–6	3
y	1	–3	6

We have drawn the graph of $x - y + 3 = 0$ by plotting the points (– 2, 1), (– 6, – 3) and (3, 6). As shown in the figure some of the other solutions of $x - y + 3 = 0$ are : $x = 1, y = 4; x = -1, y = 2$



The point $x = 0$ and $y = 3$ is on the graph. Hence, $x = 0, y = 3$ is a solution of the equation.

MISCELLANEOUS

Solved Examples

Example 1 : **The cost of a book is twice the cost of a fountain pen. Write a linear equation in two variables to represent this statement.**

SOLUTION : Let the cost of the book = ₹ x and cost of the fountain pen = ₹ y.

Since cost of a book = Twice the cost of pen

$\therefore\ x = 2y \Rightarrow x - 2y = 0$

Example 2 : **Express y in terms of x, given that $2y - 4x = 7$. Check whether $(-1, -1)$ is a solution of the line.**

SOLUTION : Given equation is

$$2y - 4x = 7$$

$$\Rightarrow\ 2y = 7 + 4x$$

$$\Rightarrow\ y = \frac{7+4x}{2}$$

Now substituting $x = -1, y = -1$ in the equation, we get

$$-1 = \frac{7+4(-1)}{2} \Rightarrow -1 = \frac{7-4}{2} \Rightarrow -1 = \frac{3}{2} \text{ which is not true}$$

$\therefore$ LHS $\neq$ RHS

Therefore point $(-1, -1)$ does not lie on the line $2y - 4x = 7$

Example 3 : **Draw the graph of the equation $2x - y + 3 = 0$. Using the graph, find the value of y, when $x = -2$.**

SOLUTION : $2x - y + 3 = 0 \Rightarrow y = 2x + 3$ ...(i)

When $x = 0$, then $y = 2 \times 0 + 3 = 3$

When $x = 1$, then $y = 2 \times 1 + 3 = 5$

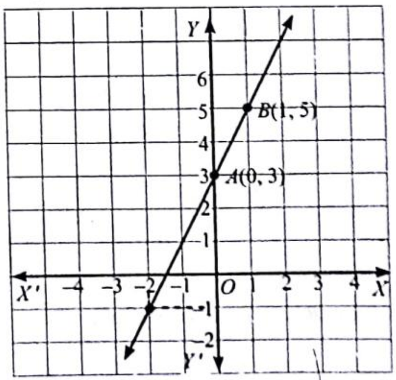
Thus, we have the following table :

x	0	1
y	3	5

Now, plot the points A (0, 3) and B (1, 5) on a graph paper

Join AB and extend it in both the directions.

Then, the line AB is the required graph of $2x - y + 3 = 0$



From the graph it is clear when $x = -2$, then $y = -1$

Example 4 : **Draw the graph of the equation $2x + 3y = 11$. From your graph, find the value of y, when $x = -2$.**

SOLUTION : $2x + 3y = 11 \Rightarrow y = \frac{(11-2x)}{2}$ ...(i)

When $x = 1$, then $y = \frac{(11-2\times1)}{3} = \frac{9}{3} = 3$

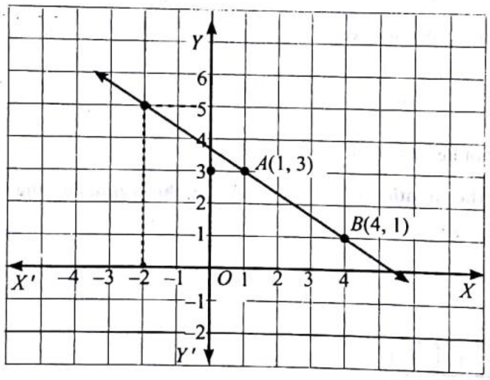
When $x = 4$, then $y = \frac{(11-2\times4)}{3} = \frac{3}{3} = 1$

Thus, we have the following table :

x	1	4
y	3	1

Now, plot the points $A(1, 3)$ and $B(4, 1)$ on a graph paper.

Join AB and extend it in both the directions. Line AB is the required graph of $2x + 3y = 11$



From the graph, it is clear that when $x = -2$, then $y = 5$.

Example 5 : **Draw the graph of the equation $y = 2x$. From your graph, find the value of x when $y = -2$.**

SOLUTION : Given equation is , $y = 2x$ ...(i)

When $x = 1$, then $y = 2 \times 1 = 2$.

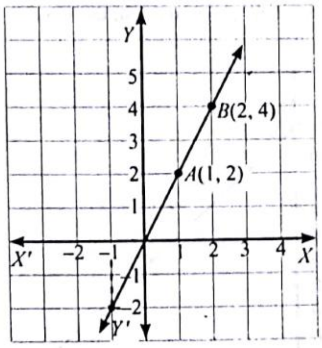
When $x = 2$, then $y = 2 \times 2 = 4$.

Thus, we have the following table :

x	1	2
y	2	4

Now, plot the points $A(1, 2)$ and $B(2, 4)$ on a graph paper.

Join AB and extend it in both the directions. Line AB is the required graph of $y = 2x$.



From the graph, it is clear that when $y = -2$, then $x = -1$.

Example 6 : Find four different solutions of the equation $x + 2y = 6$.

SOLUTION : When $x = 2$, then $2 + 2y = 6 \Rightarrow 2y = 4, \Rightarrow y = 2$

Now, let us choose $x = 0$. With this value of x, the given equation reduces to $2y = 6$ which has the unique solution $y = 3$. So $x = 0$, $y = 3$ is also a solution of $x + 2y = 6$. Similarly, taking $y = 0$, the given equation reduces to $x = 6$. So, $x = 6$, $y = 0$ is a solution of $x + 2y = 6$ as well. Finally let us take $y = 1$. The given equation now reduces to $x + 2 = 6$, whose solution is given by $x = 4$. Therefore, $x = 4$, $y = 1$ is also a solution of the given equation. So four of the infinitely many solutions of the given equation are :

$$(2, 2), (0, 3), (6, 0) \text{ and } (4, 1).$$

Example 7 : Given the point (1, 2), find the equation of a line on which it lies. How many such equations are there?

SOLUTION : Here (1, 2) is a solution of a linear equation. So, you are looking for any line passing through the point (1, 2). One example of such a linear equation is $x + y = 3$. Others are $y - x = 1$, $y = 2x$, since they are also satisfied by the coordinates of the point (1, 2). In fact, there are infinitely many linear equations which are satisfied by the coordinates of the point (1, 2).

Example 8 : Yamini and Fatima, two students of Class IX of a school, together contributed ₹ 100 towards the Prime Minister's Relief Fund to help the earthquake victims. Write a linear equation which satisfies this data. (You may take their contributions as ₹ x and ₹ y.) Draw the graph of the same.

SOLUTION : Let the contributions of Yamini and Fatima be ₹ x and ₹ y respectively.

Then according to the question

$x + y = 100$

This is the required linear equation which satisfies the given data

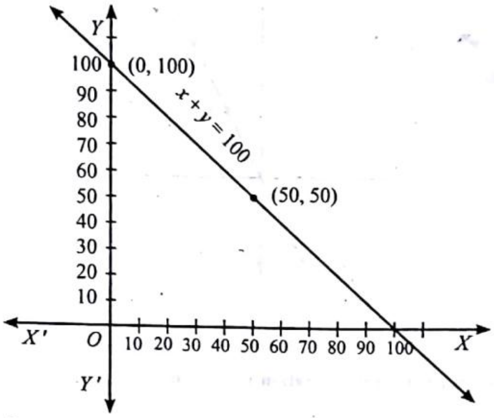
Now, we consider certain value of x and find corresponding values of y.

For $x = 0, y = 100$

$x = 50, y = 50$

x	0	50
y	100	50

We plot the points (0, 100) and (50, 50) on the graph paper and join the same to get the line which is the graph of the equation $x+y=100$.



Example 9 : In-countries like USA and Canada, temperature is measured in Fahrenheit, where as in countries like India, it is measured in Celsius. Here is a linear equation that converts Fahrenheit to Celsius.

$$F=\left(\frac{9}{5}\right)C+32$$

(i) Draw the graph of the linear equation above using Celsius for x-axis and Fahrenheit for y-axis.

(ii) If the temperature is 30°C, what is the temperature in Fahrenheit?

(iii) If the temperature is 95°F, what is the temperature in Celsius?

(iv) If the temperature is 0°C, what is the temperature in Fahrenheit and if the temperature is 0°F, what is the temperature in Celsius?

(v) Is there a temperature which is numerically the same in both Fahrenheit and Celsius? If yes, find it.

SOLUTION : Given linear equation is $F=\left(\frac{9}{5}\right)C+32$

(i) Let the value of C is represented on x-axis and the value of F is represented on y-axis.

So, we can consider $C=x$ and $F=y$

$\therefore$ Given equation becomes $y=\frac{9}{5}x+32$

Now we consider certain values of x and find the corresponding values of y.

For $x=5$, $y=\frac{9}{5}\times(5)+32=41$

For $x=10$, $y=\frac{9}{5}\times(10)+32=50$

For $x=15$, $y=\frac{9}{5}\times(15)+32=59$

Now we plot the points $A(0, 32)$, $B(5, 41)$, $C(10, 50)$ and $D(15, 59)$ on the graph paper.

(ii) For $C = 30°$, $F = \frac{9}{5} \times 30° + 32° = 86°$

Required temperature = 86° Farenheit.

(iii) For $F = 95°\ F$, we have $95 = \frac{9}{5} \times C + 32°$

$\Rightarrow\ 95° - 32° = \frac{9}{5}\ C \Rightarrow \frac{9}{5}\ C = 63° \Rightarrow C = 35°$ Farenheit

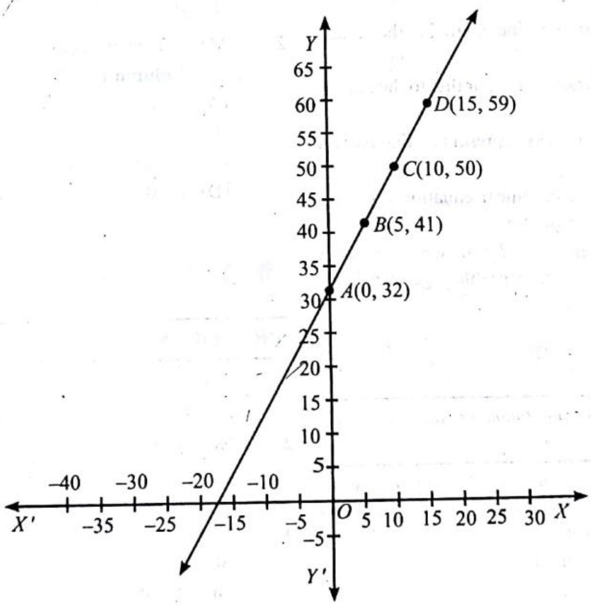
(iv) For C = 0, F = $\frac{9}{5} \times 0 + 32° = 32°$ and For F = 0° Farenheit we have $0 = \frac{9}{5} \times C + 32° \Rightarrow C = \frac{-32° \times 5}{9} = \frac{-160°}{9}$

(v) Let the temperature be $x°$ numerically. -i.e. $x°$ F = $x°$ C

$\therefore$ F = $\left(\frac{9}{5}\right)C + 32 \Rightarrow x = \frac{9}{5}x + 32$

$\Rightarrow\ -32 = \frac{9x - 5x}{5} \quad \Rightarrow x = \frac{-32 \times 5}{4} = -40°$

Hence, $-40°$F = $-40°$C.



EXERCISE 1

Fill in the Blanks

DIRECTIONS: *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

1. A linear equation in two variables has .... solutions.
2. The graph of every linear equation in two variables is a ....
3. $x = 0$ is the equation of the ....axis and $y = 0$ is the equation of the .... axis.
4. The system of equations $a + b = 3$ and $3a + 3b = 9$ is ____. (consistent/inconsistent)
5. The equations $px + qy + r = 0$ and $kpx + kqy + kr = 0$ are ______. (dependent/inconsistent)
6. The graph of $x = a$ is a straight line parallel to the .... axis.
7. The graph of $y = a$ is a straight line parallel to the .... axis
8. An equation of the type $y = mx$ represents a line passing through the ....
9. Every .... of the linear equation is a point on the graph of the linear equation.
10. An equation of the form $cy + d = 0$, where c, d are real numbers and $c \neq 0$, in the variable y geometrically represents...

True/False

DIRECTIONS: *Read the following statements and write your answer as true or false.*

1. An equation of the form $ax + by + c = 0$, where a, b and c are real numbers, such that a and b are both zero, is called a linear equation in two variables.
2. $y = 3x + 5$ has a unique solution.
3. (0, 2) is a solution for $x - 2y = 4$
4. $\left(2, \frac{1}{2}\right)$ is a solution for $x = 4y$
5. The solution of a linear equation is not affected when the same number is added to both the sides of the equation.
6. All the points (2, 0), (–3, 0), (4, 2) and (0, 5) lie on the x-axis.
7. The line parallel to the y-axis at a distance 4 units to the left of y-axis is given by the equation $x = -a$.
8. The graph of the equation $y = mx + c$ passes through the origin.

Match the Columns

DIRECTIONS: *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column I have to be matched with statements (p, q, r, s) in column II.*

1. Column II give solution for equation in column I match them correctly –

Column I	Column II
(A) $2x + y = 7$	(p) (0, 9)
(B) $x - 2y = 4$	(q) (1, 5)
(C) $x - 4y = 0$	(r) (0, 0)
(D) $px + y = 9$	(s) (4, 0)

2. Match them correctly –

Column I	Column II
(A) $x = a$	(p) x-axis
(B) $y = a$	(q) st. line parallel to x-axis
(C) $y = mx$	(r) st. line parallel to y-axis
(D) $y = 0$	(s) st. line passing through origin.

Very Short Answer Questions

DIRECTIONS: *Give answer in one word or one sentence.*

1. Express the linear equation $y - 2 = 0$ in the form $ax + by + c = 0$ and indicate the values of a, b and c
2. Determine the whether the $x = 2$, $y = -1$ is a solution of equation $3x + 5y - 2 = 0$.
3. Find two solutions for $3y + 4 = 0$
4. Find the value of k, if $x = 2$, $y = 1$ is a solution of the equation $2x + 3y = k$.
5. The cost of a notebook is twice the cost of a pen. Write a linear equation in two variables to represent this statement.
6. Give the equations of two lines passing through (2, 14). How many more such lines are there, and why?
7. Find four different solutions of the equation $x + 2y = 6$.
8. The sum of the ages of Mrs. Meenu Goyal and Dr. H.K. Goyal is 80. Write a linear equation in two variables to represent the statement.
9. Show that $x = 2$, $y = 3$ satisfy the linear equation $3x - 4y + 6 = 0$.

Short Answer Questions

DIRECTIONS: *Give answer in 2-3 sentences.*

1. (a) Verify 3 is solution of $5x - 8 = 7$.
   (b) Check $x = 2$ is a solution of $5x + 7 = 3$.
2. Given the point (2, 11), find the equation of a line on which it lies. How many such equations are there?
3. If the point (3, 4) lies on the graph of the equation $3y = ax + 7$, find the value of a.
4. Write each of the following equations in the form $ax + by + c = 0$ and indicate the values of a, b and c in each case :
   (i) $2x + 3y = 4.37$
   (ii) $x - 4 = \sqrt{3}y$ t
   (iii) $4 = 5x - 3y$
   (iv) $2x = y$.
5. Write each of the following as an equation in two variables:
   (i) $x = -5$ (ii) $y = 2$
   (iii) $2x = 3$ (iv) $5y = 2$.
6. Find two solutions for each of the following equations:
   (a) $3x + 2y = 12$
   (b) $3y + 5 = 0$
   (c) $2x + 5 = 0$
7. Express y in terms of x in the equation $2x + 3y = 11$. Find the point where the line represented by the equation $2x + 3y = 11$ cuts the y-axis.
8. Find the points where the graph of the equation $3x + 4y = 12$ cuts the x-axis and the y-axis.
9. At what point does the graph of the linear equation $x + y = 5$ meet a line which is parallel to the y-axis, at a distance 2 units from the origin and in the positive direction of x-axis?
10. Determine the point on the graph of the equations $2x + 5y = 20$ whose x-co-ordinate is $\frac{5}{2}$ times its ordinate.
11. Draw the graph of the equation represented by the straight line which is parallel to the x-axis and is 4 units above it.

Long Answer Questions

DIRECTIONS: *Give answer in four to five sentences.*

1. Draw the graph of $2x + 3y = 9$. Using the graph, check whether (3, 1) and (–1, 2) are solutions of the given equation.
2. Draw the graph of the linear equation $2x + 3y = 12$. At what points, the graph of the equation cuts the x-axis and the y-axis?
3. Draw the graph of the line $x - 2y = 3$.
4. Draw the graph of each of the following linear equations in two variables :
   (i) $x + y = 4$ (ii) $x - y = 2$
5. The taxi fare in a city is as follows : For the first kilometre, the fare is ₹ 8 and for the subsequent distance it is ₹ 5 per km. Taking the distance covered as x km and total fare as ₹ y, write a linear equation for this information, and draw its graph.
6. If the work done by a body on application of a constant force is directly proportional to the distance travelled by the body, express this in the form of an equation in two variables and draw the graph of the same by taking the constant force as 5 units. Also read from the graph the work done when the distance travelled by the body is
   (i) 2 units (ii) 0 unit
7. Solve the equation $2x + 1 = x - 3$, and represent the solution(s) on
   (i) the number line,
   (ii) the Cartesian plane.
8. Give the geometric representations of $2x + 9 = 0$ as an equation
   (i) in one variable (ii) in two variables.
9. Alka and Noori, two students of class IX, together contribute ₹ 500 towards Prime Minister's Relief fund to help the earthquake victims. Write a linear equation which satisfies this data and draw the graph of the same.

EXERCISE 2

Multiple Choice Questions

DIRECTIONS: *This section contains multiple choice questions. Each questions has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. The linear equation $2x - 5y = 7$ has
(a) A unique solution (b) Two solutions
(c) Infinitely many solutions
(d) No solution

2. The equation $2x + 5y = 7$ has a unique solution, if x, y, are:
(a) Natural numbers (b) Positive real numbers
(c) Real numbers (d) Rational numbers

3. If (2, 0) is a solution of the linear equation $2x + 3y = k$, then the value of k is
(a) 4 (b) 6
(c) 5 (d) 2

4. If $(3, -2)$ is a solution of the equation $3x - py - 7 = 0$, then the value of p is
(a) -1 (b) 1
(c) $-\frac{13}{3}$ (d) 2

5. The value of x for which $y = -4$ is a solution of the linear equation $5x - 8y = 47$ is
(a) -3 (b) 3
(c) $\frac{79}{5}$ (d) $-\frac{79}{5}$

6. A solution of the equation $2x + 5y - 3 = 0$ is
(a) $(5, -2)$ (b) $(-5, 2)$
(c) $(-1, 1)$ (d) $(1, -1)$

7. Any solution of the linear equation $2x + 0y + 9 = 0$ in two variables is of the form
(a) $\left(-\frac{9}{2}, m\right)$ (b) $\left(n, -\frac{9}{2}\right)$
(c) $\left(0, -\frac{9}{2}\right)$ (d) $(-9, 0)$

8. The graph of the linear equation $2x + 3y = 6$ cuts the y-axis at the point
(a) (2, 0) (b) (0, 3)
(c) (3, 0) (d) (0, 2)

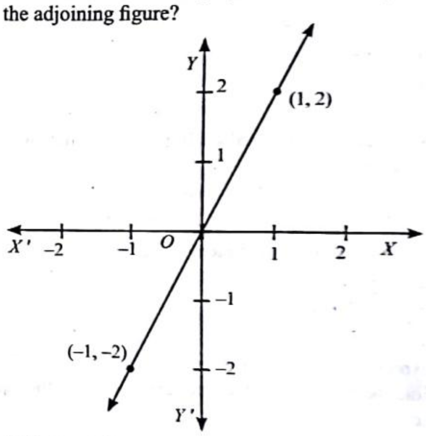
9. The equation $x = 7$, in two variables, can be written as
(a) $1.x + 1.y = 7$ (b) $1.x + 0.y = 7$
(c) $0.x + 1.y = 7$ (d) $0.x + 0.y = 7$

10. Which of the following equations represents a line parallel to x-axis?
(a) $3x + 2 = 0$ (b) $3y + 2 = 0$
(c) $3x + 2y = 0$ (d) $3x - 2y = 0$

11. Which of the following equations represents a line parallel to y-axis?
(a) $2y = 5x$ (b) $2y = 5$
(c) $2x = 5$ (d) $2x + 3y = 5$

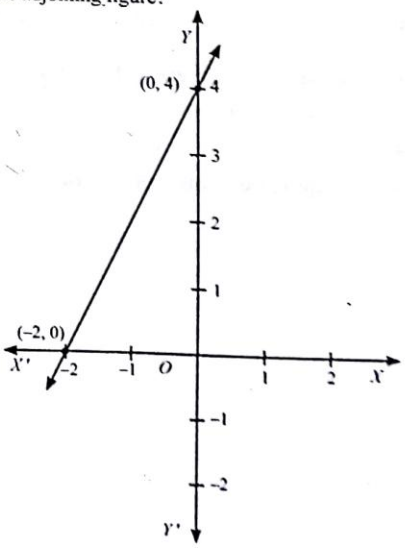
12. Any point on the line $y = x$ is of the form
(a) (a, a) (b) $(0, a)$
(c) $(a, 0)$ (d) $(a, -a)$

13. Which of the following equations has a graph shown in the adjoining figure?



(a) $2x + y = 0$ (b) $2x - y = 0$
(c) $x + 2y = 0$ (d) $x - 2y = 0$

14. Which of the following equations has a graph shown in the adjoining figure?



(a) $2x + y = 0$ (b) $y = 2x$
(c) $y = x + 4$ (d) $y = 2x + 4$

15. The graph of $y = 6$ is a line
(a) parallel to x-axis at a distance 6 units from the origin
(b) parallel to y-axis at a distance 6 units from the origin
(c) making an intercept 6 on the x-axis.
(d) making an intercept 6 on both the axes.

16. If a linear equation has solutions $(-2, 2)$, $(0, 0)$ and $(2, -2)$, then it is of the form
(a) $y - x = 0$ (b) $x + y = 0$
(c) $-2x + y = 0$ (d) $-x + 2y = 0$

17. The positive solutions of the equation $ax + by + c = 0$ always lie in the
(a) 1st quadrant (b) 2nd quadrant
(c) 3rd quadrant (d) 4th quadrant

18. The graph of the linear equation $2x + 3y = 6$ is a line which meets the x-axis at the point
(a) $(0, 2)$ (b) $(2, 0)$
(c) $(3, 0)$ (d) $(0, 3)$

19. Passing through the point $(-3, 5)$
(a) one and only one line can be drawn
(b) two and only two lines can be drawn
(c) only a finite number of lines can be drawn
(d) infinitely many lines can be drawn

20. If we multiply or divide both sides of a linear equation with a non-zero number, then the solution of the linear equation :
(a) Changes
(b) Remains the same
(c) Changes in case of multiplication only
(d) Change in case of division only

21. The point of the form (a, a) always lies on :
(a) x-axis (b) y-axis
(c) On the line $y = x$ (d) On the line $x + y = 0$

22. Which one of the following options is true, and why? $y = 3x + 5$ has
(a) a unique solution (b) only two solutions,
(c) infinitely many solutions.
(d) no soluiton

MTQ More than One Correct

DIRECTIONS: *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

1. Which of the following is/are correct?
(a) Solutions of the equation $4x + 3y = 12$ are $(0, 4)$ and $(3, 0)$.
(b) $\left(0, -\frac{4}{3}\right)$ and $\left(1, -\frac{4}{3}\right)$ are solution of the equation $3y + 4 = 0$.
(c) $(0, 0)$ is a solution of the equation $2x + 5y = 0$
(d) $(2, 3)$ is one of the solution of the equation $x + 2y = 6$.

2. Which of the following is/are correct?
(a) The equation $3x = \sqrt{5}y - 7$ has infinitely many solutions.
(b) $(-1, 1)$ is a solution of the equation $3x - 4y + 7 = 0$
(c) The linear equation $3x - y = x - 1$ has no solution.
(d) $x = 5$, $y = 2$ is a solution of the linear equation $x + 2y = 7$.

3. Which of the following is/are not correct?
(a) Any point on the y-axis is of the form $(x, 0)$
(b) The equation of x-axis is of the form $x + y = 0$
(c) Any point on the x-axis is of the form $(0, y)$
(d) The equation $y + 3 = 0$ represents a line.

4. Which of the following is/are not the solutions of the equation $x - 2y = 4$?
(a) $(4, 0)$ (b) $(\sqrt{2}, 4\sqrt{2})$
(c) $(1, 1)$ (d) $(2, 0)$

5. Which of the following is/are false statement?
(a) The solution of a linear equation is affected when the same number is added to (or subtracted from) both the sides of the equation.
(b) $x - 4 = 3\sqrt{y}$ is a linear equation in two variables.
(c) Geometrical representation of $ax + by + c = 0$ is a straight line
(d) Every solution of the linear equation is a point on the graph of the linear equation.

6. Check which of the following are solutions of the equation $2x - y = 4$?
(a) $x = 0, y = -4$ (b) $x = 3, y = 2$
(c) $x = 1, y = 1$ (d) $y = 0, x = 2$

PBQ Passage Based Questions

DIRECTIONS: *Study the given paragraph(s) and answer the following questions.*

PASSAGE-I

If the temperature of a liquid can be measured in Kelvin units as $x°K$ or in Fahrenheit units as $y°F$, the relation between the two systems of measurement of temperature is given by the linear equation

$$y = \frac{9}{5}(x - 273) + 32$$

1. Find the temperature of the liquid in Fahrenheit if the temperature of the liquid is 313°K.
(a) 105°F (b) 104°F
(c) 343°F (d) 100°F

2. If the temperature is 158°F, then the temperature in Kelvin is.
(a) 105°K (b) 104°K
(c) 343°K (d) 100°K

PASSAGE-II

The linear equation that converts Fahrenheit (F) to Celsius (C) is given by the relation

$$C = \frac{5F - 160}{9}$$

1. If the temperature is 35°C, what is the temperature in Fahrenheit is
   (a) 95°F (b) 96°F
   (c) 98°F (d) 94°F
2. What is the numerical value of the temperature which is same in both the scales?
   (a) 40 (b) -40
   (c) 50 (d) -50

Assertion & Reason

DIRECTIONS: *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and **Reason** is the **correct explanation** of **Assertion**.
(b) If both **Assertion** and **Reason** are correct, but **Reason** is **not the correct** explanation of **Assertion**.
(c) If **Assertion** is **correct** but **Reason** is **incorrect**.
(d) If **Assertion** is **incorrect** but **Reason** is **correct**.

1. **Assertion :** A linear equation $2x + 3y = 5$ has a unique solution.
   Reason : A linear equation in two variables has infinitely many solutions
2. **Assertion :** All the points (1, 0), (–1, 0), (2, 0) and (5, 0) lie on the x-axis.
   Reason : Equation of the x-axis is $y = 0$
3. **Assertion :** The point (0, 3) lies on the graph of the linear equation $3x + 4y = 12$.
   Reason : (0, 3) satisfies the equation $3x + 4y = 12$.
4. **Assertion :** The graph of every linear equation in two variables need not be a line.
   Reason : Graph of a linear equation in two variables is always a line.
5. **Assertion :** The graph of the equation $3x + y = 0$ is a line passing through the origin.
   Reason : An equation of the form $ax + by + c = 0$, where a, b, c are real numbers is called a linear equation in x and y.

Multiple Matching Questions

DIRECTIONS : *Following question has four statements (A, B, C and D) given in Column I and statements (p, q, r, s, ...) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in Column I with entries in Column II.*

1.

Column I	Column II
(A) $x + y = 0$	(p) (3/2, 3)
(B) $y - 2x = 0$	(q) (–5, 5)
(C) $3x + 5y = 0$	(r) (5, –2)
(D) $2x - y = 12$	(s) (0, 0)
	(t) (–5, 3)
	(u) (5, –3)

Hots Subjective Questions

DIRECTIONS: *Answer the following questions.*

1. Draw the graph of each of the following linear equations in two variables :
   (i) $y = 3x$ (ii) $3 = 2x + y$
2. From the choices given below, choose the equation whose graphs are given in Fig.(1) and Fig. (2).

For Fig. 1	For Fig. 2
(i) $y = x$	(i) $y = x + 2$
(ii) $x + y = 0$	(ii) $y = x - 2$
(iii) $y = 2x$	(iii) $y = -x + 2$
(iv) $2 + 3y = 7x$	(iv) $x + 2y = 6$

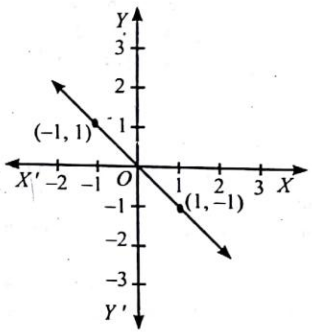


Fig 1

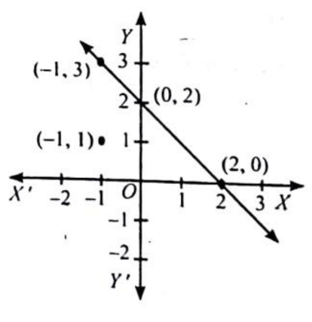


Fig. 2

3. Draw the graph of each of the following equations:
(i) $x = 0$ (ii) $y = 0$ (iii) $x = 4$ (iv) $y = 3$
(v) $y + 4 = 0$ (vi) $x + 2 = 0$

4. The autorikshaw fare in a city is charged ₹ 10 for the first kilometer and @ ₹ 4 per kilometer for subsequent distance covered. Write the linear equation to express the above statement. Draw the graph of the linear equation.

5. The work done by a body on application of a constant force is the product of the constant force and the distance travelled by the body in the direction of force. Express this in the form of a linear equation in two variables and draw its graph by taking the constant force as 3 units. What is the work done when the distance travelled is 2 units. Verify it by plotting the graph.

SOLUTIONS

EXERCISE I

FILL IN THE BLANKS

1. infinitely many
2. straight line
3. y, x
4. consistent
5. dependent
6. y
7. x
8. origin.
9. solution
10. A point on the number line.

TRUE/FALSE

1. False
2. False
3. False
4. True
5. True
6. False, the points (2, 0), (–3, 0) lie on the x-axis. The point (4, 2) lies in the first quadrant. The point (0, 5) lies on the y-axis.
7. True, since the line parallel to y-axis at a distance a units to the left to y-axis is given by the equation $x = -a$
8. False, because $x = 0, y = 0$ does not satisfy the equation.

MATCH THE COLUMNS

1. (A) → (q); (B) → (s); (C) → (r); (D) → (p)
2. (A) → (r); (B) → (q); (C) → (s); (D) → (p)

VERY SHORT ANSWER QUESTIONS

1. Given equation is $y - 2 = 0$
$\Rightarrow \quad 0.x + 1.y - 2 = 0$
Comparing with $ax + by + c = 0$, we get $a = 0$, $b = 1$, $c = -2$.

2. Given eq. is $3x + 5y - 2 = 0$ .... (1)
Taking L.H.S. $= 3x + 5y - 2 = 3 \times 2 + 5 \times (-1) - 2$
$= 6 - 5 - 2 = -1 \neq 0$
Here L.H.S. ≠ R.H.S. therefore $x = 2$, $y = -1$ is not a solution of given equation.

3. Writing the equation $3y + 4 = 0$ as $0.x + 3y + 4 = 0$, we will find that $y = -\frac{4}{3}$ for any value of x. Thus, two solutions can be given as $\left(0, -\frac{4}{3}\right)$ and $\left(1, -\frac{4}{3}\right)$

4. Since, $x = 2, y = 1$ is a solution of the equation $2x + 3y = k$, therefore these values will satisfy the equation.
So, we put $x = 2$ and $y = 1$ in the equation, we get
$2(2) + 3(1) = k \Rightarrow k = 7$.

5. Let the cost of a note book be ₹ x and the cost of a pen be ₹ y.
According to the question,
$x = 2y \Rightarrow x - 2y = 0$ which is the required linear equation in two variables.

6. The equations of two lines passing through (2, 14) can be taken as
$x + y = 16$ and $7x - y = 0$.
So, There are infinitely many such lines because through a point an infinite number of lines can be drawn.

7. (2, 2), (0, 3), (6, 0) and (4, 1).

8. $x + y = 80$

9. Substituting $x = 2, y = 3$ in the given equation, we get
LHS $= 3 \times 2 - 4 \times 3 + 6 = 0 =$ RHS.
$\therefore\ x = 2, y = 3$ satisfy $3x - 4y + 6 = 0$

SHORT ANSWER QUESTIONS

1. (a) $5x - 8 = 7$
L.H.S. $= 5x - 8 = 5 \times 3 - 8$, (on putting $x = 3$)
$= 15 - 8 = 7$
and R.H.S. $= 7$
$\therefore$ On putting $x = 3$, we find that L.H.S. = R.H.S.
Hence, the solution of the given equation is $x = 3$.
(b) $x = 2$ is not a solution of $5x + 7 = 3$ as we substituting $x = 2$ in the equation.
L.H.S. $= 5 \times 2 + 7 = 17 \neq$ R.H.S.

2. Given point (2, 11) i.e. $x = 2$ and $y = 11$
By addition, we get $x + y = 2 + 11 = 13$
By subtraction, we get $x - y = 2 - 11 = -9$
Hence $x + y = 13$ and $x - y = -9$ are two lines passing through (2, 11). Infinite number of lines may be drawn through the point (2, 11).

3. Since, the point (3, 4) lies on the graph of the equation $3y = ax + 7$, therefore this point satisfies the equation So, we put $x = 3, y = 4$ in given equation, we get
$3(4) = a(3) + 7$
$12 = 3a + 7$
$3a = 12 - 7$
$3a = 5$
$\Rightarrow a = \frac{5}{3}$

4. (i) $2x + 3y - 4.37 = 0$;
$a = 2, b = 3$ and $c = -4.37$

(ii) $x - \sqrt{3}y - 4 = 0; a = 1, b = -\sqrt{3}, c = -4$

(iii) $-5x + 3y + 4 = 0; a = -5, b = 3$ and $c = 4$
or $5x - 3y - 4 = 0; a = 5, b = -3$ and $c = -4$

(iv) $2x - y + 0 = 0; a = 2, b = -1$ and $c = 0$.

5. (i) $1.x + 0y + 5 = 0$
(ii) $0x + 1y - 2 = 0$
(iii) $2x + 0y - 3 = 0$
(iv) $0x + 5y - 2 = 0$.

6. (a) (0, 6), (4, 0)
(b) $\left(0, \frac{-5}{3}\right), \left(1, \frac{-5}{3}\right)$
(c) $\left(\frac{-5}{3}, 0\right), \left(\frac{-5}{2}, 1\right)$

7. $y = \frac{11 - 2x}{3}, \left(0, \frac{11}{3}\right)$

8. The graph of the linear equation $3x + 4y = 12$ cuts the x-axis at the point where $y = 0$. On putting $y = 0$ in the linear equation, we have $3x = 12$, which gives $x = 4$. Thus, the required point is (4, 0).

The graph of the linear equation $3x + 4y = 12$ cuts the y-axis at the point where $x = 0$. On putting $x = 0$ in the given equation, we have $4y = 12$, which gives $y = 3$. Thus, the required point is (0, 3).

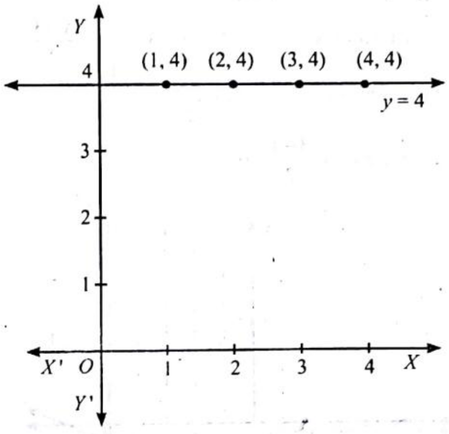
9. The co-ordinates of the points lying on the line parallel to the y-axis, at a distance 2 units from the origin and in the positive direction of the x-axis are of the form $(2, a)$. Putting $x = 2$, $y = a$ in the equation $x + y = 5$, we get $a = 3$. Thus, the required point is (2, 3)

10. As the x-coordinate of the point is $\frac{5}{2}$ times its ordinate, therefore, $x = \frac{5}{2}y$.

Now putting $x = \frac{5}{2}y$ in $2x + 5y = 20$, we get $5y = 10$.

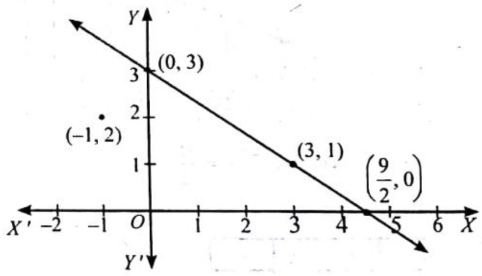
Therefore, $x = 5$. Thus, the required point is (5, 2)

11. Any straight line parallel to x-axis is given by $y = k$, where k is the distance of the line from the x-axis. Here $k = 4$. Therefore, the equation of the line is $y = 4$. To draw the graph of this equation, plot the points (1, 4) and (2, 4) and join them. This is the required graph (see fig.)



LONG ANSWER QUESTIONS

1.



2. The given equation is $2x + 3y = 12$. To draw the graph of this equation, we need at least two points lying on the graph.

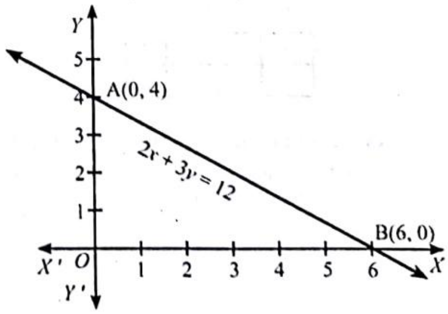
From the equation, we have $y = \frac{12 - 2x}{3}$

For $x = 0$, $y = 4$, therefore, (0, 4) lies on the graph.

For $y = 0$, $x = 6$, therefore, (6, 0) lies on the graph.

Now plot the points $A(0, 4)$ and $B(6, 0)$ and join them (see Fig.), to get the line AB. Line AB is the required graph.

You can see that the graph (line AB) cuts the x-axis at the point (6, 0) and the y-axis at the point (0, 4).

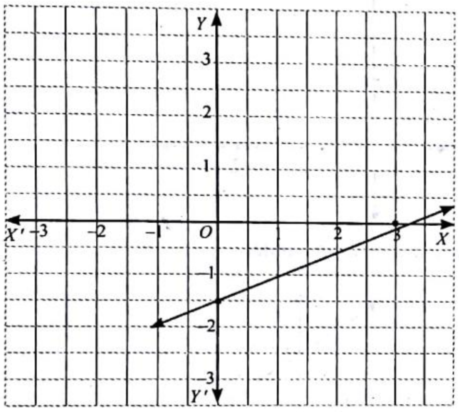


3. Here given equation is $x - 2y = 3$

Solving it for y we get, $2y = x - 3 \Rightarrow y = \frac{x-3}{2}$

Let $x = 0$, then $y = \frac{0-3}{2} = \frac{-3}{2}$,

$x = 3$, then $y = \frac{3-3}{2} = 0$.



$x = -2$, then $y = \frac{-2-3}{2} = \frac{-5}{2}$

Hence, we get,

x	0	3	–2
y	–3/2	0	–5/2

Now, we plot these points on a graph paper and join them to get a straight line.

4. (i) Given equation is $x + y = 4$ or $y = 4 - x$

Now, we take certain values of x and find their corresponding values of y

For $x = 0, y = 4 - 0 = 4$

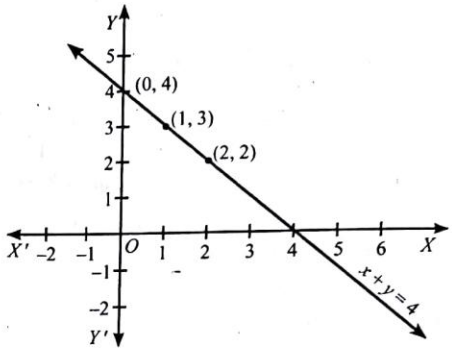
$x = 1, y = 4 - 1 = 3$

$x = 2, y = 4 - 2 = 2$

$\therefore$ Table is given below:

x	0	1	2
y	4	3	2

Now we plot the points (0, 4), (1, 3) and (2, 2) on the graph and join them to get the straight line which represents the linear equation $x + y = 4$.



(ii) Given equation is $x - y = 2$

$\Rightarrow$ $x = 2 + y$

Now take certain values of y and find their corresponding values of x.

For $y = 0, x = 2$

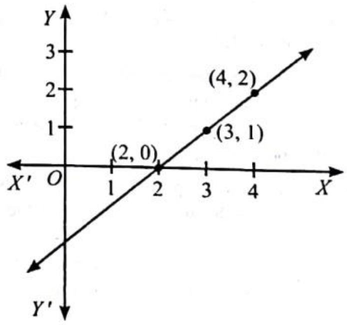
$y = 1, x = 3$

$y = 2, x = 4$

$\therefore$ The corresponding table is

x	2	3	4
y	0	1	2

Now we plot the points (2, 0), (3, 1) and (4, 2) on the graph paper and join them to get the corresponding line of the linear equation $x - y = 2$.



5. Total distance covered = x km

Total fare = ₹ y

Fare for the first kilometre = ₹ 8

Subsequent distance = $(x - 1)$ km

Fare for the subsequent distance = ₹ $5(x - 1)$

According to the question,

$y = 8 + 5(x - 1)$

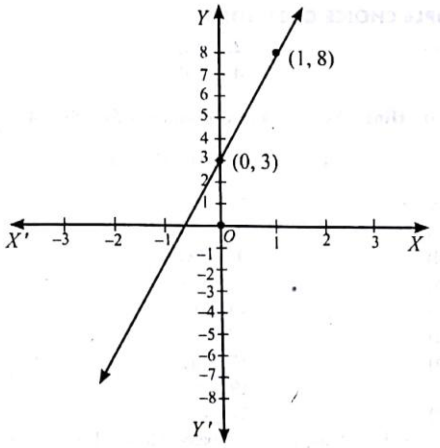
$\Rightarrow$ $y = 8 + 5x - 5$

$\Rightarrow$ $y = 5x + 3$

Table of solutions

x	0	1
y	3	8

We plot the points (0, 3) and (1, 8) on the graph paper and join the same to get the line which is the graph of the equation $y = 5x + 3$.



6. Let the work done by the constant force be y units and the distance travelled by the body be x units.

Constant force = 5 units

We have,

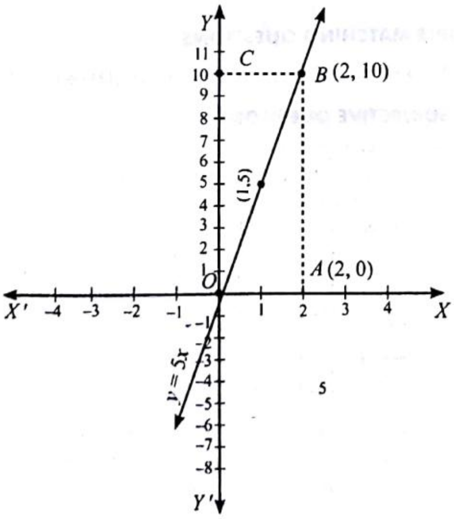
Work done = Force × Displacement

$\Rightarrow \quad y = 5x$

Table of solutions

x	0	1	2
y	0	5	10

We plot the points (0, 0) and (1, 5) on the graph paper and join the same to get the line which is the graph of the equation $y = 5x$.



(i) Let A be (2,0). Through A, draw a line parallel to OY to intersect the graph of the equation $y = 5x$ at B. Through B, draw a line parallel to OX to intersect OY at C. Then C will be (0, 10)

$\therefore$ Work done when the distance travelled by the body is 2 units = 10 units.

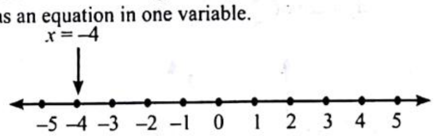
(ii) Clearly $y = 0$ when $x = 0$. So, the work done when the distance travelled by the body is 0 unit = 0 unit.

7. We solve $2x + 1 = x - 3$,

$\Rightarrow \quad 2x - x = -3 - 1$

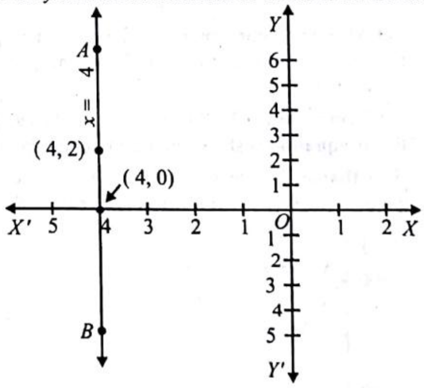
i.e., $\quad x = -4$

(i) The representation of the solution on the number line is shown in below given Fig where $x = -4$ is treated as an equation in one variable.



(ii) We know that $x = -4$ can be written as $x + 0.y = -4$, which is a linear equation in two variables x and y. Now all the values of y are permissible because $0.y$ is always 0. However, x must satisfy the equation $x = -4$. Hence, two solutions of the given equation are $x = -4, y = 0$ and $x = -4, y = 2$.

The graph of $x + 0.y = -4$ is the straight line AB parallel to the y-axis and at a distance of 4 units to the left of it.



Similarly, you can obtain a line parallel to the x-axis corresponding to equations of the type $y = 3$ or $0x + 1 . y = 3$.

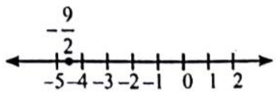
8. The given equation is

$2x + 9 = 0$

(i) **In one variable**

$2x + 9 = 0 \Rightarrow x = -\frac{9}{2}$

The representation of $2x + 9 = 0$ on the number line is as shown below:



(ii) **In two variables**

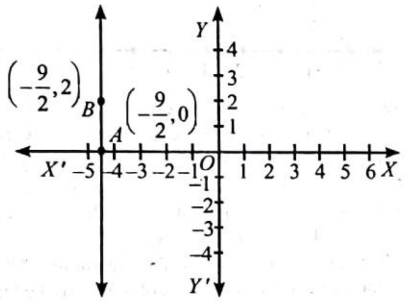
$2x + 9 = 0$ can be written as $2x + 0.y + 9 = 0$

It is a linear equation in two variables x and y. This is represented by a line. All the values of y are permissible because $0y$ is always 0. However, x must satisfy the relation $2x + 9 = 0$.

i.e., $x = -\frac{9}{2}$.

Hence, two solution of the given equation are $x = -\frac{9}{2}$, $y = 0$ and $x = -\frac{9}{2}$, $y = 2$.

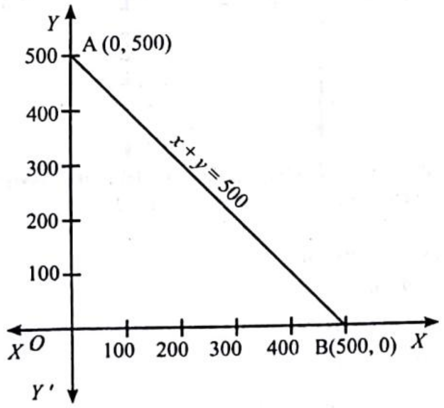
The graph AB is a line parallel to the y-axis and at a distance of $\frac{9}{2}$ units to the left of it.



9. Let Alka and Noori contribute ₹ x and ₹ y respectively, then the linear equation satisfying the given data is $x + y = 500$.

Let 1 cm represent ₹ 100 on both axes. The graph of the **linear equation** is shown in the adjoining figure.

Note that as the values of x or y cannot be negative, the graph of the given data is the line segment AB.



EXERCISE 2

MULTIPLE CHOICE QUESTIONS

1. (c) **2.** (a)
3. (a) **4.** (a)

5. (b) **Hint :** As $y = -4$ is a solution of $5x - 8y = 47$

$5x - 8(-4) = 47 \Rightarrow 5x + 32 = 47$

$\Rightarrow 5x = 15 \Rightarrow x = 3$

6. (c) **7.** (a)
8. (d) **9.** (b)
10. (b) **11.** (c)
12. (a) **13.** (b)
14. (d) **15.** (a)
16. (b) **17.** (a)
18. (c) **19.** (d)
20. (b) **21.** (c)

22. (c) $y = 3x + 5$ has infinitely many solution, because for every value of x, there is a corresponding value of y and vice-versa.

MORE THAN ONE CORRECT

1. (a), (b), and (c) **2.** (a) and (b)
3. (a, b, c) **4.** (b, c, d)
5. (a, b)
6. (a, b) ($x = 0, y = -4$), ($x = 3, y = 2$)

PASSAGE BASED QUESTIONS

1. (1) (b) 104°F (2) (c) 343°K
2. (1) (a) 95° F (2) (b) −40

ASSERTION AND REASON

1. (d) **2.** (a)
3. (a) **4.** (d)
5. (b)

MULTIPLE MATCHING QUESTIONS

1. (A) → (q, s); (B) → (p); (C) → (t), (u); (D) → (r)

HOTS SUBJECTIVE QUESTIONS

1. (i) Given equation is $y = 3x$

Now we take certain values of x and find the corresponding values of y.

When $x = 0, y = 3 \times 0 = 0$

$x = 1, y = 3 \times 1 = 3$

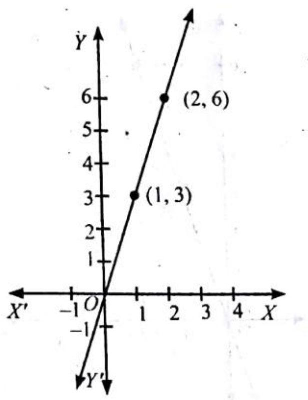
$x = 2, y = 3 \times 2 = 6$

The corresponding table is

x	0	1	2
y	0	3	6

Thus, we get the points (0, 0), (1, 3), (2, 6) and we plot them on the graph paper and join to get a straight line which is the representation of $y = 3x$.

Class 9 Part A IIT JEE KVPY NSEJS Foundation Mathematics by K K Anand Chennai SmartEdu for Standard 9



(ii) Given equation is $3 = 2x + y$

or $y = 3 - 2x$

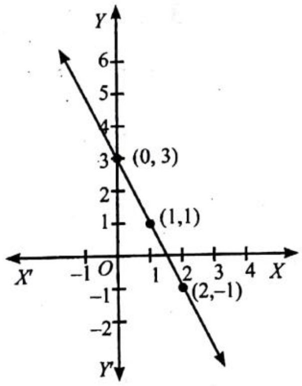
Now, we consider certain values of x and find their corresponding values of y.

When $x = 0, y = 3$

$x = 1, y = 1$

$x = 2, y = -1$

Thus, the points are (0, 3), (1, 1) and (2, – 1) and join them to get straight line which is the graphical representation of the given linear equation.



2. **For Fig. (1)**

The correct equation is (ii), Which is $x + y = 0$.

Because it passes through origin and when $x = 1$, $y = -1$ and $x = -1$ and $y = 1$

For Fig. (2)

The correct equation is (iii), Which is $y = -x + 2$.

Because, when $x = 0, y = 2$ and $x = 2, y = 0$

Thus, points are (0, 2) and (2, 0)

3. As discussed above, we have:

(i) $x = 0$ is the equation of the y-axis.

$\therefore$ the graph of $x = 0$ in the line YOY'

(ii) $y = 0$ is the equation of the x-axis.

$\therefore$ the graph of $y = 0$ is the line $X'OX$.

(iii) $x = 4$ is the line AB, parallel to the y-axis, at a distance of 4 units from it, to its right.

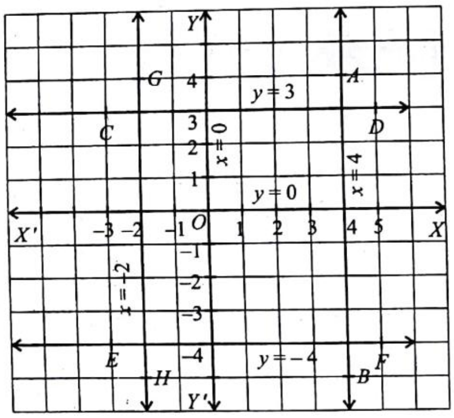
(iv) $y = 3$ is the line CD, parallel to the x-axis, at a distance of 3 units from it, above the x-axis.

(v) $y + 4 = 0 \Rightarrow y = -4$.

$y = -4$ is the line EF, parallel to the x-axis, at a distance of 4 units from its, below the x-axis.

(vi) $x + 2 = 0 \Rightarrow x = -2$.

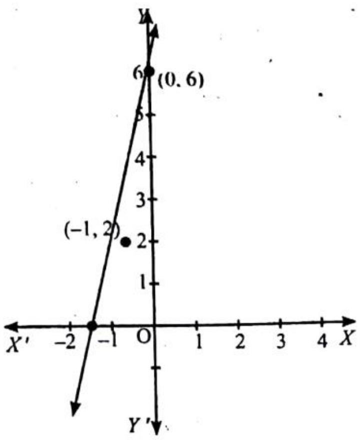
$x = -2$ is the line GH, parallel to the y-axis, at a distance of 2 units from it, to its left.



4. Let the total distance covered be x km and the fare charged ₹ y. Then for the first km, fare charged in ₹ 10 and for remaining $(x - 1)$ km fare charged is ₹ $4(x - 1)$:

Therefore, $y = 10 + 4(x - 1) = 4x + 6$

The required equation is $y = 4x + 6$. Now, when $x = 0$, $y = 6$ and when $x = -1, y = 2$. The graph is given in fig.

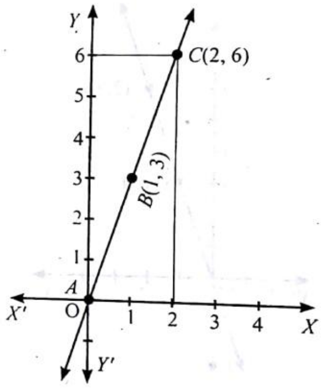


5. Work done = (constant force) × (distance)

= 3 × (distance),

i.e., $y = 3x$. where y (units) is the work done and x(units) is the distance travelled. Since $x = 2$ units (given), therefore work done = 6 units. To plot the graph of the linear equation $y = 3x$, we need at least two solutions of the equation. We see that $x = 0$, $y = 0$ satisfies the given equation also $x = 1$, $y = 3$ satisfies the equation.

Now we plot the points A(0, 0) and B(1, 3) and join AB (see Figure). The graph of the equation is a straight line. To verify from the graph, draw a perpendicular to the x-axis at the point (2, 0) meeting the graph at the point C. Clearly the coordinates of C are (2, 6). It means that the work done is 6 units.



Euclid (325 BC – 265 BC)

5

CHAPTER

Introduction to Euclid's Geometry and Present Day Geometry-Lines & Angles (Only)

INTRODUCTION

The word 'Geometry' comes from the Greek. The ancient people faced several practical problems like measurement of land, computing volumes of granaries, construction of pyramid, construction of altars (or vedis) and fire places for performing vedic rites. So the geometry was studied in various forms in every ancient civilisation, in Egypt, Babylonia, China, India, Greece etc.

Euclid (325 BC – 265 BC), a teacher of mathematics at Alexandra in Egypt, collected all the known work and arranged it in his famous treatise, called 'Elements'. He divided the 'Elements' into thirteen chapters, each called a book. Euclid's Elements form one of the most beautiful and influential works of mathematics. These books influenced the whole world's understanding of geometry. In this chapter, we shall discuss Euclid's approach of geometry and shall try to link it with the present day geometry.

Euclid began his exposition by listing 23 definitions in Book-1 of the 'Elements'.

INTRODUCTION TO EUCLID'S GEOMETRY

Euclid collected all the known work and arranged in his famous treatise called 'Elements'. He divided the Elements into thirteen chapters, each called a book. Euclid began his exposition by listing 23 definitions in Book-1 of the 'Elements'.

23 DEFINITIONS FROM BOOK-1

Definition 1

A point is that which has no part.

Definition 2

A line is breadthless length.

Definition 3

The ends of a line are points.

Definition 4

A straight line is a line which lies evenly with the points on itself.

Definition 5

A surface is that which has length and breadth only.

Definition 6

The edges of a surface are lines.

Definition 7

A plane surface is a surface which lies evenly with the straight lines on itself.

Definition 8

A plane angle is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line.

Definition 9

When the lines containing the angle are straight, the angle is called rectilinear.

Definition 10

When a straight line standing on a straight line makes the adjacent angles equal to one another, each of the equal angles is right, and the straight line standing on the other is called a perpendicular to that on which it stands.

Definition 11

An obtuse angle is an angle greater than a right angle.

Definition 12

An acute angle is an angle less than a right angle.

Definition 13

A boundary is that which is an extremity of anything.

Definition 14

A figure is that which is contained by any boundary or boundaries.

Definition 15

A circle is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure equal one another.

Definition 16

The point within the circle from which all the straight lines falling on it are equal is called the centre of the circle.

Definition 17

A diameter of the circle is any straight line drawn through the centre and terminated in both directions by the circumference of the circle, and such a straight line also bisects the circle.

Definition 18

A semicircle is the figure contained by the diameter and the circumference cut off by it. And the center of the semicircle is the same as that of the circle.

Definition 19

Rectilinear figures are those which are contained by straight lines, trilateral figures being those contained by three, quadrilateral those contained by four, and multilateral those contained by more than four straight lines.

Definition 20

Of trilateral figures, an equilateral triangle is that which has its three sides equal, an isosceles triangle is that which has two of its sides alone equal, and a scalene triangle is that which has its three sides unequal.

Definition 21

Further, of trilateral figures, a right-angled triangle is that which has a right angle, an obtuse-angled triangle is that which has an obtuse angle, and an acute-angled triangle is that which has its three angles acute.

Definition 22

Of quadrilateral figures, a square is that which is both equilateral and right-angled; a rectangle is that which is right-angled but not equilateral; a rhombus is that which is equilateral but not right-angled; and a rhomboid is that which has its opposite sides and angles equal to one another but is neither equilateral nor right-angled. And the quadrilaterals other than these be called trapezia.

Definition 23

Parallel straight lines are straight lines which being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction.

UNDEFINED TERMS

If you carefully study these definitions, you find that some of the terms like part, breadth, length, evenly, etc. need to be further explained clearly. So, to define one thing, you need to defined many other things, and may get a long chain of definitions without an end. For such reasons, mathematicians agree to leave some geometric terms undefined. However, we have an institutional feeling of these undefined terms. In Euclid's geometry a point, a line and a plane are taken as undefined. But we can represent them intuitivly and explain them with the help of 'Physical models'.

EUCLID'S POSTULATE AND COMMON NOTIONS (OR AXIOMS)

Starting with his definitions, Euclid assumed certain properties, which were not to be proved. These assumptions are actually 'obvious universal truths'. He divided them into two types: postulates and common notions (or axioms). He used the term **'postulate'** for the assumptions that were specific to geometry. Common notions often called **axioms**, on the other hand, were assumptions used throughout mathematics and not specifically linked to geometry.

Some of Euclid's notions (or axioms), not in his order, are given below:

COMMON NOTIONS (OR AXIOMS)

1. Things which equal the same thing also equal one another. [Note that, this common notion can be apply to plane figures only.]
2. If equals are added to equals, then the wholes are equal. [Note that, magnitude of only same kind can be added.]
3. If equals are subtracted from equals, then the remainders are equal. [Note that, magnitude of only same kind can be subtracted from larger one.]
4. Things which coincide with one another equal one another. [Note that, if two things are identical, then they are equal.]
5. The whole is greater than the part.
6. Things which are double of the same things are equal to one another.
7. Things which are halves of the same things are equal to one another.

POSTULATES

Euclid's five postulates are given below:

Postulate 1

A straight line may be drawn from any one point to any other point.

This first postulate says that for given any two points such as A and B, there is a unique line AB whose end points are A and B.

Postulate 2

A terminated line can be produced indefinitely.

Euclid's terminated line is now called line segment. So, according to the present day terms, the second postulate say that a line segment can be extended on either side to form a line.

Postulate 3

A circle can be drawn with any centre and radius.

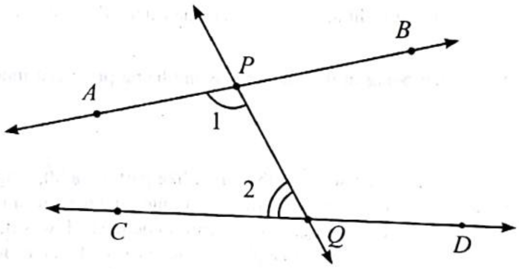
Postulate 4

All right angles equal one another.

Postulate 5

If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of the angles is less than the two right angles.

For example, in the figure, the line PQ falls on lines AB and CD such that the sum of the interior angles 1 and 2 is less than 180° on the left side of PQ. Therefore, the lines AB and CD will eventually intersect on the left side of PQ, if the produced infinitely.



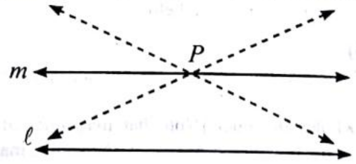
Now-a-days, 'postulates' and 'axioms' are terms that are used interchangeably and in the same sense.

Equivalent Versions of Euclid's Fifth Postulate

Euclid's fifth postulate is very significant in the history of mathematics. There are several equivalent versions of this postulate. One of them is 'Playfair's Axiom' (given by a Scottish mathematician John Playfair in 1729), as stated below:

'For every line l and for every point P not lying on l, there exists a unique line m passing through P and parallel to line l'.

From figure you can see that of all the lines passing through the point P, only line m is parallel to line l.



This result can also be stated in the following form:

Two distinct intersecting lines cannot be parallel to the same line.

CONSISTENT SYSTEM OF AXIOMS

A system of axioms is called **consistent** if it is impossible to deduce from these axioms a statement that contradicts any axiom or previously proved statement.

PROPOSITIONS OR THEOREMS

After Euclid stated his postulates and axioms, he used them to prove other results. Then using these results, he proved some more results by applying deductive reasoning. The statements that were proved are called **propositions** or **theorems**.

Euclid deduced 465 propositions in a logical chain using his axioms, postulates, definitions and theorems proved earlier in the chain.

Now we prove a theorem, which is frequently used in different results:

THEOREM

Two distinct lines cannot have more than one point in common.

Proof

For the time being, let us suppose that the two lines intersect in two distinct points, say P and Q. So, you have two lines passing through two distinct points P and Q. But this assumption clashes with the axiom that only one line can pass through two distinct points. So, the assumption that we started with, that two lines can pass through two distinct points is wrong.

Therefore, we conclude that two distinct lines cannot have more than one point in common.

NON-EUCLIDEAN GEOMETRY

All the attempts to prove Euclid's fifth postulate using first 4 postulates failed. But they led to the discovery of several other geometries, called Non-Euclidean Geometries.

Euclidean Geometry is valid only for the figures in the plane. On the curved surfaces, it fails.

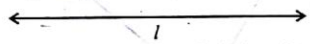
Creation of non-Euclidean geometry is considered a landmark in the history of thought because till then every one had believed that Euclid's was the only geometry and the world itself was Euclidean.

PRESENT DAY GEOMETRY – LINES AND ANGLES (ONLY)

LINE, LINE-SEGMENT AND RAY

Line is a set of infinite points that has no end point. If A and B are any two points on line l then we denote the line as $\overleftrightarrow{AB}$ and is represented as

We also denote the line by a single small English Alphabet say l and represented as



Line-Segment is a part of a line that has two end points. It is of finite length and cannot be extended further. Line segment is denoted as AB where A, B are its end points and is represented as

Ray is a part of a line with one end point. It can be extended infinitely on one side. Ray is denoted as $\overrightarrow{AB}$ where A is the end point and B is any other point on the ray is represented as

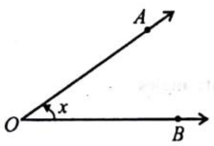
Here B is not the end point
We can extend the ray towards B from A infinitely.

ANGLE

Two rays which have common end point form an angle. The rays making an angle are called arms of the angle. Angle can be acute, obtuse, right, reflex and straight and angle is represented in degree (°)

TYPES OF ANGLES

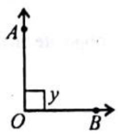
1. Acute angle :



$\angle AOB = x$

$0° < x < 90°$

2. Right angle :

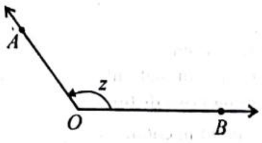


$\angle AOB = y$

$y = 90°$

[Right angle is represented as ⊾]

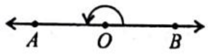
3. Obtuse angle:



$\angle AOB = z$

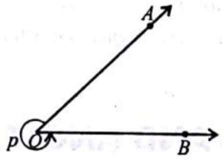
$90° < z < 180°$

4. Straight angle:



$\angle AOB = 180°$

5. Reflex angle:



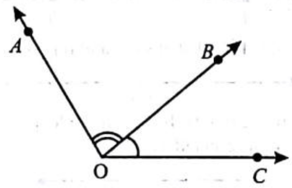
Reflex $\angle AOB = 360 - \angle AOB = p$

$180° < p < 360°$

SPECIAL TYPE OF ANGLES

ADJACENT ANGLES

Two Angles having a common vertex, a common arm and their non-common arms lying on different sides of the common arm are said to be adjacent angles.



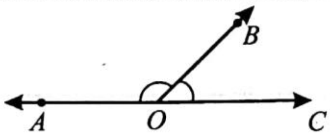
$\angle AOB$ and $\angle BOC$ are adjacent angles.

COMPLIMENTARY AND SUPPLEMENTARY ANGLES

If sum of two angles is 90° they are complimentary angles and if sum of two angles is 180° they are supplementary angles.

LINEAR PAIR OF ANGLES

If the non common arms of two adjacent angles form a line then the adjacent angles are termed as linear pair of angles.

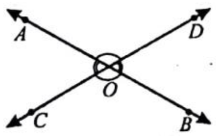


$\angle AOB$ and $\angle BOC$ are lincar pair of angles and their sum is 180°.

$\angle AOB + \angle BOC = 180°$

VERTICALLY OPPOSITE ANGLES

Angles formed on opposite side when two lines intersect are vertically opposite angles.



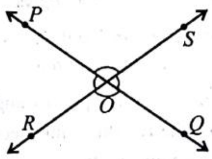
$\angle AOD$, $\angle BOC$ are vertically opposite angles and are equal.

Similarly, $\angle BOD$ and $\angle AOC$ are vertically opposite angles and are equal

$\angle AOD = \angle BOC$

$\angle AOC = \angle BOD$

Illustration 1 : In the given figure lines PQ and RS intersect each other at point O. If $\angle POR : \angle ROQ = 7:11$, find all the angles.



SOLUTION : Here

$\angle POR + \angle ROQ = 180°$

Let $\angle POR = 7x$ and $\angle ROQ = 11x$ [As their ratio are given]

So, $7x + 11x = 180$

$18x = 180 \Rightarrow x = 10$

$\Rightarrow$ $\angle POR = 7\times10 = 70°$, $\angle ROQ = 11\times10 = 110°$

$\angle SOQ = \angle POR = 70°$ [Vertically opposite angles]

$\angle POS = \angle ROQ = 110°$ [Vertically opposite angles]

PLANE

A plane is never ending flat surface which extends indefinitely in all directions. It therefore has only two dimensions-length and breadth. It has no thickness.
The surface of wall, floor and ceilling of a room and table top are some examples of a portion of plane.

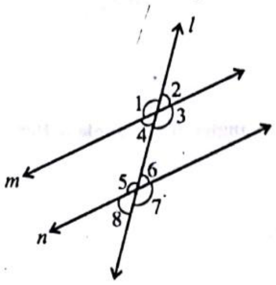
PARALLEL AND INTERSECTING LINES

Two lines in a plane are either parallel or intersecting. If the distance between two lines in a plane are same everywhere, then they are called parallel lines. Otherwise called intersecting lines. The distance between two intersecting lines is zero. If two or more lines parallel to the same line, then all the lines are parallel to each other.

m and n are parallel lines. p and q are intersecting lines. If two lines m and n are parallel then we can write it as $m \parallel n$.
If two lines p and q are intersecting lines than we can write it as $m \nparallel n$.

TRANSVERSAL LINE

A straight line intersecting two or more straight lines in distinct points is known as a transversal to the two given lines.
In the figure, the line l is a transversal which intersects two lines m and n forming eight angles
$\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$, $\angle 6$, $\angle 7$ and $\angle 8$.



RELATED TERMS

(i) Exterior Angles : Exterior angles are the angles lying outside the region between the two lines, which are intersected by a transversal. In the given figure $\angle 1, \angle 2, \angle 7$ and $\angle 8$ are exterior angles.

(ii) Interior angles : Interior angles are the angles lying inside the region between the two lines, which are intersected by a transversal. In the given figure $\angle 3, \angle 4, \angle 5$ and $\angle 6$ are interior angles.

(iii) Corresponding angles : Corresponding angles are the pair of angles lying on the same side of the transversal, both of which either lie above the two lines or below the two lines intersected by a transversal. In the given figure $\angle 1$ & $\angle 5$, $\angle 2$ & $\angle 6$, $\angle 3$ & $\angle 7$, $\angle 4$ & $\angle 8$ are pair of corresponding angles.

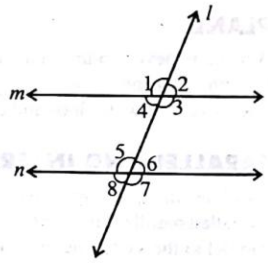
(iv) Vertically opposite angles : Vertically opposite angles are the pair of angles whose arms form two pairs of opposite rays. In the figure ($\angle 1$ & $\angle 3$), ($\angle 2$ & $\angle 4$), ($\angle 5$ & $\angle 7$) and ($\angle 6$ & $\angle 8$) are pair of vertically opposite angles.

(v) Alternate interior angles : Alternate interior angles are the pair of angles lying in the region between the two lines (intersected by a transversal) and on the opposite sides of the transversal but one below the transversal and other above the transversal. In the above figure ($\angle 4$ & $\angle 6$) and ($\angle 3$ & $\angle 5$) are pair of alternate interior angles.

(vi) Alternate exterior angles : Alternate exterior angles are the pair of angles lying outside the region between the two lines (intersected by a transversal) and on the opposite sides of the transversal but one below the transversal and other above the transversal. In the given figure ($\angle 1$ & $\angle 7$) and ($\angle 2$ & $\angle 8$) are pair of alternate exterior angles.

Special Case :

If lines m and n are parallel i.e. $m \parallel n$ then



(i) Corresponding angles are equal i.e.
$\angle 1 = \angle 5, \angle 2 = \angle 6, \angle 3 = \angle 7$ and $\angle 4 = \angle 8$

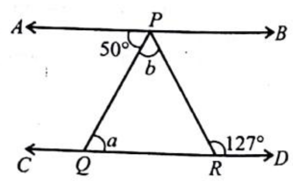
(ii) Alternate interior angles are equal i.e.
$\angle 4 = \angle 6$ and $\angle 3 = \angle 5$

(iii) Alternate exterior angles are equal i.e.
$\angle 1 = \angle 7$ and $\angle 2 = \angle 8$

(iv) Exterior angles on the same side of the transversal are supplementary i.e.
$\angle 1 + \angle 8 = 180°$ and $\angle 2 = \angle 7 = 180°$

(v) Interior angles on the same side of the transversal are supplementary i.e.
$\angle 4 + \angle 5 = 180°$ and $\angle 3 + \angle 6 = 180°$

Illustration 2 : In the given figure if $AB \| CD$, $\angle APQ = 50°$ and $\angle PRD = 127°$, find a and b



SOLUTION : In the given figure,

$\angle APQ = \angle PQR$ {alternate interior opposite angles}

$\Rightarrow$ $a = 50°$

Also $\angle APR = \angle PRD$ {alternate interior opposite angles}

$50 + b = 127$

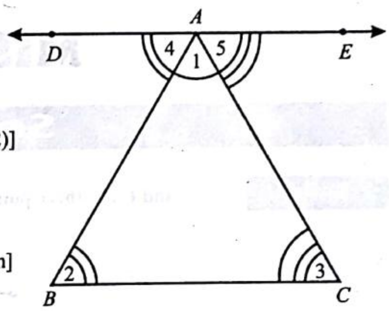
$b = 127 - 50 = 77°$

THEOREM 1 : Prove that the sum of the three angles of a triangle is 180°.

Given: A triangle ABC.

To prove: $\angle A + \angle B + \angle C = 180°$.

Construction: Through A, draw a line DE parallel to BC



PROOF : Since $DE \parallel BC$, therefore,

$\angle 2 = \angle 4$ ...(1) [Alternate $\angle$s]

and $\angle 3 = \angle 5$ ...(2) [Alternate $\angle$s]

$\therefore$ $\angle 2 + \angle 3 = \angle 4 + \angle 5$ ...(3) [Adding (1) and (2)]

Adding $\angle 1$ to both sides of (3), we have

$\angle 1 + \angle 2 + \angle 3 = \angle 1 + \angle 4 + \angle 5$ ...(4)

But $\angle 4 + \angle 1 + \angle 5 = 180°$ ...(5) [Linear Pair Axiom]

From (4) and (5), we have

$\angle 1 + \angle 2 + \angle 3 = 180°$, i.e., $\angle A + \angle B + \angle C = 180°$

Thus, the sum of the three angles of a triangle is 180°.

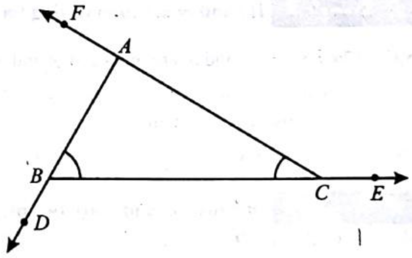
THEOREM 2 : If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.

Given : A triangle ABC of which sides AB, BC and CA are produced.

To prove : $\angle BAF = \angle ABC + \angle ACB$

$\angle CBD = \angle BAC + \angle ACB$

and $\angle ACE = \angle BAC + \angle ABC$



PROOF : $\angle BAF + \angle BAC = 180°$ ...(i) [Linear pair axiom]

In ΔABC, $\angle BAC + \angle ABC + \angle ACB = 180°$ ...(ii)

From equation (i) and (ii), we get

$\angle BAF + \angle BAC = \angle BAC + \angle ABC + \angle ACB$

$\Rightarrow$ $\angle BAF = \angle ABC + \angle ACB$

Similarly we can prove that

$\angle CBD = \angle BAC + \angle ACB$

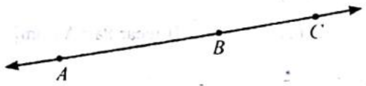
and $\angle ACE = \angle BAC + \angle ABC$

Note : **Questions based on Euclid's Geometry are marked with '$*$' in Miscellaneous Solved Examples and both Exercises-1 and 2**

MISCELLANEOUS

Solved Examples

Example 1 : If A, B and C are three points on a line, and B lies between A and C (Fig.), then prove that $AB + BC = AC$.



SOLUTION : In the figure given above, AC coincides with $AB + BC$.

Also, Euclid's Axiom (4) says that things which coincide with one another are equal to one another. So, it can be deduced that $AB + BC = AC$.

Note that in this solution, it has been assumed that there is a unique line passing through two points.

Example 2 : Consider the following statement : There exists a pair of straight lines that are everywhere equidistant from one another. Is this statement a direct consequence of Euclid's fifth postulate? Explain.

SOLUTION : Take any line l and a point P not on l. Then, by Playfair's axiom, which is equivalent to the fifth postulate, we know that there is a unique line m through P which is parallel to l.

Now, the distance of a point from a line is the length of the perpendicular from the point to the line. This distance will be the same for any point on m from l and any point on l from m. So, these two lines are everywhere equidistant from one another.

Example 3 : If l and m are intersecting lines, $l \parallel p$ and $m \parallel q$, show that p and q also intersect.

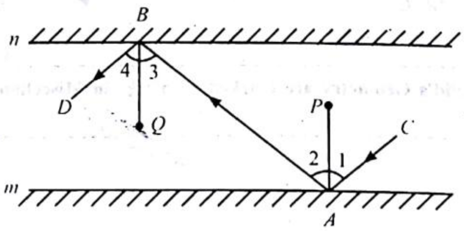
SOLUTION : Since l and m are intersecting and $l \parallel p$

$\Rightarrow$ m and p intersect.

Now, m and p intersect and $m \parallel q$.

$\Rightarrow$ p and q intersect.

Example 4 : In figure, m and n are two plane mirrors parallel to each other. Show that the incident ray CA is parallel to the reflected ray BD.



SOLUTION : Through A and B draw respectively, $AP \perp m$ and $BQ \perp n$, so that $AP \parallel BQ$

$\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

But $\angle 2 = \angle 3$

$\therefore$ $\angle 1 = \angle 4$

or $2\angle 1 = 2\angle 4$, i.e., $\angle CAB = \angle ABD$

But they form a pair of alternate angles $\therefore CA \parallel BD$

EXERCISE 1

FIB Fill in the Blanks

DIRECTIONS: *Complete the following statements with an appropriate word/term to be filled in the blank space(s).*

***1.** .... are the assumptions which are obvious universal truths.

***2.** Things which are equal to the same thing are .... to one another.

***3.** If equals are subtracted from equals, the remainder are ....

***4.** Things which are double of the same things are .... one another.

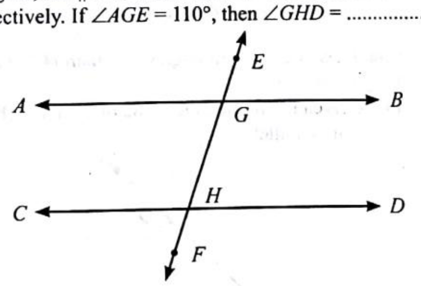
***5.** Two distinct intersecting lines cannot be .... to the same line.

***6.** 'For every line l and for every point P not lying on l, there exists a unique line m passing through P and parallel to ......

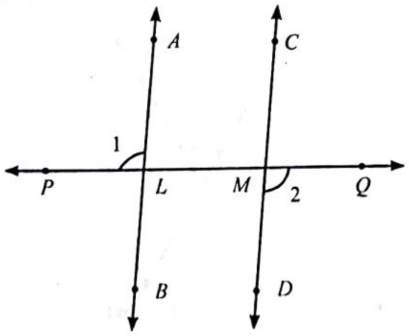
7. If a transversal intersects two parallel lines, then the sum of the interior angles on the same side of the transversal is ....

***8.** Lines which are parallel to the same line are .... to each other.

9. In figure, $AB \parallel CD$, transversal EF cuts them at G and H respectively. If $\angle AGE = 110°$, then $\angle GHD =$ ....



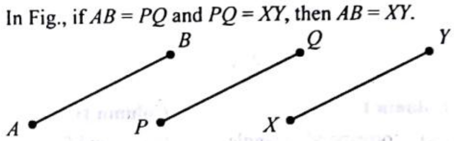
10. In figure, a transversal PQ intersects two parallel line AB and CD at L and M respectively. If $\angle 1 = 95°$, then $\angle 2 =$ ....



T/F True/False

DIRECTIONS: *Read the following statements and write your answer as true or false.*

***1.** In Fig., if $AB = PQ$ and $PQ = XY$, then $AB = XY$.



***2.** Only one line can pass through a single point.

***3.** If two circles are equal, then their radii are equal.

***4.** A terminated line can be produced indefinitely on both the sides.

***5.** There are an infinite number of lines which pass through two distinct points.

6. Three lines are concurrent if they have a common point.

***7.** A line segment has one end-point only

8. Four lines are concurrent if they have a common point.

9. A segment has no length.

10. Three points will be collinear only when they lie on a line.

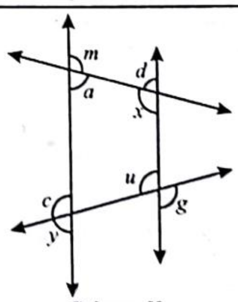
11. A ray has a finite length.

12. Two lines will meet in one point only when they are parallel.

MTC Match the Columns

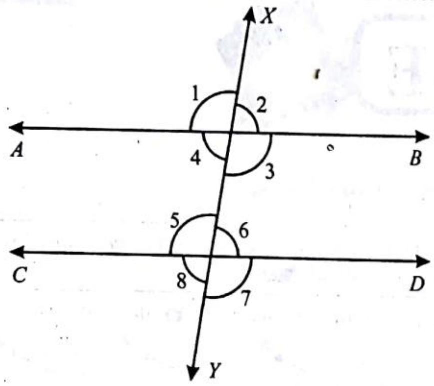
DIRECTIONS: *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column I have to be matched with statements (p, q, r, s) in column II.*

1.



Column I	Column II
(A) angles m and y	(p) alternate exterior pair
(B) angles a and d	(q) alternate interior pair
(C) angles d and u	(r) corresponding pair
(D) angles c and g	(s) vertical pair

2. For the figure shown, Match column I and II correctly



Column I	Column II
(A) corresponding angles	(p) 1 and 5
(B) alternate interior angles	(q) 4 and 6
(C) alternate exterior angles	(r) 1 and 7
(D) interior angles on same side of the transversal	(s) 4 and 5

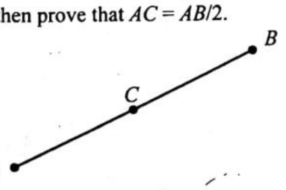
3. Match them correctly

Column I	Column II
(A) Only one line can pass through a	(p) one point
(B) Infinite number of line can pass through	(q) two point
(C) Two distinct lines cannot have more than.... in common	(r) same side
(D) For line segment ....are required.	(s) opposite side

VSAQ Very Short Answer Questions

DIRECTIONS: *Give answer in one word or one sentence.*

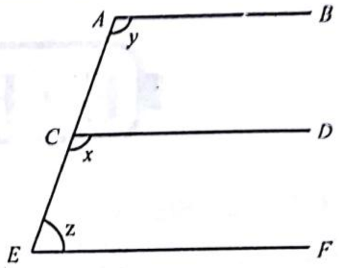
***1.** If a point C lies between two points A and B such that $AC = BC$, then prove that $AC = AB/2$.



2. An angle is 16° more than its complement. What is its measure ?

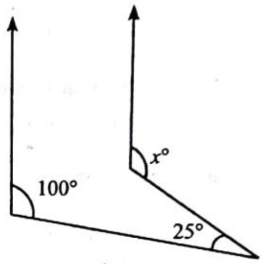
3. Two parallel lines are cut by a transversal such that one of the interior angle is 57°. Find each of the other interior angles.

4. In the given figure, AB, CD and EF are parallel. If $\angle z = 70°$, find the value of x and y.

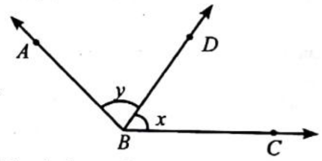


***5.** Why is Axiom 5, in the list of Euclid's axioms, considered a 'universal truth'? (Note that the question is not about the fifth postulate).

***6.** Find $\angle x$:

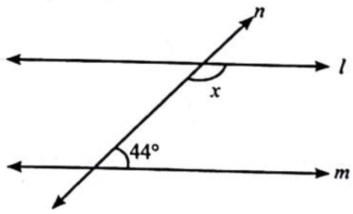


7. For what value of $x + y$ in the given figure, will ABC be a line ? Justify your answer.

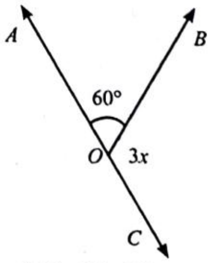


8. Can a triangle have all angles less than 60°? Give reason for your answer.

9. In the given figure, find the value of x for which the lines l and m are parallel.



10. In the given figure, AOC is a line, find x.



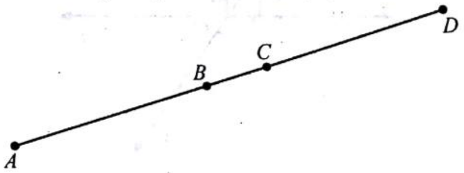
11. In ΔABC, $\angle B = 105°$, $\angle C = 50°$. Find $\angle A$.

SAQ *Short Answer Questions*

DIRECTIONS: *Give answer in 2-3 sentences.*

***1.** Let point C is called a mid-point of line segment AB. Prove that every line segment has one and only one mid-point.

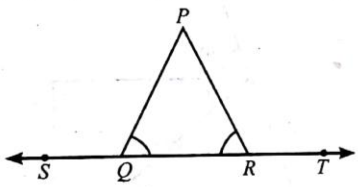
***2.** If $AC = BD$, then prove that $AB = CD$.



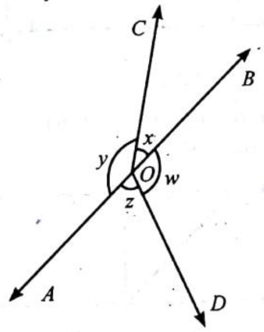
***3.** Does Euclid's fifth postulate imply the existence of parallel lines? Explain.

4. The supplement of an angle is one-fifth of itself. Determine the angle and its supplement.

5. In figure, $\angle PQR = \angle PRQ$, then prove that $\angle PQS = \angle PRT$.

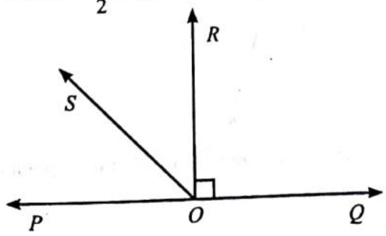


6. In figure, if $x + y = w + z$, then prove that AOB is a line.

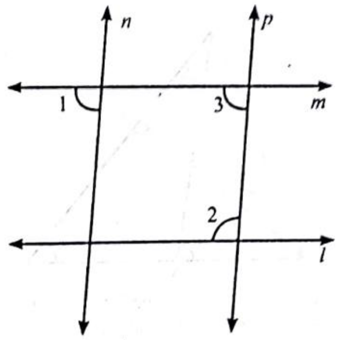


7. In figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that

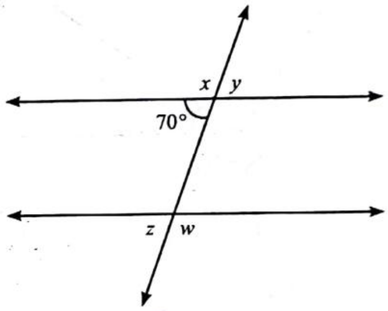
$$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$$



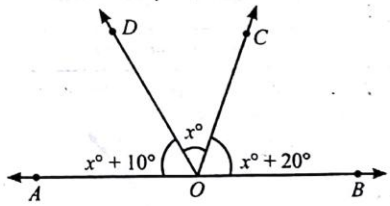
8. In figure if $l \parallel m$, $n \parallel p$, and $\angle 1 = 85°$ then find $\angle 2$.



9. From the adjoining diagrams, calculate $\angle x$, $\angle y$, $\angle z$ and $\angle w$.



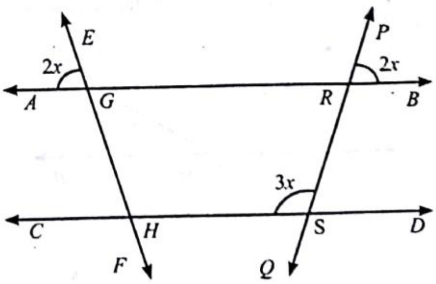
10. In the following figure, find $\angle x$. Further find $\angle BOC$, $\angle COD$ and $\angle AOD$.



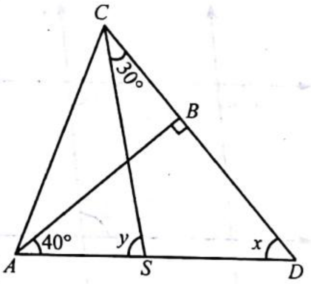
11. In the figure, find the values of

(i) x (ii) $\angle GHS$

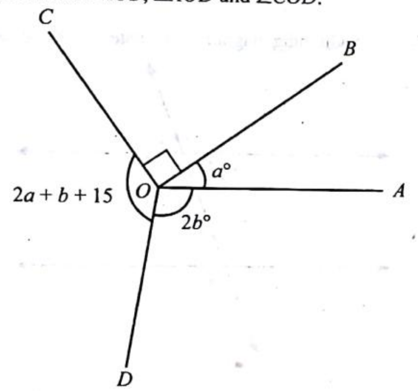
(iii) $\angle PRG$ (iv) $\angle RSD$



12. In figure, if $AB \perp CD$, $\angle BAD = 40°$ and $\angle SCD = 30°$, find x and y.



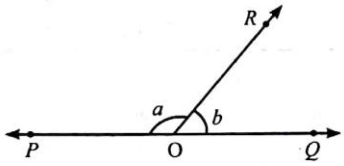
13. In the given figure, $2b - a = 65°$ and $\angle BOC = 90°$, find the measure of $\angle AOB$, $\angle AOD$ and $\angle COD$.



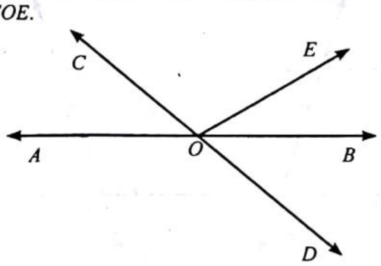
LAQ Long Answer Questions

DIRECTIONS: *Give answer in four to five sentences.*

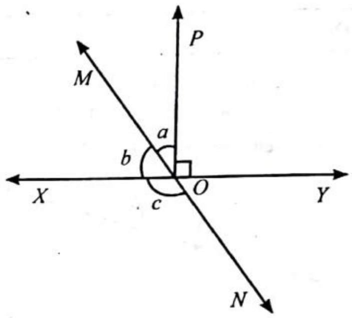
1. In figure, $\angle POR$ and $\angle QOR$ form a linear pair. If $a - b = 80°$, find the values of a and b.



2. In figure, Lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70°$ and $\angle BOD = 40°$, find $\angle BOE$ and reflex $\angle COE$.

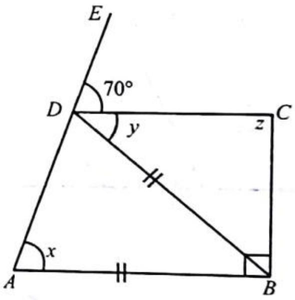


3. In figure, lines XY and MN intersect at O. If $\angle POY = 90°$ and $a : b = 2 : 3$, find c.



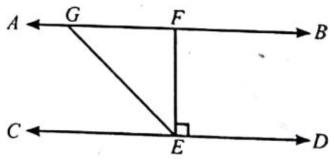
4. It is given that $\angle XYZ = 64°$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$.

5. From the adjoining diagram $AB \parallel DC$, calculate

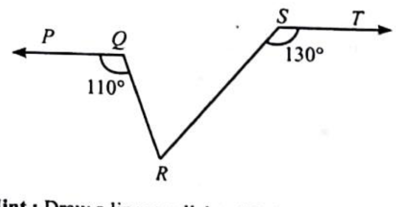


(i) $\angle x$ (ii) $\angle y$ (iii) $\angle z$

6. In figure, if $AB \parallel CD$, $EF \perp CD$ and $\angle GED = 126°$, find $\angle AGE$, $\angle GEF$ and $\angle FGE$.

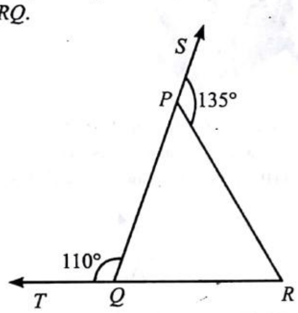


7. In figure, if $PQ \parallel ST$, $\angle PQR = 110°$ and $\angle RST = 130°$, find $\angle QRS$.

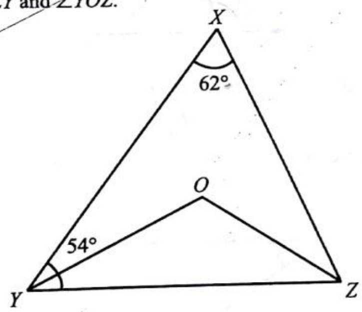


[Hint : Draw a line parallel to ST through point R.]

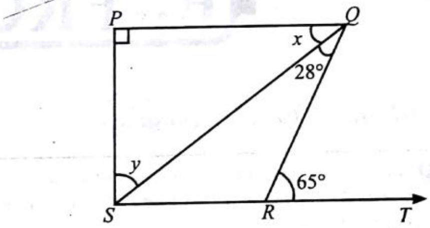
8. In figure, sides QP and RQ of ΔPQR are produced to points S and T respectively. If $\angle SPR = 135°$ and $\angle PQT = 110°$, find $\angle PRQ$.



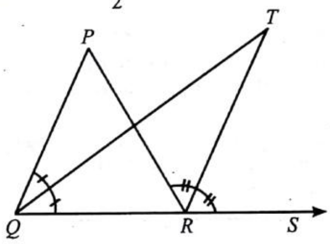
9. In figure, $\angle X = 62°$, $\angle XYZ = 54°$. If YO and ZO are the bisectors of $\angle XYZ$ and $\angle XZY$ respectively of ΔXYZ, find $\angle OZY$ and $\angle YOZ$.



10. In figure, if $PQ \perp PS$, $PQ \parallel SR$, $\angle SQR = 28°$ and $\angle QRT = 65°$, then find the values of x and y.



11. In figure, the side QR of ΔPQR is produced to a point S. If the bisectors of $\angle PQR$ and $\angle PRS$ meet at point T, then prove that $\angle QTR = \frac{1}{2} \angle QPR$



EXERCISE 2

Multiple Choice Questions

DIRECTIONS: *This section contains multiple choice questions. Each questions has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. If two lines intersected by a transversal, then each pair of corresponding angles so formed is –

(a) Equal (b) Complementary
(c) Right angle (d) None of these

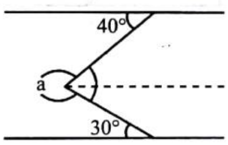
2. An angle is 14° more than its complementary angle then angle is –

(a) 38° (b) 52°
(c) 50° (d) None of these

3. If one angle of triangle is equal to the sum of the other two then triangle is –

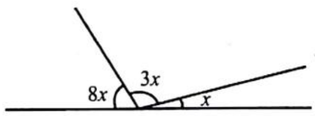
(a) acute a triangle (b) obtuse triangle
(c) right triangle (d) None

4. In the figure, angle a is



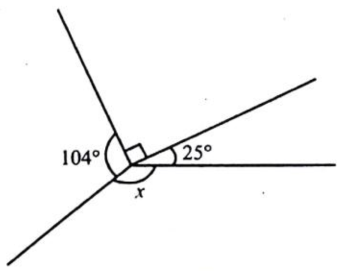
(a) 290° (b) 70°
(c) 105° (d) 45°

5. Value of $\angle x$ =



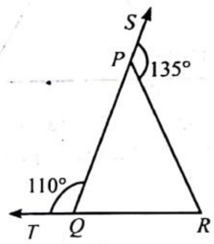
(a) 270° (b) 70°
(c) 15° (d) 45°

6. Value of $\angle x$ =



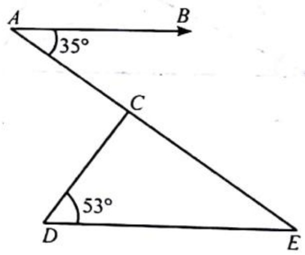
(a) 141° (b) 70°
(c) 105° (d) 45°

7. In figure, sides QP and RQ of $\Delta\ PQR$ are produced to points S and T respectively. If $\angle SPR = 135°$ and $\angle PQT = 110°$, then the value of $\angle PRQ$ is



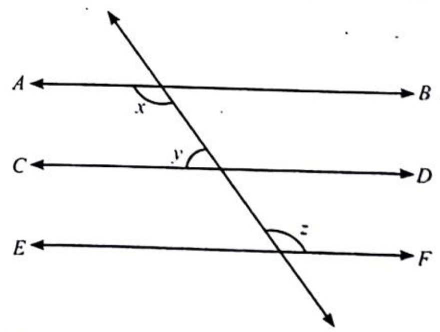
(a) 65° (b) 35°
(c) 75° (d) 30°

8. In figure, if $AB \parallel DE$, $\angle BAC = 35°$ and $\angle CDE = 53°$, $\angle DCE = ?$



(a) 102° (b) 92°
(c) 80° (d) 72°

9. In figure, if $AB \parallel CD$, $CD \parallel EF$ and $y : z = 3 : 7$, $x = ?$



(a) 112° (b) 116°
(c) 96° (d) 126°

MTQ More than One Correct

DIRECTIONS: *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE or MORE may be correct.*

*1. Which of the following is/are correct?
  (a) A plane surface is a surface which lies evenly with the straight line on itself.
  (b) A surface is that which has length and breadth only.
  (c) A line is breadthless length.
  (d) A point is that which has no part.

*2. Which of the following is/are Euclid's axioms?
  (a) Things which are equal to the same thing are never equal to one another.
  (b) If equals are added to equals, the wholes are equal.
  (c) The whole is greater than the part.
  (d) Things which are halves of the same things are equal to one another.

*3. Which of the following is/are Euclid's postulates?
  (a) A straight line may be drawn from any one point to any other point.
  (b) A circle can be drawn with any centre and any radius.
  (c) A terminated line cannot be produced indefinitely.
  (d) All right angles are never equal to one another.

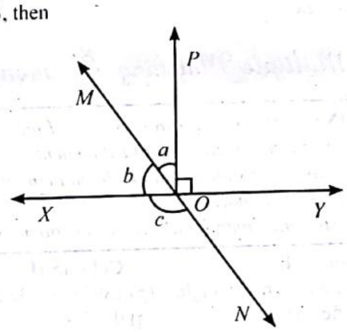
*4. Which of the following is/are incorrect?
  (a) A triangle whose sides are equal, is called a scalene triangle.
  (b) A triangle, each of whose angle is less than 90°, is called an acute triangle.
  (c) If all sides of a polygon are different, it is called a regular polygon.
  (d) A triangle with one of its angles greater than 90°, is known as an obtuse triangle.

5. Which is/are correct among the following given options?
  (a) An angle whose measure is more than 180, but less than 360°, is called reflex angle.
  (b) If the sum of two angles is 90°, then these angles are called supplementary angles.
  (c) If the sum of two angles is 180°, these angles are called complementary angles.
  (d) The sum of angles forming a linear-pair is 360°.

PBQ Passage Based Questions

DIRECTIONS: *Study the given paragraph(s) and answer the following questions.*

In figure, lines XY and MN intersect at O. If $\angle POY = 90°$ and $a : b = 2 : 3$, then



1. Measure of angle 'a' is
  (a) 54° (b) 36°
  (c) 126° (d) none of these
2. Measure of angle 'b' is
  (a) 126° (b) 36°
  (c) 54° (d) none of these
3. Measure of angle 'c' is
  (a) 36° (b) 126°
  (c) 54° (d) none of these

A&R Assertion & Reason

DIRECTIONS: *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.
(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.
(c) If **Assertion** is **correct** but **Reason** is **incorrect**.
(d) If **Assertion** is **incorrect** but **Reason** is **correct**.

1. **Assertion :** Sum of the pair of angles (like 120°, 60°) is supplementary.
   Reason : Two angles, the sum of whose measures is 180°, are called supplementary angles.
2. **Assertion :** If an angle formed by two intersecting lines is 60°, then its vertically opposite angle is 60°
   Reason : If two lines intersect each other, then the vertically opposite angles are equal.
3. **Assertion :** If angles 'a' and 'b' form a linear pair of angles and $a = 40°$, then b = 150°.
   Reason : Sum of linear pair of angles is always 180°.

4. **Assertion :** A triangle can have two obtuse angles.
Reason : Sum of the three angles in a triangle is always 180°.

5. **Assertion :** If two internal opposite angles of a triangle are equal and external angle is given to be 110°, then each of the equal internal angle is 55°.
Reason : A triangle with one of its angle 90°, is called a right triangle.

MMQ Multiple Matching Question

DIRECTIONS : *Following question has four statements (A, B, C and D) given in Column I and statements (p, q, r, s, ...) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in column I with entries in column II.*

1.

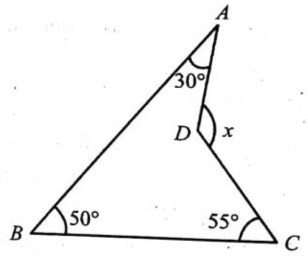
Column I	Column II
(A) Complementary angle	(p) $180° < \theta < 360°$
(B) Reflex angle	(q) $\theta = 100°$
(C) Obtuse triangle	(r) $0 < \theta < 90°$
(D) supplementary angle	(s) (35°, 55°)
	(t) $\theta = 240°$
	(u) (140°, 40°)

HOTS Hots Subjective Questions

DIRECTIONS: *Answer the following questions.*

1. The sides BC, CA and AB of a ΔABC are produced in order, forming exterior angles $\angle ACD$, $\angle BAE$ and $\angle CBF$. Show that $\angle ACD + \angle BAE + \angle CBF = 360°$.

2. Find $\angle ADC$ from the given figure.



3. If the sides of an angle are respectively parallel to the sides of another angle, then prove that these angles are either equal or supplementary.

SOLUTIONS

EXERCISE-1

FILL IN THE BLANKS

1.	Axioms	2.	equal	3.	equal	4.	equal
5.	parallel	6.	l	7.	180°	8.	parallel
9.	70°	10.	25°				

TRUE/FALSE

1.	True	2.	False	3.	True	4.	True
5.	False	6.	True	7.	False	8.	False
9.	False	10.	True	11.	False	12.	False

MATCH THE COLUMNS

1. (A) → p; (B) → q; (C) → r; (D) → p
2. (A) → p; (B) → q; (C) → r; (D) → s
3. (A) → p; (B) → p; (C) → p; (D) → q

VERY SHORT ANSWER QUESTIONS

1. $AC = BC$

   $\Rightarrow \; AC + AC = BC + AC$

   [Equals are added to equals]

   $\Rightarrow \; 2AC = AB \Rightarrow AC = AB/2$

2. Let one angle $= a°$

   Then second angle $= (a + 16)°$

   Now, $a + (a + 16) = 90°$

   $\Rightarrow 2a + 16 = 90° \Rightarrow a = 37°$

   Second angle $= 37° + 16 = 53°$

3. 123°, 57°, 123°
4. $x = y = 110°$
5. This is true for any thing in any part of the world, this is a universal truth.
6. 125°
7. For A, B, C must be lie in a line $x + y = 180°$.
8. No, angle sum must be equal to 180°
9. $x + 44° = 180°$ *i.e.*, $x = 136°$.
10. 40°
11. 25°

SHORT ANSWER QUESTIONS

1. **Given:** C is mid-point of AB.

   To Prove: C is the only mid-point of AB.

   Let a line AB have two mid-points, say, C and D. Then,

   $AC = \frac{1}{2}AB$ and ....(1)

   $AD = \frac{1}{2}AB$ ....(2)

   From (1) and (2)

   $AC = AD$. ($\because$ Things which are equal to the same thing are equal to one another)

2. $AC = BD$ ... (1)

   $AC = AB + BC$ [B lies between A and C] ... (2)

   $BD = BC + CD$ [C lies between B and D] ... (3)

   Substituting (2) and (3) in (1), we get

   $AB + BC = BC + CD$

   $\Rightarrow \; AB = CD$ [Subtracting equals from equal]

3. If a straight line falls on two straight lines m and n such that the sum of the interior angles on one side of is two right angles, then by Euclid's fifth postulate the lines will not meet on this side of. Next, we know that the sum of the interior angles on the other side of line will also be two right angles. Therefore, they will not meet on the other side also. So, the lines m and n never meet and are, therefore, parallel.

4. Let the measure of the angle be $x°$, then the measure of its supplementary angle is $180° - x°$.

   It is given that $180 - x = \frac{1}{5}x$

   $\Rightarrow \; 5(180° - x) = x$

   $\Rightarrow \; 900 - 5x = x \Rightarrow 900 = 5x + x$

   $\Rightarrow \; 900 = 6x \Rightarrow 6x = 900 \Rightarrow x = \frac{900}{6} = 150$

5. By linear pair axiom we have

   $\angle PQS + \angle PQR = 180°$ ...(1)

   Also, Ray RP stands on line ST

   $\therefore \; \angle PRQ + \angle PRT = 180°$ (Linear Pair Axiom) ...(2)

   From (1) and (2), we obtain

   $\angle PQS + \angle PQR = \angle PRQ + \angle PRT$

   Now, Given $\angle PQR = \angle PRQ$

   $\therefore \; \angle PQS = \angle PRT$.

6. $x + y = w + z$ ...(1) (Given)

   $\because$ The sum of all the angles round a point is equal to 360°

   $\therefore \; x + y + w + z = 360°$

   $\Rightarrow \; x + y + x + y = 360°$ (Using (1))

   $\Rightarrow \; 2(x + y) = 360°$

   $\Rightarrow \; x + y = \frac{360°}{20}$

   $\Rightarrow \; x + y = 180°$

   $\therefore \; AOB$ is a line.

   (If the sum of two adjacent angles is 180°, then the non-common arms of the angles form a line)

7. Since $OR \perp PQ$

$\therefore \angle QOR = \angle POR = 90°$ ...(1)

and $\angle QOS = \angle QOR + \angle ROS$ ...(2)

Also, $\angle POS = \angle POR - \angle ROS$ ...(3)

Subtracting (2) and (3),

$\Rightarrow \angle QOS - \angle POS = (\angle QOR - \angle POR) + 2\angle ROS$

$= 2\angle ROS$ (Using (1))

$\Rightarrow \angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$

8. $\because n \parallel p$ and m is transversal

$\therefore \angle 1 = \angle 3 = 85°$ (Corresponding angles)

Also, $m \parallel \ell$ & p is transversal

$\therefore \angle 2 + \angle 3 = 180°$ ($\because$ Consecutive interior angles)

$\Rightarrow \angle 2 + 85° = 180°$

$\Rightarrow \angle 2 = 180° - 85°$

$\Rightarrow \angle 2 = 95$

9. $\angle y = 70°$ ..... (vertical opp. angle)

$\angle x + 70 = 180°$

.... (adjacent angles on a st. line or linear pair)

$\therefore \angle x = 180 - 70 = 110°$

$\angle z = 70°$ .....(corresponding angles)

$\angle z + \angle w = 180°$

..... (adjacent angles on a st. line or linear pair)

$\therefore 70 + \angle w = 180°$

$\therefore \angle w = 180° - 170° = 110°$

10. $x = 50°, 70°, 50°, 60°$

11. (i) 36° (ii) 72°

(iii) 108° (iv) 108°

12. $x = 50°, y = 80°$

13. 35°, 100°, 135°

LONG ANSWER QUESTIONS

1. $\because \angle POR$ and $\angle QOR$ form a linear pair

$\therefore \angle POR + \angle QOR = 180°$ (Linear pair axiom)

or $a + b = 180°$ .... (1)

But $a - b = 80°$ [Given] .... (2)

Adding eqs (1) and (2), we get

$2a = 260° \quad \therefore \quad a = \frac{260}{2} = 130°$

Substituting the value of a in (1), we get

$130° + b = 180°$

$b = 180° - 130° = 50°$

2. $\angle AOC = \angle BOD$ (Vertically Opposite Angles)

But Given $\angle BOD = 40°$ ...(1)

$\therefore \angle AOC = 40°$ (Alternate angle) ...(2)

Now, Given $\angle AOC + \angle BOE = 70°$

$\Rightarrow \angle BOE = 70° - 40°$ (from 2)

$\Rightarrow \angle BOE = 30°$

Again, Reflex $\angle COE = \angle COD + \angle BOD + \angle BOE$

$\because$ Ray OA stands on line CD

$= \angle COD + 40° + 30°$(Using (1) and (2))

$= 180° + 40° + 30°$

$= 250°$ ($\because$ Ray OA stands on line CD)

3. Since Ray OP stands on line XY and from Linear Pair Axiom

$\therefore \angle POX + \angle POY = 180°$

Given $\angle POY = 90°$

$\Rightarrow \angle POX = 90°$

$\Rightarrow \angle POM + \angle XOM = 90°$

$\Rightarrow a + b = 90°$ (from figure) ...(1)

Now, we have

$a : b = 2 : 3$

$\Rightarrow \frac{a}{b} = \frac{2}{3} \Rightarrow \frac{a}{2} = \frac{b}{3} = k$ (say) $\Rightarrow a = 2k$ and $b = 3k$

Putting the values of a and b in (1), we get

$2k + 3k = 90°$

$\Rightarrow k = 18°$

$\therefore \left.\begin{array}{l} a = 2k = 2(18°) = 36° \\ b = 3k = 3(18°) = 54° \end{array}\right\}$ ...(2)

Again, Ray OX stands on Line MN

$\therefore \angle XOM + \angle XON = 180°$ (Linear Pair Axiom)

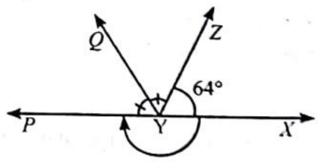
$\Rightarrow b + c = 180°$

$\Rightarrow 54° + c = 180°$ (Using (2))

$\Rightarrow c = 180° - 54°$

$\Rightarrow c = 126°$

4. $\angle XYZ + \angle ZYP = 180°$ (Linear Pair Axiom and YZ stands on line PX)



Given, $\angle XYZ = 64°$

$\therefore 64° + \angle ZYP = 180°$

$\Rightarrow \angle ZYP = 116°$ ...(1)

Also, Ray YQ bisects $\angle ZYP$

$\therefore \angle PYQ = \angle ZYQ = \frac{1}{2} \angle ZYP$

$= \frac{1}{2}(116°) = 58°$ (Using (1)) ...(2)

$\therefore$ Reflex $\angle QYP = 360° - 58° = 302°$

($\because$ The sum of all the angles round a point is equal to 360°)

Again, $\angle XYQ = \angle XYZ + \angle ZYQ$

$= 64° + 58°$

$= 122°$.

($\because \angle XYZ = 64°$ (given) and $\angle ZYQ = 58°$ [From (2)])

5. $\angle x = \angle EDC = 70°$ ..... (corresponding angles)

Now, $\angle ADB = x = 70°$ ... $[\because AD = DB]$

In ΔABD, $\angle ABD = 180 - \angle x - \angle x$

$= 180 - 70 - 70 = 40°$

$\Rightarrow \angle BDC = \angle ABD = 40°$ (alternate angles)

$\Rightarrow \angle y = 40°$

Since, $AB \parallel DC$

$\Rightarrow \angle z + 90 = 180°$

$\Rightarrow \angle z = 180 - 90 = 90°$

6. (i) $\angle AGE = \angle GED = 126°$ (Alternate Interior Angles)

(ii) We have $\angle GED = 126°$

$\angle GEF + \angle FED = 126°$

$\Rightarrow \angle GEF + 90° = 126°$ (From the figure, $\angle FED = 90°$)

$\Rightarrow \angle GEF = 126° - 90° = 36°$

(iii) Since, CD is a line

$\therefore$ By linear sum property, we have

$\angle GEC + \angle GEF + \angle FED = 180°$

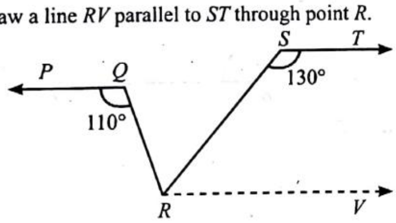
$\Rightarrow \angle GEC + 36° + 90° = 180°$

$\Rightarrow \angle GEC = 180° - 126° = 54°$

Now, $\angle FGE = \angle GEC = 54°$

(Alternate Interior Angles)

7. Draw a line RV parallel to ST through point R.



$\therefore \angle RST + \angle SRV = 180°$

($\because$ Sum of the consecutive interior angles on the same side of the transversal is 180°)

$\Rightarrow \angle SRV = 180° - 130° = 50°$ ....(1)

Also, $\angle QRV = \angle PQR = 110°$

(Alternate Interior Angles)

$\Rightarrow \angle QRS + \angle SRV = 110°$

$\Rightarrow \angle QRS = 110° - 50° = 60°$ Using (1)

8. $\angle PQT + \angle PQR = 180°$ ...(linear pair)

$\Rightarrow 110° + \angle PQR = 180°$

$\Rightarrow \angle PQR = 180° - 110° = 70°$ ...(1)

Again, $\angle SPR + \angle QPR = 180°$

$135° + \angle QPR = 180°$ ($\because QS$ is a line)

$\Rightarrow \angle QPR = 180° - 135° = 45°$ ...(2)

In ΔPQR,

$\angle PQR + \angle QPR + \angle PRQ = 180°$

(By Δ's angle sum property)

From (1) and (2)

$70° + 45° + \angle PRQ = 180°$

$\Rightarrow \angle PRQ = 180° - 115° = 65°$

9. From ΔXYZ, we have

$\angle XYZ + \angle YZX + \angle ZXY = 180°$

(By Δ's angle sum prpoerty)

$\Rightarrow 54° + \angle YZX + 62° = 180°$

$\Rightarrow \angle YZX = 180° - 116° = 64°$ ...(1)

Given YO is the bisector of $\angle XYZ$

$\therefore \angle XYO = \angle OYZ = \frac{1}{2}\angle XYZ = \frac{1}{2}(54°) = 27°$ ...(2)

Similarly

$\angle XZO = \angle OZY = \frac{1}{2} \angle YZX = \frac{1}{2}(64)° = 32°$ ... (3)

($\because ZO$ is the bisector of $\angle YZX$)

Now, consider ΔOYZ, which gives

$\angle OYZ + \angle OZY + \angle YOZ = 180°$ (By angle sum property)

[Using (2) and (3)]

$\Rightarrow 27° + 32° + \angle YOZ = 180°$

$\Rightarrow \angle YOZ = 180° - 59° = 121°$.

10. As we know the exterior angle is equal to the sum of the two interior opposite angles

$\therefore \angle QRT = \angle RQS + \angle QSR$

$\Rightarrow 65° = 28° + \angle QSR$

$\Rightarrow \angle QSR = 65° - 28° = 37°$

Also given $PQ \perp SP$

$\therefore \angle QPS = 90°$

Also $PQ \parallel SR$ gives

$\angle QPS + \angle PSR = 180°$

($\because$ The sum of consecutive interior angles on the same side of the transversal is 180°)

$\angle PSR = 90°$

$\Rightarrow \angle PSQ + \angle QSR = 90° \Rightarrow y + 37° = 90°$

$\Rightarrow y = 90° - 37° = 53°$

Now, from ΔPQS, we have

$\angle PQS + \angle QSP + \angle QPS = 180°$

(By angle sum property of a Δ)

$\Rightarrow x + y + 90° = 180°$

$\Rightarrow x + 53° + 90° = 180°$

$\Rightarrow x = 180° - 143° = 37°$.

11. Since , $\angle TRS$ is an exterior angle of ΔTQR

$\therefore \angle TRS = \angle TQR + \angle QTR$ ...(1)

($\because$ The exterior angle is equal to sum of the two interior opposite angles)

Again $\angle PRS$ is an exterior angle of ΔPQR

$\therefore \angle PRS = \angle PQR + \angle QPR$ ...(2)

$\Rightarrow 2\angle TRS = 2\angle TQR + \angle QPR$

($\because QT$ is the bisector of $\angle PQR$ and RT is the bisector of $\angle PRS$)

$\Rightarrow 2(\angle TRS - \angle TQR) = \angle QPR$ ...(3)

From (1),

$\angle TRS - \angle TQR = \angle QTR$ ...(4)

From (3) and (4), we obtain

$2\angle QTR = \angle QPR$

$\Rightarrow \angle QTR = \frac{1}{2}\angle QPR$

EXERCISE - 2

MULTIPLE CHOICE QUESTIONS

1. (d) 2. (b) 3. (c) 4. (a) 5. (c)
6. (a) 7. (a) 8. (b) 9. (d)

MORE THAN ONE CORRECT

1. (a, b, c, d) 2. (b, c, d)
3. (a, b) 4. (a, c) 5. (b, c, d)

PASSAGE BASED QUESTIONS

1. (b) $\angle a = 36°$ 2. (c) $\angle b = 54°$
3. (b) $\angle c = 126°$

ASSERTION AND REASON

1. (a) 2. (a) 3. (d)
4. (d) 5. (b)

MULTIPLE MATCHING QUESTIONS

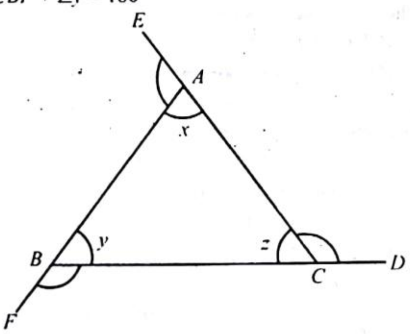
1. (A) → s; (B) → p, t; (C) → q; (D) → u

HOTS SUBJECTIVE QUESTIONS

1. $\angle ACD + \angle z = 180°$

$\angle BAE + \angle x = 180°$

$\angle CBF + \angle y = 180°$



$\angle ACD + \angle BAE + \angle CBF + \angle x + \angle y + \angle z = 540°$ ..(1)

Now put, $\angle x + \angle y + \angle z = 180°$ in eqn. (1)

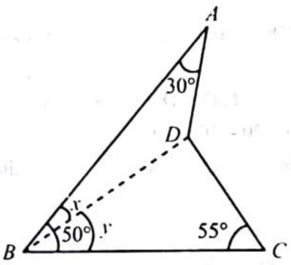
We get $\angle ACD + \angle BAE + \angle CBF = 360°$

2. Construction: Join BD

Now in, ΔABD

$\angle x + \angle ADB + 30° = 180°$ ...(1)

In ΔBDC, $\angle y + \angle BDC + 55° = 180°$ ...(2)



By adding (1) and (2)

$\angle x + \angle ADB + 30° + \angle y + \angle BDC + 55° = 360°$

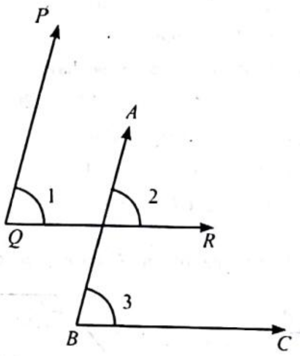
Also, $\angle x + \angle y = 50°$

Then $\angle ADB + \angle BDC + 135° = 360°$

$\Rightarrow \angle ADB + \angle BDC = 225°$

i.e., $x = \angle ADC = 135°$

3. **Case I :**



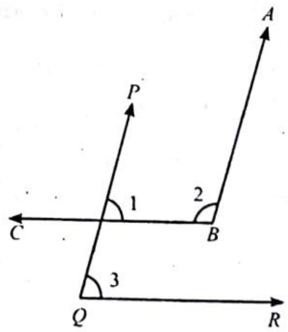
As $QR \parallel BC$

$\therefore \angle 2 = \angle 3$ and $PQ \parallel AB$

Then $\angle 1 = \angle 2$

Hence, $\angle 1 = \angle 2 = \angle 3$

Case II:



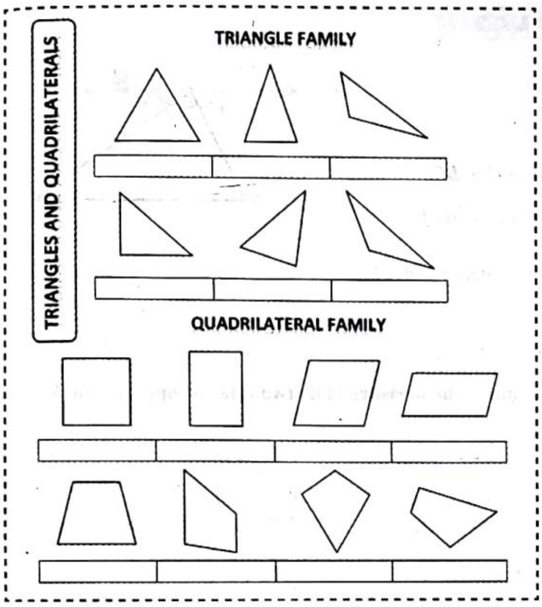
As $QR \parallel CB$

$\therefore \angle 1 = \angle 3$

but $\angle 1 + \angle 2 = 180°$ then

$\angle 3 + \angle 2 = 180°$ supplementary.

($\because \angle 1 = \angle 3$)



6
CHAPTER

Triangles and Quadrilaterals

INTRODUCTION

A triangle is a closed figure formed by three intersecting lines. In triangle ABC denoted by ΔABC; AB, BC, CA are three sides and A, B, C are three vertices [Fig. (i)]

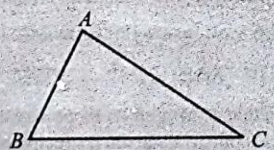


Fig. (i)

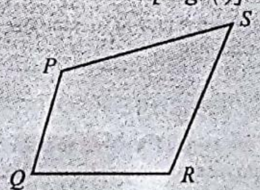


Fig. (ii)

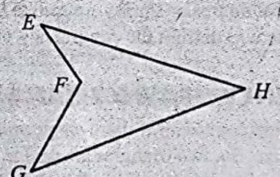


Fig. (iii)

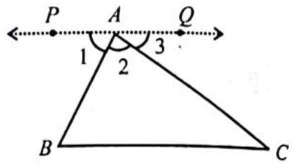
If there are four points in a plane and no three out of these four points are collinear, then on joining these four points in pair in same order, we obtained four sided closed figures (ii) and (iii), called quadrilateral. In this chapter, we will consider only quadrilateral of the type given in figure (ii), called convex quadrilateral (in which any part of line segment joining any two points in side the quadrilateral does not lie outside the quadrilateral) but not as given in fig. (iii), called concave quadrilateral (in which some part of some line segments joining any two points inside the quadrilateral lie outside the quadrilateral).

In quadrilateral PQRS [Fig. (ii)] ; PQ, QR, RS SP are four sides and P, Q, R, S are four vertices.

In this chapter, we shall study about the congruence of triangles, some more properties of triangles and inequality in a triangle.

We shall also study about different types of quadrilaterals, their properties and specially those of parallelograms.

TRIANGLES



THEOREM 1 : The sum of the angles of a triangle is 180°.

Given : A triangle ABC.

To Prove : $\angle A + \angle B + \angle C = 180°$

Construction : Draw a line PQ passing through the vertex A and parallel to BC.

PROOF : $\angle 1 + \angle 2 + \angle 3 = 180°$ ...... (i) [Linear Pair Axiom]

Since $PQ \parallel BC$

$\therefore$ $\angle 1 = \angle B$ and $\angle 3 = \angle C$ .... (ii) [Alternate interior angles]

From equation (i) and (ii), we get

$\angle B + \angle 2 + \angle C = 180°$

i.e. $\angle B + \angle A + \angle C = 180°$

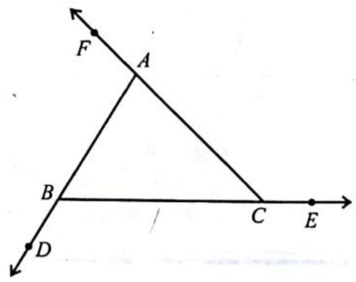
THEOREM 2 : The measure of an exterior angle is equal to the sum of the measure of its two interior opposite angles.

Given : A triangle ABC of which sides AB, BC and CA are produced.

To Prove : $\angle BAF = \angle ABC + \angle ACB$

$\angle CBD = \angle BAC + \angle ACB$

and $\angle ACE = \angle BAC + \angle ABC$



PROOF : $\angle BAF + \angle BAC = 180°$ ...... (i) [Linear Pair Axiom]

In ΔABC,

$\angle BAC + \angle ABC + \angle ACB = 180°$ .... (ii)

From equation (i) and (ii), we get

$\angle BAF + \angle BAC = \angle BAC + \angle ABC + \angle ACB$

$\Rightarrow$ $\angle BAF = \angle ABC + \angle ACB$

Similarly we can prove that

$\angle CBD = \angle BAC + \angle ACB$

and $\angle ACE = \angle BAC + \angle ABC$

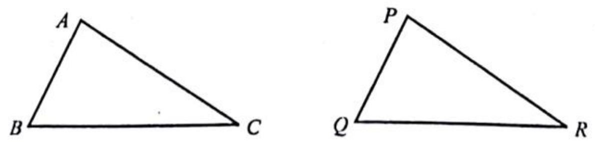
CONGRUENCY OF TWO PLANE GEOMETRICAL FIGURES (OR SHAPES)

Two geometrical figures or shapes are said to be congruent if they have same size and same shape i.e. exactly overlap each other. In this chapter, we will discuss the congruency of two triangles.

CONGRUENCY OF TWO TRIANGLES

Two triangles are congruent if there shape and size are same. The three angles of a triangle determine its shape and its three sides determine its size. So, two triangles are said to be congruent if three sides and three angles of a triangle are respectively equal to the corresponding sides and angles of other triangle.

In other words, two triangles are congruent if each one of them can be made to superpose on the other, so as to cover it exactly.



If ΔABC is congruent to ΔPQR, we wirte $\Delta ABC \cong \Delta PQR$, in which sides AB, BC, CA are corresponding sides of sides PQ, QR, RP respectively and $\angle A$, $\angle B$, $\angle C$ are corresponding angles of $\angle P$, $\angle Q$, $\angle R$ respectively. If two triangles are congruent, then their corresponding parts (sides and angles) are equal.

$\therefore$ $AB = PQ$, $BC = QR$, $CA = RP$, $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$

Note that we write in short 'CPCT' for corresponding parts of congruent triangles.

Now check which of the triangles Fig.(ii) to (v), given below are congruent to triangle ABC *Fig (i)*.

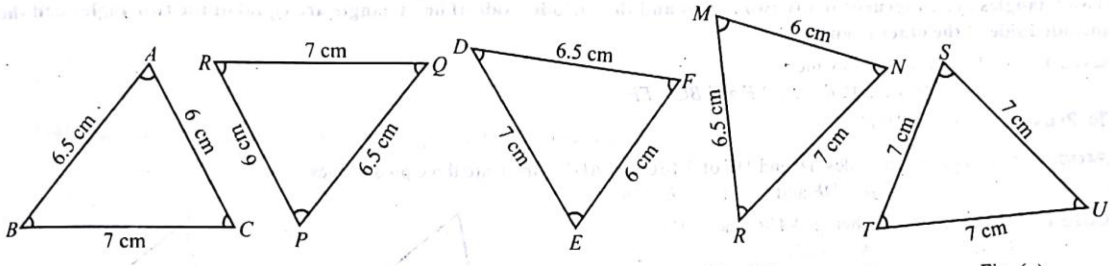


Fig. (i) Fig. (ii) Fig. (iii) Fig. (iv) Fig. (v)

Cut out each of the triangles Fig. (ii) to (v). Turn them around and try to cover ΔABC. You will observe that $\Delta ABC \cong \Delta PQR$; because sides AB and PQ, BC and QR, CA and RP ; and also $\angle A$ and $\angle P$, $\angle B$ and $\angle Q$, $\angle C$ and $\angle R$ overlap each other and hence they are corresponding to each other. Similarly we can see that ΔDEF and ΔMNR are congruent to ΔABC but ΔSTU is not congruent to ΔABC because ΔABC and ΔSTU do not cover to each other.

$\Delta ABC \cong \Delta PQR$ can be also written as $\Delta ACB \cong \Delta PRQ$, because in this case the correspondence of sides and angles between the two triangles does not change.

But $\Delta ABC \cong \Delta PQR$ can not be written as $\Delta ABC \cong \Delta PRQ$, because in this case the correspondence of sides and angles between two the two triangles is as follow.

$AB = PR$, $BC = RQ$, $CA = QP$, $\angle A = \angle P$, $\angle B = \angle R$ and $\angle C = \angle Q$, which is different as for $\Delta ABC \cong \Delta PQR$.

Therefore, to write the congruency of triangles in symbolic form, it is necessary to keep in mind the exact sides correspondence of sides and angles between the two triangles.

SUFFICIENT CONDITIONS (OR RULE OR CRITERIA) FOR CONGRUENCE OF TRIANGLES

In the previous section, we have studied that two triangles are congruent if and only if the corresponding sides and the corresponding angles of two triangles are equal. In this section we shall prove that if three properly choosen conditions out of the six conditions (3 sides and 3 angles of one triangle equal to corresponding 3 sides and 3 angles of other triangle are equal) are satisfied, then the other three are automatically satisfied and hence the two triangles are congruent.

The followings are the sufficient conditions for congruency of two triangles.

(i) Side-Angle-Side
(ii) Angle-Side-Angle
(iii) Side-Side-Side and
(iv) Right Angle-Hypotenus-Side

Let us now discuss these three conditions (or criteria) which ensure the congruence of two triangles.

(I) SIDE-ANGLE-SIDE (SAS) RULE FOR CONGRUENCY OF TWO TRIANGLES

If two triangles have two sides and the included angle of the one equal to the corresponding sides and the included angle of the other, then the triangles are congruent.

Given : ΔABC and ΔDEF in which $AB = DE$, $AC = DF$ and $\angle A = \angle D$

To Prove : $\Delta ABC \cong \Delta DEF$

PROOF : Place ΔABC over ΔDEF such that A falls on D and AB falls along DE.

Since $AB = DE$, so B falls on E.

Since, $\angle A = \angle D$, so AC will fall along DF.

Also, $AC = DF$

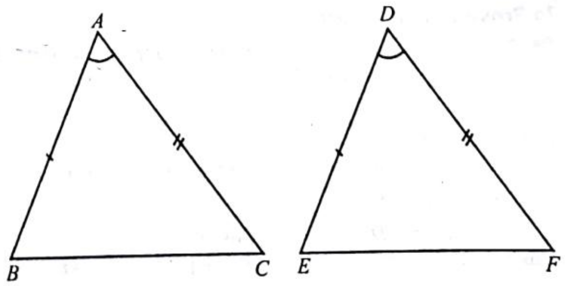
$\therefore$ C will fall on F.

Thus, AC will coincide with DF.

And, therefore, BC will coincide with EF.

$\therefore$ ΔABC coincides with ΔDEF.

Hence, $\Delta ABC \cong \Delta DEF$



(II) ANGLE-SIDE-ANGLE (ASA) RULE FOR CONGRUENCY OF TWO TRIANGLES

Two triangles are congruent if any two angles and the included side of one triangle are equal of the two angles and the included side of the other triangle.

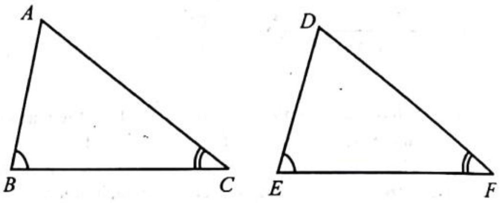
Given : ΔABC and ΔDEF, in which

$\angle ABC = \angle DEF$, $\angle ACB = \angle DFE$ and $BC = EF$

To Prove : $\Delta ABC \cong \Delta DEF$

PROOF : On comparing the sides AB and DE of ΔABC and ΔDEF, there are three possibilities :

(i) $AB = DE$ (ii) $AB < DE$ and (iii) $AB > DE$



Case (i) : When $AB = DE$, then in ΔABC and ΔDEF

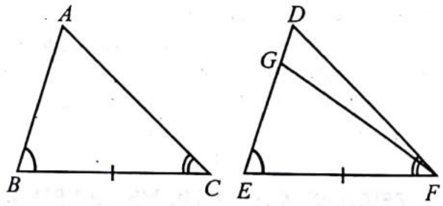
$AB = DE$ (Assumed)

$\angle ABC = \angle DEF$ (Given)

$BC = EF$ (Given)

Hence, $\Delta ABC \cong \Delta DEF$ [By SAS criterian].

Case (ii) : When $AB < DE$, then we take point G on DE such that $AB = GE$ and join GF as shown in figure.



Now in ΔABC and ΔGEF

$AB = GE$ (Assumed)

$BC = EF$ (Given)

$\angle ABC = \angle GEF$ (Given) $[\because \angle GEF = \angle DEF]$

$\therefore$ $\Delta ABC \cong \Delta GEF$ (By SAS Rule)

Hence $\angle ACB = \angle GFE$ .... (1)

But $\angle ACB = \angle DFE$ (Given) .... (2)

Therefore from (1) and (2), we have

$\angle DFE = \angle GFE$

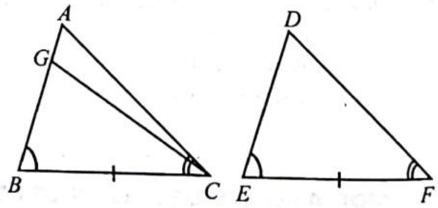
which is impossible unless GF coincide with DF and hence G coincides with D.

$\therefore$ $AB = DE$

Hence, $\Delta ABC \cong \Delta DEF$ [By SAS criterian]

Case (iii) : When $AB > DE$, then we take a point G on AB such that $BG = DE$ and show as in case (ii) that G must coincide with A,

i.e., $AB = DE$ and hence, $\Delta ABC \cong \Delta DEF$ (SAS rule)



Hence in all three cases $\Delta ABC \cong \Delta DEF$

Since the sum of the three interior angles of a triangle is 180°, therefore if two angles of one triangle are equal to the two angles of another triangle, then the third angle of the first triangle will automatically be equal to the third angle of the second triangle. On the basis of this result, the following corollary arise.

COROLLARY (ANGLE-ANGLE-SIDE)

Two triangles are congruent if any two pairs of angles and one pair of corresponding sides are equal.

Given : In ΔABC and ΔDEF,

$\angle A = \angle D$, $\angle B = \angle E$ and $BC = EF$

To Prove : $\Delta ABC \cong \Delta DEF$

PROOF : The sum of three interior angle of a triangle is 180°, then

$\angle A + \angle B + \angle C = 180°$ .....(1)

$\angle D + \angle E + \angle F = 180°$ .....(2)

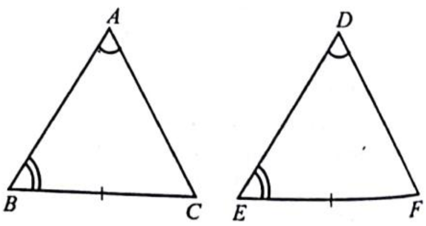
From (1) and (2),

$\angle A + \angle B + \angle C = \angle D + \angle E + \angle F$ .....(3)

$\angle B = \angle E$

$\angle A = \angle D$ (Given)

Hence, $\angle C = \angle F$ [By (3)] ..... (4)



Now in ΔABC and ΔDEF

$\angle B = \angle E$ (Given)

$BC = EF$ (Given)

$\angle C = \angle F$ [By (4)]

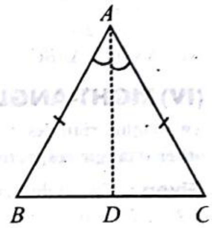
Hence, $\Delta ABC \cong \Delta DEF$ (By ASA rule)

THEOREM 3 : The angles opposite to equal sides of a triangle are equal.

Given : ΔABC in which $AB = AC$.

To Prove : $\angle B = \angle C$

Construction : Draw AD, bisector of angle $\angle BAC$ which meets BC at D.



PROOF : In ΔABD and ΔACD

$AB = AC$ (Given)

$\angle BAD = \angle CAD$ (By construction)

$AD = AD$ (Common side)

Therefore $\Delta ABD \cong \Delta ACD$ (By SAS Rule)

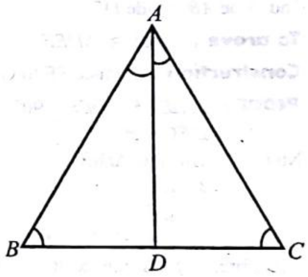
Hence corresponding angles $\angle B = \angle C$

THEOREM 4 : The sides opposite to equal angles of a triangles are equal.

Given : ΔABC, in which $\angle B = \angle C$

To prove : $AB = AC$

Construction : Draw AD, the bisector of angle $\angle BAC$ which meets BC at D.



PROOF : In ΔABD and ΔACD

$\angle B = \angle C$ (Given)

$AD = AD$ (Common side)

$\angle BAD = \angle CAD$ (By construction)

Therefore $\Delta ABD \cong \Delta ACD$ (By ASA)

Hence corresponding sides, $AB = AC$

(III) SIDE-SIDE-SIDE (SSS) RULE FOR CONGRUENCY OF TWO TRIANGLES

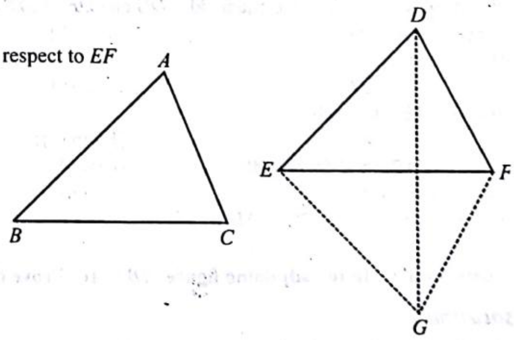
Two triangles are congruent if the three sides of one triangle are equal to the three sides of the other triangle.

Given : ΔABC and ΔDEF in which

$AB = DE$, $BC = EF$ and $AC = DF$

To Prove: $\Delta ABC \cong \Delta DEF$

Construction : Draw a line segment EG at the other side of ED with respect to EF such that $EG = AB$ and $\angle ABC = \angle FEG$ then join GF and DG.



PROOF : In ΔABC and ΔGEF

$AB = GE$ (By construction)

$\angle ABC = \angle GEF$ (By construction)

$BC = EF$ (Given)

Hence by SAS rule,

$\Delta ABC \cong \Delta GEF$

Hence the corresponding sides and angles are equal

$\therefore$ $\angle A = \angle EGF, AC = GF$ .... (1)

Now $AB = EG$ (By construction)

$AB = DE$ (Given)

$\therefore$ $GE = DE$ .... (2)

Similarly $AC = GF$ [From (1)]

and $AC = DF$ (Given)

$\therefore$ $GF = DF$ .... (3)

Now in ΔEDG the opposite angle of equal sides EG and DE are equal

$\therefore$ $\angle EDG = \angle EGD$ .... (4)

Similarly in ΔFDG the opposite angle of equal sides GF and DF are equal

$\angle GDF = \angle DGF$ .... (5)

Adding equation (4) and (5), we get

$\angle EDG + \angle GDF = \angle EGD + \angle DGF$

$\Rightarrow \quad \angle EDF = \angle EGF$ .... (6)

From eq. (1), $\angle A = \angle EGF$ .... (7)

Hence, from (6) and (7), we have

$\angle A = \angle EDF$ .... (8)

In ΔABC and ΔDEF

$AB = DE$ (Given)

$\angle A = \angle EDF$ [By (8)]

$AC = DF$ (Given)

By SAS rule, $\Delta ABC \cong \Delta DEF$

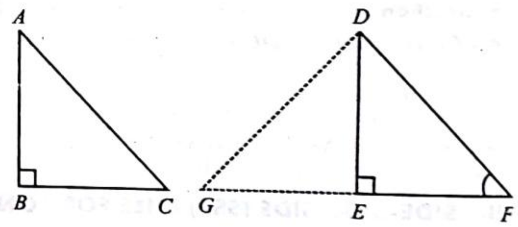
(IV) RIGHT-ANGLE-HYPOTENUSE-SIDE (RHS) RULE FOR CONGRUENCY OF TWO TRIANGLES

Two right triangles are congruent if the hypotenuse and a side of one triangle are equal to hypotenuse and a side of the other triangle respectively.

Given : Two right triangles ΔABC and ΔDEF, in which
$\angle B = \angle E = 90°$
Hypotenuse AC = Hypotenuse DF
and Side AB = Side DE

To prove : $\Delta ABC \cong \Delta DEF$

Construction : Produce FE to G such that $GE = BC$ and join G and D.



PROOF : In ΔDEF $\angle DEF = 90°$

$\therefore \quad \angle DEG = 90°$ .... (1)

Now in ΔABC and ΔDEG

$AB = DE$ (Given)

$BC = GE$ (By construction)

$\angle ABC = \angle DEG = 90°$ [By (1)]

Therefore, by SAS rule, $\Delta ABC \cong \Delta DEG$

Hence the corresponding sides and angles are equal

$\therefore \quad AC = DG$ and $\angle C = \angle G$ .... (2)

But given that $AC = DF$ .... (3)

From (2) and (3) $DG = DF$ .... (4)

$\because$ Angles opposite to the equal sides DG and DF of ΔDGF are equal

Hence $\angle G = \angle F$ .... (5)

Therefore, from (2) and (5)

$\angle C = \angle F$ .... (6)

Now in ΔABC and ΔDEF

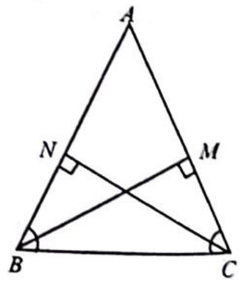
$\angle C = \angle F$ [From (6)]

$\angle ABC \cong \angle DEF = 90°$ (Given)

and $AB = DE$ (Given)

Hence by ASA rule, $\Delta ABC \cong \Delta DEF$

Illustration 1 : In the adjoining figure, $AB = AC$. Prove that $BM = CN$



SOLUTION :

In ΔABC, $AB = AC$ (given)

$\therefore \quad \angle ABC = \angle ACB$ (angles opposite to equal sides are equal)

In Δs BCM and CBN,

$\angle N = \angle M$ (each = 90°)

$\angle ABC = \angle ACB$ (from above)

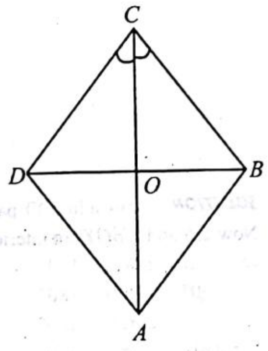
$BC = BC$ (common)

$\therefore \quad \Delta BCM \cong \Delta CBN$ (A.A.S. rule of congruency)

$\Rightarrow \quad BM = CN$ (CPCT)

Illustration 2 : In a quadrilateral $ABCD$, AC bisects $\angle C$ and $BC = CD$.

Prove that : (i) $AB = AD$ (ii) AC is the perpendicular bisector of BD.



SOLUTION :

(i) In triangles ADC and ABC,

$CD = CB$ (Given)

$CA = CA$ (Common)

$\angle DCA = \angle BCA$ (Given)

$\therefore$ $\Delta DCA \cong \Delta BCA$ (By SAS rule of congruency)

$\therefore$ $AD = AB$

(ii) In triangles DCO and BCO

$CD = CB$ (Proved in (i) above)

$CO = CO$ (common)

$\angle DCO = \angle BCO$ (Given)

$\therefore$ $\Delta DCO \cong \Delta BCO$ (By SAS rule of congruency)

$\therefore$ $\angle COD = \angle COB$

Since $\angle COD$ and $\angle COB$ are linear pair angles, therefore $\angle DOC + \angle BOC = 180°$

$\Rightarrow$ $\angle COD + \angle COD = 180°$

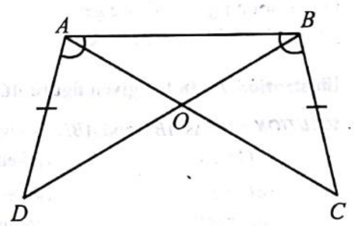
$\Rightarrow$ $\angle COD = 90°$

$\therefore$ CO is perpendicular to DB

$\therefore$ CA is perpendicular to DB

Illustration 3 : In the adjoining figure, ABC and BAD are two triangles on the same base AB such that $BC = AD$ and $\angle ABC = \angle BAD$.

Prove that : (i) AC = BD (ii) $\angle$ACB = $\angle$BDA (iii) CO = DO



SOLUTION : In Δs ABC and BAD, we have :

$AB = BA$ (Common)

$BC = AD$ (Given)

$\angle ABC = \angle BAD$ (Given)

$\therefore$ $\Delta ABC \cong \Delta BAD$ (SAS rule]

$\therefore$ $AC = BD$ and $\angle ACB = \angle BDA$ and therefore, $\angle OCB = \angle ODA$

Again, in Δs BOC and AOD, we have :

$BC = AD$ (Given)

$\angle OCB = \angle ODA$ (Proved)

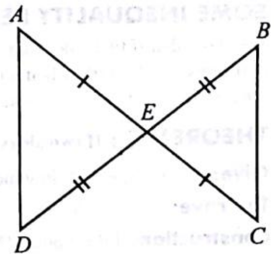
and $\angle BOC = \angle AOD$ (Vertically opposite angles]

$\therefore$ $\Delta BOC \cong \Delta AOD$ (AAS rule)

Hence, $CO = DO$.

Illustration 4 : In figure, $AE = EC$ and $DE = BE$. Prove that

(i) $\angle AED = \angle CEB$ (ii) $\angle A = \angle C$



SOLUTION : Given that as in figure

$AE = EC$

$DE = BE$ .... (1)

In ΔAED and ΔBEC

$AE = EC$ (Given)

$\angle AED = \angle CEB$ (Vertically opposite angles)

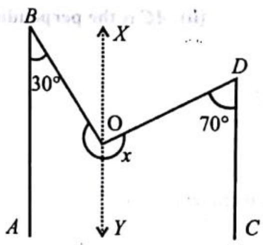
$DE = EB$ (Given)

Then by SAS Rule

$\Delta AED \cong \Delta CEB$

Hence, $\angle A = \angle C$ and $\angle D = \angle B$ (By CPCT)

Illustration 5 : In the given figure, $AB \parallel CD$ and $\angle ABO = 30°$, $\angle ODC = 70°$, find x.



SOLUTION : Draw a line XY passing through O parallel to AB and CD.

Now $\angle B$ and $\angle BOY$ are interior angles on the same side of transversal OA between parallel lines AB and XY.

$\Rightarrow \quad \angle B + \angle BOY = 180°$

$30° + \angle BOY = 180°$

$\angle BOY = 150°$

Similarly $\angle D$ and $\angle DOY$ are interior angles on the same side of transversal OD between parallel lines XY and CD.

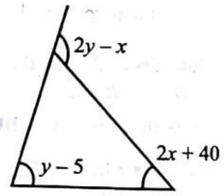
$\Rightarrow \quad \angle D + \angle DOY = 180°$

$70° + \angle DOY = 180°$

$\angle DOY = 180° - 70° = 110°$

Now, $x = \angle BOY + \angle DOY = 150° + 110° = 260°$

Illustration 6 : In the given figure find y, if $x = 5°$



SOLUTION : We know that in a triangle exterior angle = sum of two interior opposite angles.

$2y - x = y - 5 + 2x + 40$

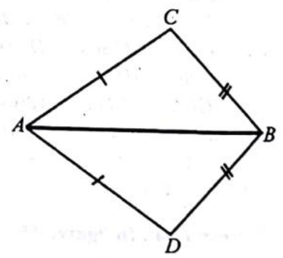
$2y - y = 2x + 35 + x$

$y = 3x + 35$

On substituting $x = 5°$, we get

$y = 3 \times 5 + 35 = 15 + 35 = 50°$

Illustration 7 : In the given figure $AC = AD$ and $CB = DB$ then prove that AB bisects $\angle A$.



SOLUTION : In Δs ABC and ABD, we have

$AC = AD$ (given)

$BC = BD$ (given)

$AB = AB$ (common)

So, by SSS rule

$\Delta ABC \cong \Delta ABD$

$\therefore \quad \angle BAC = \angle BAD$ (corresponding parts of congruent triangles are equal)

$\therefore \quad AB$ bisects $\angle A$.

SOME INEQUALITY RELATIONS IN A TRIANGLE

We know that if two sides of a triangle are equal then the angles opposite to them are also equal and vice-versa. What happens the two sides of a triangle when angles opposite to them are unequal and vice-versa ? We get the answer of such type of questions in the form of following three theorems.

THEOREM 5 : If two sides of a triangle are unequal, then the angle opposite to the longer side is larger (or greater).

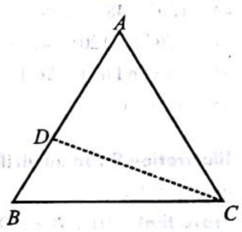
Given: A triangle ABC in which $AB > AC$

To Prove: $\angle C > \angle B$

Construction: Take a point D on AB such that $AC = AD$ join CD.

PROOF : In ΔACD, $AC = AD$

Therefore, $\angle ACD = \angle ADC$ .... (1)



But $\angle ADC$ is an exterior angle of ΔBDC

$\therefore \quad \angle ADC > \angle B$ .... (2)

From (1) and (2), we have

$\angle ACD > \angle B$ .... (3)

By figure, $\angle ACB > \angle ACD$ .... (4)

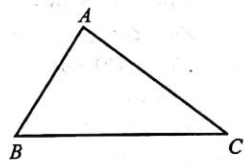
From (3) and (4), we have

$\angle ACB > \angle ACD > \angle B$

$\Rightarrow \quad \angle ACB > \angle B$

$\Rightarrow \quad \angle C > \angle B$

THEOREM 6 : In a triangle, the side opposite to the longer (greater) angle is longer.



Given: A triangle ABC in which $\angle B > \angle C$

To Prove: $AC > AB$

PROOF: We have the following three possibilities for sides AB and AC of ΔABC.

(i) $AC = AB$ (ii) $AC < AB$ and (iii) $AC > AB$

Case (i) : If $AC = AB$:

If $AC = AB$, then opposite angles of equal sides are equal. Hence, $\angle B = \angle C$.

But it is given that $\angle B > \angle C$

Hence $AC \neq AB$

Case (ii) : If $AC < AB$:

We know that the angle opposite to longer side is larger.

$\therefore \quad AC < AB \Rightarrow \angle C > \angle B$,

which is also contrary to given $\angle B > \angle C$

Hence, $AC \nless AB$

Case (iii) : If $AC > AB$:

We are left only this possibility which must be true.

Hence, $AC > AB$.

THEOREM 7 : The sum of any two sides of a triangle is greater than its third side.

Given : A triangle ABC.

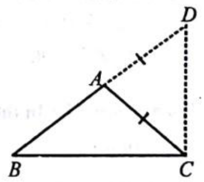
To Prove :

(i) $AB + BC > AC$

$BC + AC > AB$

$AC + AB > BC$

Construction : Produce BA to D, such that $AD = AC$ and join DC.



PROOF : In ΔADC, by construction $AD = AC$, then opposite angles are equal,

$\therefore \quad \angle ACD = \angle ADC$ .... (1)

Now, $\quad \angle BCD > \angle ACD$ .... (2)

From (1) and (2), we have

$\angle BCD > \angle ACD = \angle ADC$

Therefore, $BD > BC$ [side opposite to larger angle in a triangle is longer]

$\Rightarrow \quad BA + AD > BC \quad [\because BD = BA + AD]$

$\Rightarrow \quad BA + AC > BC \quad$ [By construction $AD = AC$]

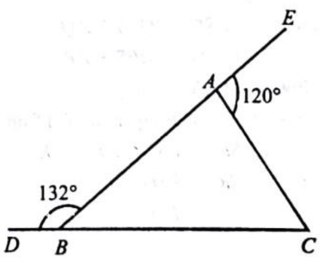
$\Rightarrow \quad AB + AC > BC$

Similarly, we can show that

$AB + BC > AC$

$BC + AC > AB$

Illustration 8 : In figure, $\angle DBA = 132°$ and $\angle EAC = 120°$. Show that $AB > AC$



SOLUTION : As DBC is a straight line,

$132° + \angle ABC = 180°$

$\Rightarrow \quad \angle ABC = 180° - 132° = 48°$

For ΔABC, $\angle EAC$ is an exterior angle

$120° = \angle ABC + \angle BCA$ (Ext. $\angle$ = sum of two opp. int. $\angle s$)

$\Rightarrow$ $120° = 48° + \angle BCA$

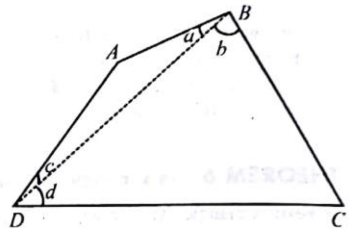
$\Rightarrow$ $\angle BCA = 120° - 48° = 72°$

Thus, we find that $\angle BCA > \angle ABC$

$\Rightarrow$ $AB > AC$ (side opposite to greater angle is greater)

Illustration 9 : In quadrilateral $ABCD$, AB is the shortest side and DC is the longest side.

Prove that : (i) $\angle B > \angle D$ (ii) $\angle A > \angle C$



SOLUTION : Join B and D

In ΔABD, $AD > AB$ (given, AB is the shortest side)

$\therefore$ $\angle a > \angle c$ .... (1) (angle opposite to greater side is greater)

In ΔBCD, $CD > BC$ (given, CD is the longest side)

$\therefore$ $\angle b > \angle d$ .... (2) (angle opposite to greater side is greater)

$\therefore$ $\angle a + \angle b > \angle c + \angle d$ (Adding (1) and (2))

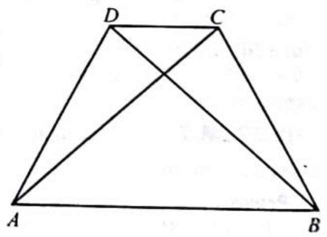
$\Rightarrow$ $\angle B > \angle D$

Similarly, by joining AC, it can be proved that $\angle A > \angle C$.

Illustration 10 : In figure, $ABCD$ is a quadrilateral. Show that

(i) $AB + BC + CD + DA > 2AC$

(ii) $AB + BC + CD + DA > AC + BD$



SOLUTION : We know that the sum of the two sides of a triangle is greater than its third side.

In ΔABC, $AB + BC > AC$ .... (1)

In ΔADC, $AD + DC > AC$ .... (2)

In ΔABD, $AB + AD > BD$ .... (3)

In ΔBCD, $BC + CD > BD$ .... (4)

Adding (1) and (2), we get

$AB + BC + AD + CD > 2\,AC$ .... (i)

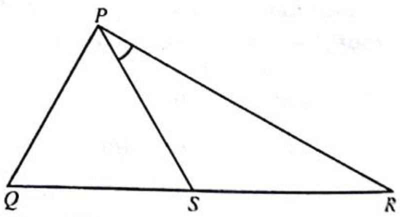
Again adding (1), (2), (3) and (4), we get

$2\,(AB + BC + DO + AD) > 2\,(AC + BD)$

$\Rightarrow$ $AB + BC + DC + AD > AC + BD$ .... (ii)

Illustration 11 : In the given figure, $PR > PQ$ and PS bisects $\angle QPR$.

Prove that : $\angle PSR > \angle PSQ$.



SOLUTION : In ΔPQR

$PR > PQ$ (given)

$\Rightarrow$ $\angle Q > \angle R$ [angle opposite to larger side is greater]

$\Rightarrow$ $\angle Q - \angle R > 0$

Also, $\angle PSR = \angle QPS + \angle Q$ .... (1)

$\angle PSQ = \angle RPS + \angle R$ .... (2) [exterior angle is equal to sum of interior opposite angles]

Now, $\angle QPS = \angle RPS$ [PS bisects $\angle QPR$]

On subtracting the equations (2) from (1), we get

$\angle PSR - \angle PSQ = \angle Q - \angle R$

$\Rightarrow$ $\angle PSR - \angle PSQ > 0$

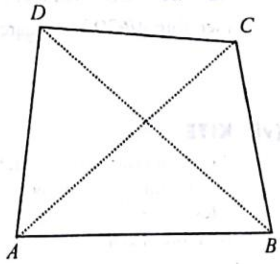
$\Rightarrow$ $\angle PSR > \angle PSQ$

QUADRILATERALS

A quadrilateral is a closed figure obtained by joining four point in a plane (with no three points collinear) in an order.

Since, 'quad' means 'four' and 'lateral' means 'sides', therefore 'quadrilateral' means 'a figure bounded by four sides'.

Every quadrilateral has : (i) Four vertices (ii) Four sides (iii) Four angles and (iv) Two diagonals.



The given figure shows a quadrilateral $ABCD$, which has

(i) four vertices namely : A, B, C and D.

(ii) four sides namely : AB, BC, CD and DA.

(iii) four angles namely : $\angle A$, $\angle B$, $\angle C$ and $\angle D$.

(iv) Two diagonals, namely : AC and BD.

A diagonal is a line segment obtained on joining the opposite vertices. Thus on joining opposite vertices A, and C, we get the diagonals AC. In the same way, on joining the opposite vertices B and C, we get the diagonals BD.

Adjucent sides : Two sides of a quadrilateral having a common end point are called its adjacent or consecutive sides. (AB, BC), (BC, CD), (CD, DA) and (DA, AB) are four pair of its adjacent sides.

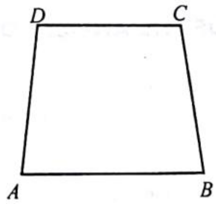
Opposite sides : Two sides of a quadrilateral having no common end point are called its opposite sides.

Adjacent Angles : Two angles of a quadrilateral having a common arm are called its adjacent angles. ($\angle A$, $\angle B$), ($\angle B$, $\angle C$), ($\angle C$, $\angle D$) and ($\angle D$, $\angle A$) are four pairs of adjacent angles.

Opposite angels : Two angles of a quadrilateral having no common arm are called its opposite angles. ($\angle A$, $\angle C$) and ($\angle B$, $\angle D$) are two pair of opposite angles.

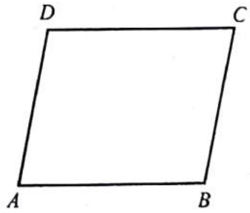
TYPES OF QUADRILATERALS

(i) TRAPEZIUM



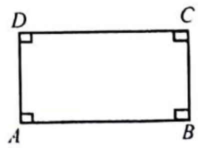
It is a quadrilateral in which one pair of opposite sides are parallel.
In the quadrilateral $ABCD$, sides AB and DC are parallel, therefore it is a trapezium.
If non-parallel sides of a trapezium are equal , it is known as isosceles trapezium.

(ii) PARALLELOGRAM



It is a quadrilateral in which both the pairs of opposite sides are parallel.
The adjoining figure shows a quadrilateral $ABCD$ in which AB is parallel to DC and AD is parallel to BC, therefore $ABCD$ is a parallelogram.

(iii) RECTANGLE



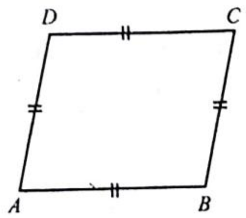
It is a quadrilateral whose each angle is 90°.
The adjoining figure shows a quadrilateral $ABCD$ in which each angle is 90°
i.e., $\angle A = 90° = \angle B = \angle C = \angle D$

(A) $\angle A + \angle B = 90° + 90° = 180° \Rightarrow AD \parallel BC$

(B) $\angle B + \angle C = 90° + 90° = 180° \Rightarrow AB \parallel DC$

Therefore rectangle $ABCD$ is a parallelogram.
Hence, A rectangle is a parallelogram also.

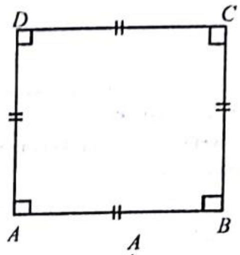
(iv) RHOMBUS



It is a quadrilateral whose all the sides are equal.
The adjoining figure shows a quadrilateral $ABCD$ in which $AB = BC = CD = DA$, therefore it is a rhombus.

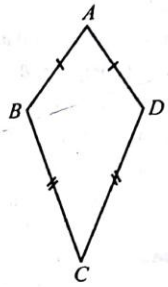
(v) SQUARE

It is a quadrilateral whose all the sides are equal and each angle is 90°.
The adjoining figure shows a quadrilateral $ABCD$ in which
$AB = BC = CD = DA$ and $\angle A = \angle B = \angle C = \angle D = 90°$,
therefore $ABCD$ is a square.



(vi) KITE

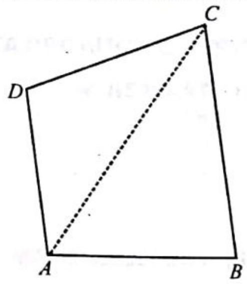
It is a quadrilateral in which two pairs of adjacent sides are equal.
The adjoining figure shows a quadrilateral $ABCD$ in which adjacent sides AB and AD are equal and also adjacent sides BC and CD are equal therefore $ABCD$ it is a kite or kite shaped figure.



Note :
(i) Square, rectangle and rhombus are all parallelograms.
(ii) Kite and trapezium are not parallelograms.
(iii) A square is a rectangle.
(iv) A square is a rhombus.
(v) A parallelogram is a trapezium.

SUM OF THE ANGLES OF A QUADRILATERAL

Consider a quadrilateral $ABCD$. Join A and C to get the diagonal AC which divides the quadrilateral $ABCD$ into two triangles ABC and ADC.



We know the sum of the angles of each triangle is 180° (2 right angles).

$\therefore$ In ΔABC, $\angle CAB + \angle B + \angle BCA = 180°$ ...(i)

In ΔADC, $\angle DAC + \angle D + \angle DCA = 180°$ ...(ii)

On adding (i) and (ii), we get

$(\angle CAB + \angle DAC) + \angle B + \angle D + (\angle BCA + \angle DCA) = 180° + 180°$

$\angle A + \angle B + \angle D + \angle C = 360°$

Thus, the sum of the angles of a quadrilateral is 360° (4-right angles)

Illustration 12 : The angle of a quadrilateral are in the ratio 2 : 3 : 7 : 6. Find the measure of each angle of the quadrilateral.

SOLUTION : Let the measure of the angles of the given quadrilateral be $(2x)°$, $(3x)°$, $(7x)°$, $(6x)°$ respectively.

Then, $2x + 3x + 7x + 6x = 360$ [$\because$ The sum of the angles of a quadrilateral is 360°]

$\Rightarrow$ $18x = 360$

$\Rightarrow$ $x = 20$

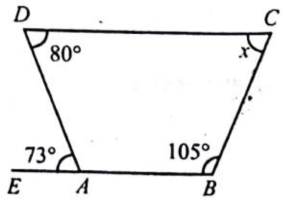
$\therefore$ First angle $= (2x)° = (2 \times 20)° = 40°$

Second angle $= (3x)° = (3 \times 20)° = 60°$

Third angle $= (7x)° = (7 \times 20)° = 140°$

Fourth angle $= (6x)° = (6 \times 20)° = 120°$

Illustration 13 : Using the information given in figure, calculate the value of x.



SOLUTION : Since, EAB is a straight line

$\therefore$ $\angle DAE + \angle DAB = 180°$

$\Rightarrow$ $73° + \angle DAB = 180°$

i.e., $\angle DAB = 180° - 73° = 107°$

Since, the sum of the angles of quadrilateral $ABCD$ is $360°$

$\therefore \quad 107° + 105° + x + 80° = 360°$

$\Rightarrow \quad 292° + x = 360°$

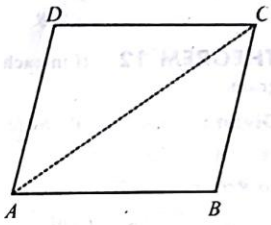
and, $x = 360° - 292° = 68°$

THEOREM 8 : A diagonal of a parallelogram divides the parallelogram into two congruent triangles.

Given : A parallelogram $ABCD$.

To Prove: A diagonal divides the parallelogram into two congruent triangles i.e., if diagonal AC is drawn then $\Delta ABC \cong \Delta CDA$ and if diagonal BD is drawn then $\Delta ABD \cong \Delta CDB$

Construction: Join A and C.



PROOF : Since, $ABCD$ is a parallelogram

$AB \parallel DC$ and $AD \parallel BC$.

In ΔABC and ΔCDA,

$\angle BAC = \angle DCA$ [Alternate angles]

$\angle BCA = \angle DAC$ [Alternate angles]

and $AC = AC$ [Common side]

$\therefore \quad \Delta ABC \cong \Delta CDA$ [By ASA rule]

Similarly, we can prove that

$\Delta ABD \cong \Delta CDB$

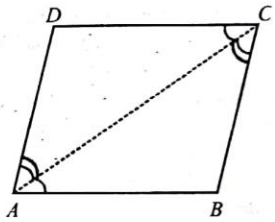
THEOREM 9 : In a parallelogram, opposite sides are equal.

Given : A parallelogram $ABCD$ in which $AB \parallel DC$ and $AD \parallel BC$.

To Prove : Opposite sides are equal

i.e., $AB = DC$ and $AD = BC$.

Construction : Join A and C



PROOF : In ΔABC and ΔCDA

$\angle BAC = \angle DCA$ [Alternate angels]

$\angle BCA = \angle DAC$ [Alternate angels]

$AC = AC$ [Common]

$\therefore \quad \Delta ABC \cong \Delta CDA$ [By A.S.A rule]

$\Rightarrow \quad AB = DC$ and $BC = AD$ [By C.P.C.T]

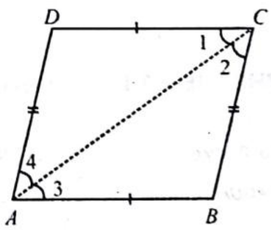
THEOREM 10 : If each pair of opposite sides of a quadrilateral is equal, then it is a parallelogram.

Given : A quadrilateral $ABCD$ in which $AB = DC$ and $AD = BC$.

To Prove: $ABCD$ is a parallelogram

i.e., $AB \parallel DC$ and $AD \parallel BC$.

Construction: Join A and C



PROOF : In ΔABC and ΔCDA

$AB = DC$ [Given]

$BC = AD$ [Given]

and $AC = AC$ [Common]

$\therefore \quad \Delta ABC \cong \Delta CDA$ [By S.S.S. rule]

$\Rightarrow \quad \angle 1 = \angle 3$ [By C.P.C.T]

and $\angle 2 = \angle 4$ [By C.P.C.T.]

But $\angle 1, \angle 3$ and $\angle 2, \angle 4$ are pair of alternate angles and whenever alternate angles are equal, the lines are parallel.

$\therefore \quad AB \parallel DC$ and $AD \parallel BC$.

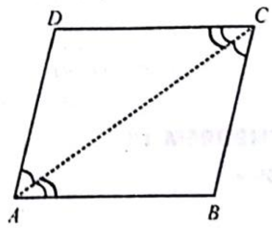
$\Rightarrow \quad ABCD$ is a parallelogram.

THEOREM 11 : In a parallelogram, opposite angles are equal.

Given: A parallelogram $ABCD$ in which $AB \parallel DC$ and $AD \parallel BC$.

To Prove : Opposite angles are equal i.e., $\angle A = \angle C$ and $\angle B = \angle D$.

Construction : Draw diagonal AC.



PROOF : In ΔABC and ΔCDA,

$\angle BAC = \angle DCA$ [Alternate angles] ...(i)

$\angle BCA = \angle DAC$ [Alternate angles] ...(ii)

$AC = AC$ [Common]

$\therefore \quad \Delta ABC \cong \Delta CDA$ [By A.S.A. rule]

$\Rightarrow \quad \angle B = \angle D$ [By C.P.C.T]

Also, on adding (i) and (ii), we get

$\angle BAC + \angle DAC = \angle BCA + \angle DCA$

$\Rightarrow \quad \angle BAD = \angle DCB$ i.e., $\angle A = \angle C$

$\therefore \quad \angle A = \angle C$ and $\angle B = \angle D$

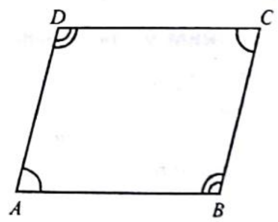
THEOREM 12 : If in each pair of opposite angles of a quadrilateral opposite angles are equal, then it is a parallelogram.

Given : A quadrilateral $ABCD$ in which opposite angles are equal.

i.e., $\angle A = \angle C$ and $\angle B = \angle D$.

To Prove : $ABCD$ is a parallelogram

i.e., $AB \parallel DC$ and $AD \parallel BC$.



PROOF : Since, the sum of the angles of a quadrilateral is 360°

$\Rightarrow \quad \angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow \quad \angle A + \angle D + \angle A + \angle D = 360°$ [$\angle A = \angle C$ and $\angle B = \angle D$]

$\Rightarrow \quad 2\angle A + 2\angle D = 360°$

$\Rightarrow \quad \angle A + \angle D = 180°$

But this is the sum of interior angles on the same side of transversal AD and whenever the sum of interior angles on the same side of the transversal is 180°, the lines are parallel.

$\therefore \quad AB \parallel DC$

In the same way,

$\angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow \quad \angle A + \angle B + \angle A + \angle B = 360°$ [$\angle A = \angle C$ and $\angle B = \angle D$]

$\Rightarrow \quad 2\angle A + 2\angle B = 360°$

$\Rightarrow \quad \angle A + \angle B = 180°$

But this is also the sum of interior angles on the same side of transversal AB.

$\Rightarrow \quad AD \parallel BC$

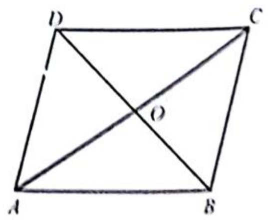
Since, $AB \parallel DC$ and $AD \parallel BC$

$\Rightarrow \quad ABCD$ is a parallelogram.

THEOREM 13 : The diagonals of a parallelogram bisect each other.

Given : A parallelogram $ABCD$. Its diagonals AC and BD intersect each other at point O.

To Prove : Diagonals AC and BD bisect each other i.e., $OA = OC$ and $OB = OD$



PROOF : Since, $AB \parallel DC$ and BD is transversal

$\therefore \quad \angle ABO = \angle CDO$ [Alternate angles]

Since, $AB \parallel DC$ and AC is transversal,

$\therefore \quad \angle BAO = \angle DCO$ [Alternate angles]

Also, $AB = DC$ [Opp. sides of a parallelogram]

$\therefore$ In ΔAOB and ΔCOD :

$\angle ABO = \angle CDO$

$\angle BAO = \angle DCO$ and $AB = DC$

$\Rightarrow \quad \Delta AOB \cong \Delta COD$ [By A.S.A.]

$\Rightarrow \quad OA = OC$ and $OB = OD$ [By C.P.C.T.]

THEOREM 14 : If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

Given: A quadrilateral $ABCD$ whose diagonals AC and BD bisect each other at point O

i.e., $OA = OC$ and $OB = OD$

To Prove: $ABCD$ is a parallelogram i.e., $AB \parallel DC$ and $AD \parallel BC$.

PROOF : In ΔAOB and ΔCOD

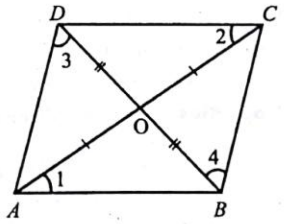
$OA = OC$ [Given]

$OB = OD$ [Given]

and, $\angle AOB = \angle COD$ [Vertically opposite angles]

$\Rightarrow$ $\Delta AOB \cong \Delta COD$ [By S.A.S.]

$\Rightarrow$ $\angle 1 = \angle 2$ [By C.P.C.T.]



But these are alternate angles and whenever alternate angles are equal, the lines are parallel.

$\therefore$ AB is parallel to DC i.e., $AB \parallel DC$

In the same way $\Delta AOD \cong \Delta COB$ [By S.A.S.]

$\Rightarrow$ $\angle 3 = \angle 4$

But these are also alternate angles

$\Rightarrow$ $AD \parallel BC$

Now, $AB \parallel DC$ and $AD \parallel BC$

$\Rightarrow$ $ABCD$ is a parallelogram.

THEOREM 15 : A quadrilateral is a parallelogram, if a pair of opposite sides is equal and parallel.

Given: A quadrilateral $ABCD$ in which $AB \parallel DC$ and $AB = DC$.

To Prove : $ABCD$ is a parallelogram

i.e., $AB \parallel DC$ and $AD \parallel BC$.

Construction : Join A and C.

PROOF : Since AB is parallel to DC and AC is transversal

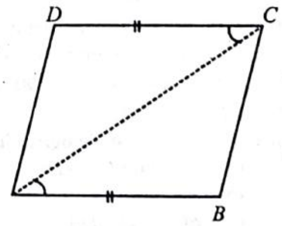
$\angle BAC = \angle DCA$ [Alternate angles]

$AB = DC$ [Given]

and $AC = AC$ [Common side]

$\Rightarrow$ $\Delta BAC \cong \Delta DCA$ [By S.A.S.]

$\Rightarrow$ $\angle BCA = \angle DAC$ [By C.P.C.T.]



But these are alternate angles and whenever alternate angles are equal, the lines are parallel.

$\Rightarrow$ $AD \parallel BC$

Now, $AB \parallel DC$ (given) and $AD \parallel BC$

$\Rightarrow$ $ABCD$ is a parallelogram.

Note : (i) In order to prove that given quadrilateral is a parallelogram, prove that
- (a) Opposite angles of the quadrilateral are equal, or
- (b) Diagonals of the quadrilateral bisect each other, or
- (c) A pair of opposite sides is parallel and is of equal length, or
- (d) Opposite sides are equal.

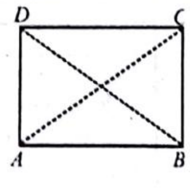
(ii) Every diagonal divides the parallelogram into two congruent triangles.

SPECIAL TYPES OF PARALLELOGRAM

RECTANGLE

A parallelogram is a rectangle if any of its angles is a right angle (90°).

The adjacent figure is a parallelogram $ABCD$ with angle $A = 90°$. Therefore, $ABCD$ is rectangle.



Reason : When $\angle A = 90°$

$\Rightarrow$ $\angle C = \angle A = 90°$ [Opp. angles of a parallelogram are equal]

Since, the adjacent angles of a parallelogram are supplementary

$\angle A + \angle B = 180° \Rightarrow 90 + \angle B = 180° \Rightarrow \angle B = 90°$

Similarly, $\angle D = 90°$

As, $\angle A = \angle B = \angle C = \angle D = 90°$, $ABCD$ is a rectangle.

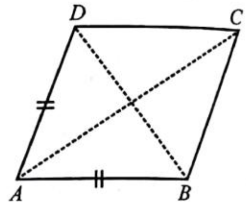
Since every rectangle is a parallelogram, therefore, it has all the properties of a parallelogram. Additional properties of a rectangle are :

All the (interior) angles of a rectangle are right angles.

$\angle A = \angle B = \angle C = \angle D = 90°$

Properties : The diagonals of a rectangle are equal i.e.
$AC = BD$.

RHOMBUS



A parallelogram is a rhombus if any pair of its adjacent sides are equal.

The given figure is a parallelogram $ABCD$ with adjacent sides AB and AD equal.

Reason : Since, the opposite sides of a parallelogram are equal, therefore $AB = DC$ and $AD = BC$.

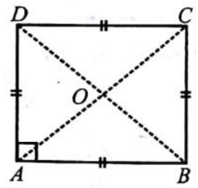
Now, if adjacent sides AB and AD are also equal them all the four sides AB, BC, CD and DA are also equal to each other i.e., $AB = BC = CD = DA$.

$\Rightarrow$ $ABCD$ is a rhombus.

Properties: Since every rhombus is a parallelogram, therefore, it has all the properties of a parallelogram. Additional properties of a rhombus are :

- All the sides of a rhombus are equal.
  In the above diagram, $AB = BC = CD = DA$.
- The diagonals of a rhombus intersect at right angles.
  In the above diagram, $AC \perp BD$.
- The diagonals bisect the angles of a rhombus.
  In the above diagram, diagonal AC bisects $\angle A$ as well as $\angle C$ and diagonals BD bisects $\angle B$ as well as $\angle D$.

SQUARE : A parallelogram is a square, if any pair of its adjacent sides are equal and any of its angle is 90°.



The adjacent figure shows a parallelogram $ABCD$ with $AB = BC$ and $\angle A = 90°$

$\therefore$ $ABCD$ is a square.

Reason : Since, opposite angles of a parallelogram are equal, therefore $\angle C = \angle A = 90°$

Since, in a parallelogram sum of its adjacent angles is 180°

$\therefore$ $\angle A + \angle B = 180° \Rightarrow 90° + \angle B = 180° \Rightarrow \angle B = 90°$

Also, $\angle D = \angle B = 90°$

Since, $ABCD$ is a parallelogram its opposite sides are equal

$\therefore$ $AB = DC$ and $BC = AD$

$\because$ $AB = BC$

$\therefore$ $AB = BC = CD = DA$

Also, it is proved above that each angle of the given parallelogram $ABCD$ is 90°

$\therefore$ $ABCD$ is a square

Properties :

(a) All the sides are equal i.e., $AB = BC = CD = DA$

(b) Each of the angles is equal to 90°
i.e., $\angle A = 90° = \angle B = \angle C = \angle D$

(c) Diagonals are equal i.e., $AC = BD$

(d) Diagonals bisect each other at right angle i.e., $OA = OC = \frac{1}{2}AC$ and $OB = OD = \frac{1}{2}BD$
and $\angle AOB = \angle BOC = \angle COD = \angle DOA = 90°$

(e) Diagonals bisects the angles of vertex i.e., AC bisects $\angle A$ and $\angle C$ and BD bisects $\angle B$ and $\angle D$

DIAGONAL PROPERTIES OF SPECIAL TYPES OF PARALLELOGRAM

Properties	Parallelogram	Rectangle	Rhombus	Square
Diagonals bisect each other	√	√	√	√
Diagonals are equal	–	√	–	√
Diagonals bisect vertex angles	√	√	√	√
Diagonals are perpendicular to each other	–	–	√	√
Diagonals form 4 equal triangles	–	–	√	√
Diagonals from 4 congruent triangles	–	–	√	√

VENN-DIAGRAM OF THE RELATIONS BETWEEN SPECIAL TYPE PARALLELOGRAMS

Venn-Diagram

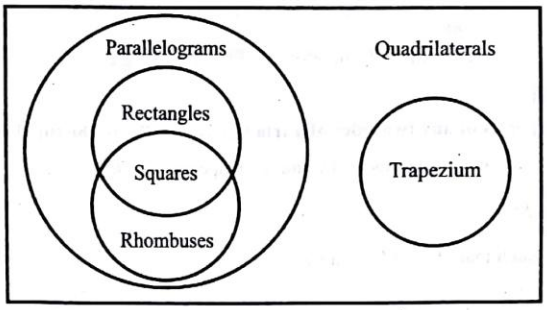
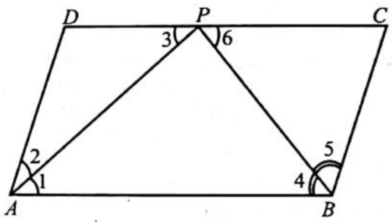


Illustration 14 : In the given figure, $ABCD$ is a parallelogram, AP bisects angle A and BP bisects angle B. Prove that $AB = 2AD$



SOLUTION : Since, AP bisects $\angle A \Rightarrow \angle 1 = \angle 2$

Also, $AB \parallel DC$ and AP is transversal

$\therefore \quad \angle 1 = \angle 3$ (Alternate angles)

$\therefore \quad \angle 2 = \angle 3$

In triangle ADP, $\angle 2 = \angle 3$

$\therefore \quad AD = DP$ .... (1) [Sides opp. to equal angles]

Similarly, $\angle 4 = \angle 5$ and $\angle 4 = \angle 6$ [Alternate angles]

$\Rightarrow \quad \angle 5 = \angle 6 \Rightarrow BC = PC$ .... (2)

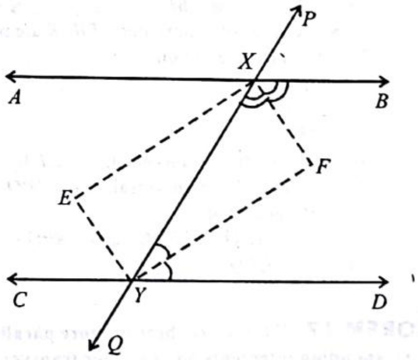
$\therefore \quad AD + BC = DP + PC$ [Adding (1) and (2)]

$\Rightarrow \quad AD + AD = DC$ [Opp. sides of a parallelogram are equal]

$\Rightarrow \quad DC = 2AD \Rightarrow AB = 2AD$

Illustration 15 : AB and CD are two parallel lines and a transversal PQ intersects AB at X and CD at Y. Prove that the bisectors of the interior angles from a rectangle.

SOLUTION : In the figure, $AB \parallel CD$ and PQ is transversal. Bisectors of interior angles at point X and Y form a quadrilateral $XEYF$.



Since, XE bisects angle AXY, $\angle EXY = \frac{1}{2}\angle AXY$

Since, XF bisects angle BXY, $\angle FXY = \frac{1}{2}\angle BXY$

$\therefore \quad \angle EXY + \angle FXY = \frac{1}{2}\angle AXY + \frac{1}{2}\angle BXY$

$\Rightarrow \quad \angle EXF = \frac{1}{2}(\angle AXY + \angle BXY)$

$= \frac{1}{2}\angle AXB = \frac{1}{2} \times 180° = 90°$

Similarly, it can be proved that $\angle EYF = 90°$

Also, $\angle EXY = \frac{1}{2}\angle AXY$ and $\angle EYX = \frac{1}{2}\angle CYX$

$\therefore \quad \angle EXY + \angle EYX = \frac{1}{2}\angle AXY + \frac{1}{2}\angle CYX = \frac{1}{2}(\angle AXY + \angle CYX)$

$= \frac{1}{2} \times 180°$ [Co-interior angles are supplementary]

$= 90°$

In ΔEXY,

$\angle E + \angle EXY + \angle EYX = 180°$

$\Rightarrow$ $\angle E + 90° = 180°$ $[\angle EXY + \angle EYX = 90°]$

$\Rightarrow$ $\angle E = 180° - 90° = 90°$

Similarly, it can be proved that $\angle F = 90°$

Since, each angle of the quadrilateral $XEYX$ is 90°, therefore $XEYF$ is a rectangle.

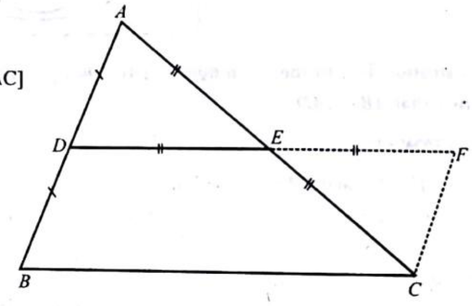
16. MID POINT THEOREM

The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.

Given : A ΔABC in which D and E are the mid-points of AB and AC respectively. DE is joined.

To prove : $DE \| BC$ and $DE = \frac{1}{2} BC$

Construction : Produce DE to F such that $DE = EF$. Join CF.



PROOF : In Δs AED and CEF, we have

$ED = EF$ (by construction)

$EA = EC$ [$\because$ E is the mid point of AC]

and $\angle AED = \angle CEF$ (vert. opp. $\angle$s)

$\therefore$ $\Delta AED \cong \Delta CEF$ (S.A.S.)

So, $AD = CF$ and $\angle ADE = \angle EFC$ [C.P.C.T.]

Now, $AD = CF$ and $AD = DB$ together imply that $DB = CF$.

Also, $\angle ADE = \angle EFC \Rightarrow AD \| CF$ [$\because$ $\angle ADE$ & $\angle EFC$ are alt. $\angle$s]

$\Rightarrow DB \| CF$

Thus, $DB \| CF$ and $DB = CF$

$\therefore$ $BCFD$ is a parallelogram

Hence, $DF \| BC$ and $DF = BC$.

But, D, E, F are collinear and $DE = EF$.

$\therefore$ $DE \| BC$ and $DE = \frac{1}{2} BC$.

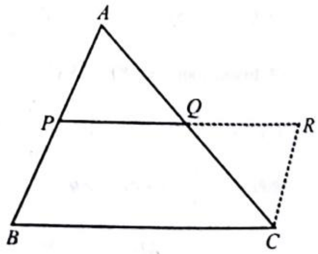
CONVERSE OF MID POINT THEOREM

The line drawn through the mid-point of one side of a triangle parallel to the another side, bisects the third side.

Given : A triangle ABC in which P is the mid-point of side AB and PQ is parallel to BC.

To prove : PQ bisects the third side AC i.e., $AQ = QC$.

Construction : Through C, draw CR parallel to BA, which meets PQ produced at point R.



PROOF : Since, $PQ \| BC$ i.e., $PR \| BC$ [Given]

and $CR \| BA$ i.e., $CR \| BP$ [By construction]

$\therefore$ Opposite sides of quadrilateral $PBCR$ are parallel.

$\Rightarrow$ $PBCR$ is a parallelogram.

$\Rightarrow$ $BP = CR$

Also, $AP = PB$ [As, P is mid-point of AB]

$\therefore$ $CR = AP$

$AB \| CR$ and AC is transversal, $\Rightarrow \angle PAQ =$ alternate $\angle RCQ$

$AB \| CR$ and PR is transversal, $\Rightarrow \angle APQ =$ alternate $\angle CRQ$

In ΔAPQ and ΔCRQ

$AP = CR$, $\angle PAQ = \angle RCQ$ and $\angle APQ = \angle CRQ$

$\Rightarrow$ $\Delta APQ \cong \Delta CRQ$ (A.S.A.)

$\Rightarrow$ $AQ = CQ$ (CPCT)

THEOREM 17: If there are three or more parallel lines and the intercepts made by them on a transversal are equal, then the corresponding intercepts on any other transversal are also equal.

Given : Three parallel lines ℓ, m and n i.e., $\ell \| m \| n$.

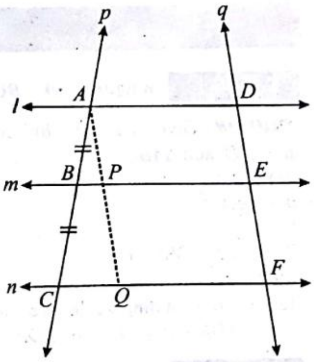
A transversal p meets these parallel lines at points A, B and C respectively such that $AB = BC$.

Another transversal q also meets these parallel lines at ℓ, m and n at points D, E and F respectively.

To Prove : $DE = EF$

Construction : Through point A, draw a line parallel to DEF ; which meets BE at point P and CF at point Q.

PROOF : In ΔACQ, B is mid-point of AC and BP is parallel to CQ and we know that the line through the mid-point of one side of a triangle and parallel to another side bisects its third side.



$\therefore$ $AP = PQ$ .... (i)

$\because$ $AP \parallel DE$ and $AD \parallel PE$ [By construction)

$\Rightarrow$ $APED$ is a parallelogram.

$\therefore$ $AP = DE$ .... (ii)

and $PQ \parallel EF$ and $PE \parallel QF$ [By construction)

$\Rightarrow$ $PQFE$ is a parallelogram

$\Rightarrow$ $PQ = EF$ .... (iii)

From (i), (ii) and (iii) we get, $DE = EF$

Illustration 16 : *ABCD* is a parallelogram in which *E* and *F* are the mid-points of the sides *AB* and *CD* respectively. Prove that the segments *CE* and *AF* trisect the diagonal *BD*.

SOLUTION : AF and CE intersect BD at P and Q respectively.

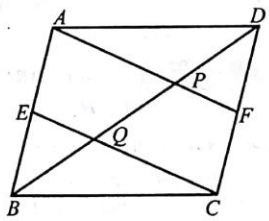
Clearly, $AE \parallel CF$ and $AE = CF$.

$\therefore$ $AECF$ is a parallelogram. So, $FA \parallel CE$.

In ΔCDQ, F is the mid-point of CD and $FP \parallel CQ$. [$\because FA \parallel CE$]

So, P is the mid-point of QD.

Consequently, $QP = PD$ ..... (i)



In ΔBPA, E is the mid-point of AB and $EQ \parallel AP$

So, Q is the mid point of BP.

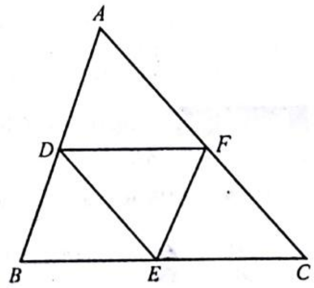
Consequently, $BQ = QP$ ..... (ii)

From (i) and (ii),

Hence, $BQ = QP = PD$.

Illustration 17 : In ΔABC, *D*, *E* and *F* are respectively the mid-points of sides *AB*, *BC* and *CA*. Show that ΔABC is divided into four congruent triangles by joining *D*, *E* and *F*.

SOLUTION : As D and E mid-points of sides AB and BC respectively of the triangle ABC.



$\therefore$ $DE \parallel AC$

Similarly, $DF \parallel BC$ and $EF \parallel AB$

Therefore, $ADEF$, $BDFE$ and $DFCE$ are all parallelograms.

Now, DE is a diagonal of the parallelogram $BDFE$,

therefore, $\Delta BDE \cong \Delta FED$

Similarly, $\Delta DAF \cong \Delta FED$

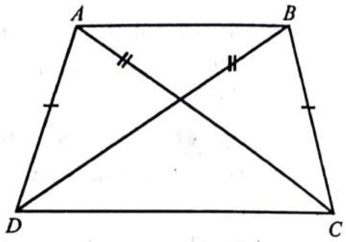
and $\Delta EFC \cong \Delta FED$

So, all the four triangles are congruent.

MISCELLANEOUS

Solved Examples

Example 1 : **In figure, $AD = BC$ and $BD = CA$. Prove that $\angle ADB = \angle BCA$ and $\angle DAB = \angle CBA$**



SOLUTION : Given that $AD = BC$ and $BD = CA$

In ΔABD and ΔABC,

$\left.\begin{array}{l} AD = BC \\ BD = CA \end{array}\right\}$ (Given)

$AB = AB$ (Common)

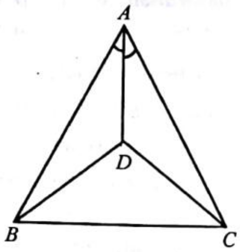
Therefore by SSS rule,

$\Delta ABD \cong \Delta BAC$

Hence corresponding angles are equal,

i.e. $\angle ADB = \angle BCA$ and $\angle DAB = \angle CBA$

Example 2 : **In figure, $AB = AC$. D is a point in the interior of ΔABC such that $\angle DBC = \angle DCB$. Prove that AD bisects $\angle BAC$.**



SOLUTION : In ΔBDC, $\angle DBC = \angle DCB$ then the opposite sides are equal.

i.e. $CD = BD$ .... (1)

Now in ΔABD and ΔACD

$BD = CD$ [by (1)]

$AD = AD$ (common side)

$AB = AC$ (Given)

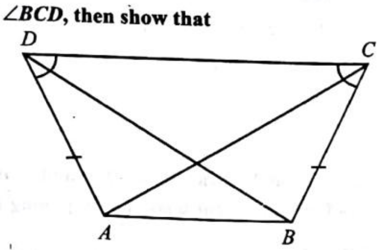
Therefore by SSS rule,

$\Delta ABD \cong \Delta ACD$

Consequently, $\angle BAD = \angle CAD$

$\Rightarrow$ AD bisects $\angle BAC$

Example 3 : **In figure, $ABCD$ is a quadrilateral in which $BC = AD$ and $\angle ADC = \angle BCD$, then show that (i) $AC = BD$ (ii) $\angle ACD = \angle CDB$.**



SOLUTION : In figure, it is given that

$BC = AD$ and $\angle ADC = \angle BCD$

Hence in ΔADC and ΔBCD

$AD = BC$ (Given)

$CD = CD$ (common)

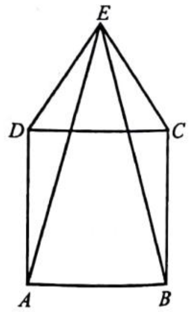
$\angle ACD = \angle BCD$ (Given)

Therefore, by SAS rule $\Delta ADC \cong \Delta BCD$

Hence corresponding sides and angles are equal.

i.e., $AC = BD$ and $\angle ACD = \angle CDB$

Example 4 : **In figure, $ABCD$ is a square and Δ CDE is an equilateral triangle. Prove that $AE = BE$.**



SOLUTION : ΔCDE is a equilateral triangle.

$CD = DE = CE55555$ .... (1)

$\angle DEC = \angle EDC = \angle DCE = 60°$ .... (2)

and $ABCD$ is a square

$\angle ADC = \angle BCD = 90°$

On adding $\angle EDC$ to both sides

$\angle ADC + \angle EDC = \angle BCD + \angle EDC$

$\Rightarrow$ $\angle EDA = \angle ECB$ .... (3) ($\because \angle EDC = \angle DCE$)

Now in ΔADE and ΔBCE

$AD = BC$ (sides of the square)

$\angle EDA = \angle ECB$ [by (3)]

$DE = EC$ [by (1)]

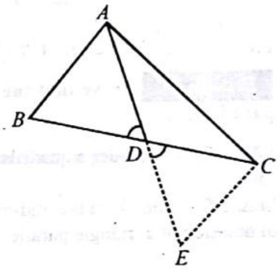
Therefore by SAS rule

$\Delta ADE \cong \Delta BCE$

Consequently corresponding sides are equal,

i.e., $AE = BE$.

Example 5 : **In figure, AD is a median of ΔABC. Prove that $AB + AC > 2AD$.**



SOLUTION : Produce AD to E such that $AD = DE$ and join CE.

In ΔADB and ΔEDC

$AD = DE$ (By construction)

$BD = DC$ (Given)

and $\angle ADB = \angle EDC$ (Vertically opposite angle)

Therefore, $\Delta ADB \cong \Delta EDC$ (By SAS rule)

Consequently $AB = CE$

Now in ΔACE

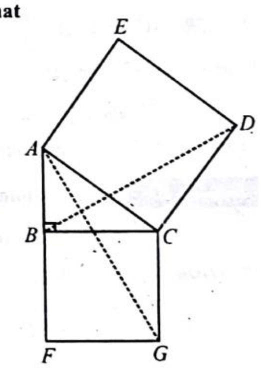
$AC + CE > AE$

$\Rightarrow$ $AC + AB > AE$ [$\because CE = AB$]

$\Rightarrow$ $AC + AB > 2AD$ [$\because AE = 2AD$]

Example 6 : **In the figure, ΔABC is right angled at B. $ACDE$ and $BFGC$ are squares. Prove that**

(i) $\Delta BCD \cong \Delta ACG$ (ii) $BD = AG$



SOLUTION : $\angle BCA = \angle BCA$ (same angle)

$\Rightarrow$ $90° + \angle BCA = 90° + \angle BCA$ (adding 90° to both sides)

$\Rightarrow$ $\angle ACD + \angle BCA = \angle GCB + \angle BCA$ (each angle in a square = 90°)

$\Rightarrow$ $\angle BCD = \angle ACG$

In Δs BCD and ACG

$CD = AC$ (sides of square $ACDE$)

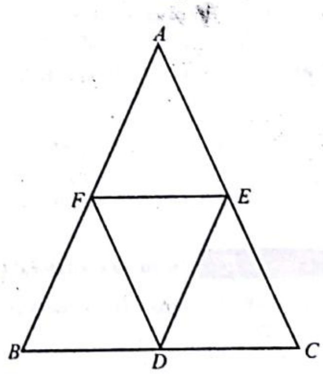
$BC = CG$ (sides of square $BFGC$)

$\angle BCD = \angle ACG$ (from above)

(i) $\Delta BCD \cong \Delta ACG$ (By SAS rule)

(ii) $BD = AG$

Example 7 : **In an equilateral triangle ABC the mid point of the side BC, CA and AB are D, E and F, respectively. Prove that ΔDEF is an equilateral triangle.**



SOLUTION : In ΔABC, D, E and F are midpoint of BC, CA and AB respectively

Thus $DE = \frac{1}{2} AB$ ....(1)

$EF = \frac{1}{2} BC$ ....(2)

$FD = \frac{1}{2} AC$ ....(3)

But ΔABC is an equilateral triangle

Hence, $AB = BC = AC$

From (1), (2) and (3), we get

$DE = EF = FD$

Therefore ΔDEF is an equilateral triangle.

Example 8 : **$ABCD$ is a parallelogram, P and Q are the points on the diagonal BD such that $DP = BQ$. Prove that $APCQ$ is a parallelogram.**

SOLUTION : In ΔAPD and ΔCQB,

$AD = BC$ (Opposite sides of parallelogram)

$\angle ADP = \angle CBQ$ (Alternate angles)

$DP = BQ$ (Given)

So, from side-angle-side congruency

$\Delta APD \cong \Delta CQB$

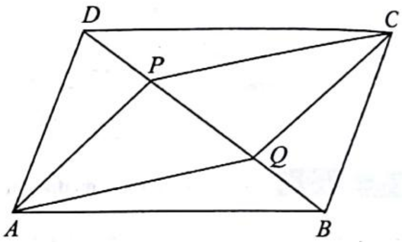
Thus, corresponding sides of congruent triangles are equal.

Therefore $AP = CQ$ .... (1)

Similarly in ΔCPD and ΔAQB

$CP = AQ$

From (1) and (2), we get $APCQ$ is a parallelogram.



Example 9 : **Prove that the quadrilateral formed by joining the mid points of the adjacent sides of a quadrilateral is a parallelogram.**

SOLUTION : Consider a quadrilateral $ABCD$, in which P, Q, R and S are the mid points of sides respectively.

Join AC

In ΔABC, P and Q are the mid-points of the sides AB and BC respectively. Since we know that the line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side.

So, $PQ \parallel AC$ and $PQ = \frac{1}{2}AC$

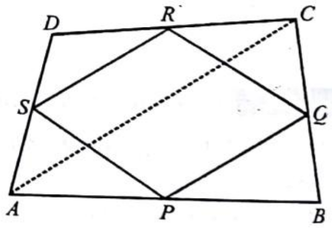
Similarly, S and R are the mid points of the sides AD and DC respectively.

$\therefore$ $SR \parallel AC$ and $SR = \frac{1}{2}AC$ .... (2)

From (1) and (2), we get

$PQ \parallel SR$ and $PQ = SR = \frac{1}{2}AC$

Therefore $PQRS$ is parallelogram.



Example 10 : **In the adjoining kite, diagonals intersect at O. If $\angle ABO = 32°$ and $\angle OCD = 40°$, find**

(i) $\angle ABC$ **(ii) $\angle ADC$** **(iii) $\angle BAD$**

SOLUTION : Given, $ABCD$ is a kite.

(i) As diagonal BD bisects $\angle ABC$,

$\angle ABC = 2\angle ABO = 2 \times 32° = 64°$

(ii) $\angle DOC = 90°$ [diagonals intersect at right angles]

$\angle ODC + 40° + 90° = 180°$ [sum of angles in ΔOCD]

$\Rightarrow$ $\angle ODC = 180° - 40° - 90° = 50°$

As diagonal BD bisects $\angle ADC$,

$\angle ADC = 2\angle ODC = 2 \times 50° = 100°$

(iii) As diagonal BD bisects $\angle ABC$

$\angle OBC = \angle ABO = 32°$

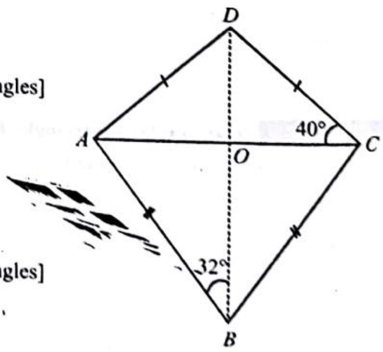
$\angle BOC = 90°$ [diagonals intersect at right angles]

$\angle OCB + 90° + 32° = 180°$ [sum of angles in ΔOBC]

$\Rightarrow$ $\angle OCB = 180° - 90° - 32° = 58°$

$\angle BCD = \angle OCD + \angle OCB = 40° + 58° = 98°$

$\therefore$ $\angle BAD = \angle BCD = 98°$ [In kite $ABCD$, $\angle A = \angle C$]



Example 11 : **In an quadrilateral (kite) $ABCD$ there is a point 'O' inside it such that $OB = OD$. Prove that 'O' lies on AC.**

SOLUTION : In the ΔAOD and the ΔAOB.

$AD = AB$ (given)

$OD = OB$ (given)

$OA = OA$ (common)

$\therefore$ $\Delta AOD \cong \Delta AOB$

$\therefore$ $\angle 1 = \angle 2$ .... (1) (C.P.C.T.E)

Similarly in the ΔCDO and ΔCBO

$\angle 3 = \angle 4$ .... (2)

Adding eqs (1) and (2), we get

$\angle 1 + \angle 3 = \angle 2 + \angle 4$

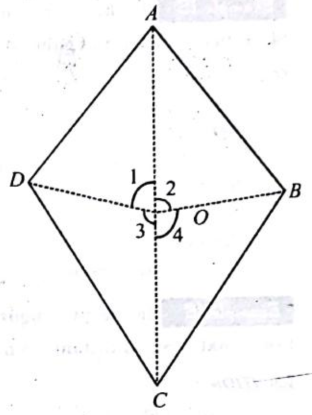
But $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360°$

$\therefore$ $2(\angle 1 + \angle 3) = 360°$

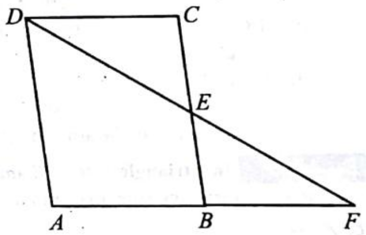
$\Rightarrow$ $\angle 1 + \angle 3 = 180°$

$\therefore$ $\angle AOC = 180°$

Hence 'O' lies on AC.



Example 12 : **In the figure, $ABCD$ is a parallelogram. E is the midpoint of BC, DE and AB, when produced meet at F. Prove that $AF = 2AB$.**



SOLUTION: In the ΔCDE and the ΔBFE,

$\angle DEC = \angle BEF$ (vert. opp. angles)

$\angle DCE = \angle EBF$ (int. alt. angles)

$CE = EB$

$\therefore$ $\Delta CDE \cong \Delta BFE$

$\therefore$ $DC = BF$

But $DC = AB$ (opp. sides of $a \parallel$ gm)

$\therefore$ $AB = BF$

$\therefore$ $AB + BF = BF + BF = AB + AB$ ($\therefore$ $AB = BF$)

$AF = 2AB$

Example 13 : **The angles of a quadrilaterals are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilaterals.**

SOLUTION : Given ratio of angles are 3 : 5 : 9 : 13

Let four angles of the quadrilateral are $3k$, $5k$, 9k and $13k$.

We know that sum of four angles of a quadrilateral is 360°

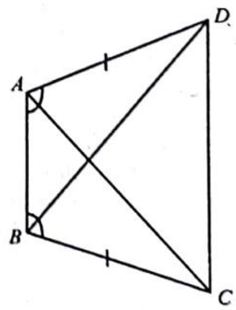
Therefore, $3k + 5k + 9k + 13k = 360°$

$\Rightarrow$ $30k = 360°$ $\Rightarrow$ $k = 12$

Therefore four angles are $(3 \times 12)°$, $(5 \times 12)°$, $(9 \times 12)°$ and $(13 \times 12)°$ or 36°, 60°, 108°, 156°.

Example 14 : **In the given figure, $AD = BC$ and $\angle DAB = \angle CBA$, Prove that:**

(i) $AC = BD$ **(ii) $\angle BAC = \angle ABD$**



SOLUTION : In Δs ABC and BAD,

$BC = AD$ (given)

$\angle ABC = \angle BAD$ (given)

$AB = AB$ (common)

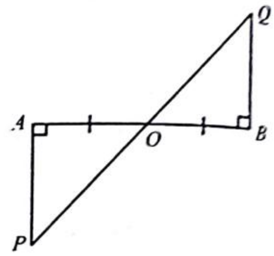
So, by SAS rule,

$\Delta ABC \cong \Delta BAD$,

$\therefore$ $BD = AC$

and $\angle BAC = \angle ABD$ [Corresponding parts of congruent triangles are equal]

Example 15 : **In the given figure $PA \perp AB$, $QB \perp AB$ and $OA = OB$. Show that O is the mid point of PQ.**



SOLUTION : In Δs AOP and BOQ,

$\angle A = \angle B = 90°$

$AO = BO$ (given)

$\angle AOP = \angle BOQ$ (vertically opposite angles)

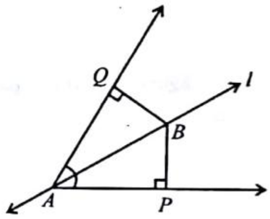
$\Rightarrow$ By ASA rule

$\Delta AOP \cong \Delta BOQ$

$\therefore$ $PO = QO$ [Corresponding parts of congruent triangles are equal]

$\Rightarrow$ O is the mid point of PQ.

Example 16 : **In the given figure, l is the bisector of $\angle PAQ$, $BQ \perp AQ$, $BP \perp AP$. Prove that B is equidistant from the arms of $\angle PAQ$.**



SOLUTION :

In Δs ABQ and ABP,

$\angle AQB = \angle APB = 90°$

$\angle QAB = \angle PAB$ [line l bisects $\angle PAQ$]

$AB = AB$ (common)

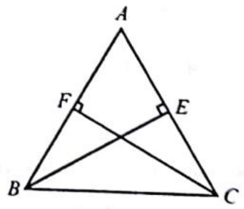
$\Rightarrow$ By AAS rule

$\Delta ABQ \cong \Delta ABP$

$\Rightarrow$ $BQ = BP$ [CPCT]

Hence B is equidistant from the arms of $\angle PAQ$.

Example 17 : **In a triangle ABC, BE and CF are two equal altitudes. Using RHS congruency rule, prove that ΔABC is isosceles.**



SOLUTION :

In right Δs BCE and CBF

$BC = BC$ (common hypotenuse)

$BE = CF$ (given)

So, by RHS property

$\Delta CBE \cong \Delta BCF$

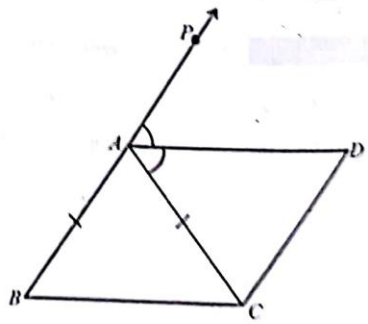
$\Rightarrow$ $\angle BCE = \angle CBF$ [CPCT]

$\Rightarrow$ $AB = AC$ [sides opposite to equal angles of a triangle are equal]

Hence, ΔABC is isosceles.

Example 18 : **ABC is an isosceles in which $AB = AC$. AD bisects exterior angle PAC and $CD \parallel AB$. Show that**

(i) $\angle DAC = \angle BCA$ and (ii) $ABCD$ is a parallelogram.



SOLUTION :

(i) ΔABC is isosceles in which $AB = AC$ (Given)

So, $\angle ABC = \angle ACB$ (angles opposite to equal sides)

Also, $\angle PAC = \angle ABC + \angle ACB$ (Exterior angle of a triangle)

or $\angle PAC = 2\angle ACB$ .... (1)

Now, AD bisects $\angle PAC$

So, $\angle PAC = 2\angle DAC$ .... (2)

Therefore, $2\angle DAC = 2\angle ACB$ [From (1) and (2)]

or $\angle DAC = \angle ACB$

(ii) Now, these equal angles form a pair of alternate angles because line segments BC and AD are intersected by a transversal AC.

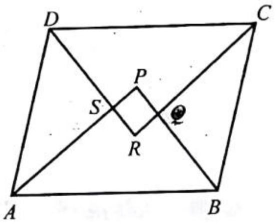
So, $BC \parallel AD$

Also, $BA \parallel CD$ (Given)

Now, both pairs of opposite sides of quadrilateral $ABCD$ are parallel.

So, $ABCD$ is a parallelogram.

Example 19: **Show that the bisectors of angles of a parallelogram form a rectangle.**



SOLUTION :

Let P, Q, R and S be the points of intersection of the bisectors of $\angle A$ and $\angle B$, $\angle B$ and $\angle C$, $\angle C$ and $\angle D$, and $\angle D$ and $\angle A$ respectively of parallelogram $ABCD$

Since DS bisects $\angle D$ and AS bisects $\angle A$, therefore,

$\angle DAS + \angle ADS = \frac{1}{2}\angle A + \frac{1}{2}\angle D$

$= \frac{1}{2}(\angle A + \angle D) = \frac{1}{2} \times 180° = 90°$ ($\angle A$ and $\angle D$ are interior angles on the same side of the transversal)

Also, $\angle DAS + \angle ADS + \angle DSA = 180°$

or $90° + \angle DSA = 180°$

or $\angle DSA = 90°$

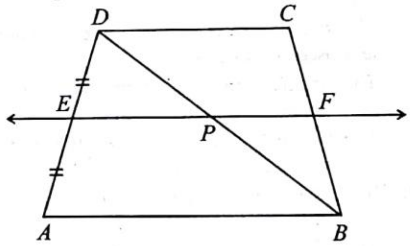
or $\angle PSR = \angle DSA = 90°$ (Being vertically opposite to $\angle DSA$)

Similarly, it can be shown that

$\angle SPQ = \angle PQR = \angle QRS = 90°$

So, $PQRS$ is a rectangle.

Example 20: **$ABCD$ is a trapezium in which $AB \parallel DC$, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (as shown). Prove that F is the mid-point of BC.**



SOLUTION :

Given line EF is parallel to AB and $AB \parallel DC$

$\therefore$ $EF \parallel AB \parallel DC$.

In ΔABD, E is the mid-point of AD

EP is parallel to AB (As $EF \parallel AB$)

$\therefore$ P is mid-point of side BD. [Since, the line through the mid-point of a side of a triangle and parallel to the other side, bisects the third side]

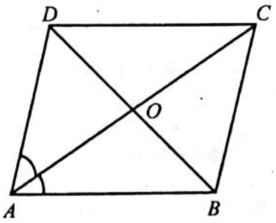
Now, in ΔBCD, P is mid-point of BD and, PF is parallel to DC [As $EF \parallel DC$]

$\therefore$ F is mid-point of BC

Example 21: **If a diagonal of a parallelogram bisects one of the angle of the parallelogram, prove that it also bisects the second angle and then the two diagonals are perpendicular to each other.**

SOLUTION :

Consider $a \parallel$ gm $ABCD$ in which the diagonals AC and BD intersect at O. And, $\angle BAO = \angle DAO$.



Since $AD \parallel BC$ and AC intersects them.

$\therefore$ $\angle BCO = \angle OAD$ (alt. int. $\angle$s)

Also, $AB \parallel DC$ and CA intersects them

$\therefore$ $\angle DCO = \angle OAB$ (alt. int. $\angle$s)

But, $\angle OAD = \angle OAB$ (Given)

$\therefore$ $\angle BCO = \angle DCO$

Also, $\angle A = \angle C$ (opp. $\angle$s of a $\parallel$gm)

$\Rightarrow$ $\frac{1}{2}\angle A = \frac{1}{2}\angle C$

$\Rightarrow$ $\angle OAD = \angle DCO$.

Now, in ΔACD, $\angle DCO = \angle OAD \Rightarrow AD = CD$

But, $AD = BC$ and $CD = AB$

$\therefore$ $AB = CD = AD = BC$

So, $ABCD$ is a rhombus.

Now, diagonals of a rhombus bisect each other at right angle. So, $AC \perp BD$.

EXERCISE 1

Fill in the Blanks

DIRECTIONS: *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

1. In ΔABC, $\angle A > \angle B$ and $\angle B > \angle C$, then smallest side is ....
2. If $\angle C$ is right angle in ΔABC, then larger side is .....
3. The angles opposite of equal sides of a triangle are ....
4. If two angles and the included side of one triangle are equal to two angles and the included side of the other triangle, then the two triangles are ....
5. In a triangle, angle opposite to the longer side is ....
6. Sum of any two sides of a triangle is greater than the .... side.
7. The triangle formed by joining the mid-point of the side of an isosceles triangles is ....
8. The triangle formed by joining the mid-points of the sides of a right triangle is ....
9. The figure formed by joining the mid-points of the consecutive sides of a quadrilateral is ....
10. If the diagonals of a parallelogram are equal, then it is a ....
11. Parallelograms on the same base and between the same .... are equal in area.
12. Triangles with equal .... and having any side of one equal to any side of the other have equal corresponding altitudes.
13. Triangles on the same base and between the same parallels are .... in area.
14. The area of a triangle = $\frac{1}{2}$ base × ....
15. A median divides a triangle into .... triangles of equal area.

True / False

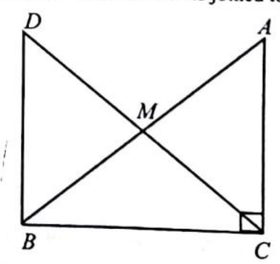
DIRECTIONS: *Read the following statements and write your answer as true or false.*

1. If two sides of a triangle are unequal, the greater side has the greater angle opposite to it.
2. Scalene triangle may be an acute-angled triangle.
3. An isosceles triangle may be a right angled triangle.
4. An obtuse angled triangle may be an equilateral triangle.
5. A right angled triangle may be a scalene triangle.
6. The angles of a triangle are in the ratio 2 : 1 : 3. Triangle is a right angled triangle.
7. In a parallelogram, the diagonals are equal.
8. In a parallelogram, the diagonals intersect at right angles.
9. If all angles of a quadrilateral are equal, it is a parallelogram.
10. Every square is a rectangle.
11. Every rectangle is a parallelogram.
12. A trapezoid is a parallelogram.
13. The sum of the interior angles of a quadrilateral is 180°.
14. If the diagonals of a quadrilateral divide it into four triangles which are equal in area, then the quadrilateral must be a parallelogram.
15. The three altitudes of an equilateral triangle are equal in length.
16. The areas of the triangles having a common side are proportional to their altitude to the common side.
17. If triangles of equal area have a common base, then their vertices must lie on a line parallel to the base.
18. If P is any point in the interior of a rectangle $ABCD$, then Area(ΔPAB) + Area(ΔPCD) = Area(ΔPBC) + Area (ΔPDA).

Match the Columns

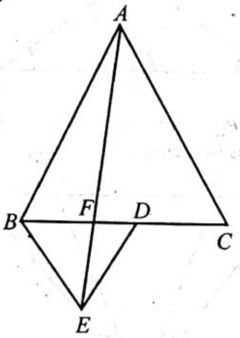
DIRECTIONS: *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column I have to be matched with statements (p, q, r, s) in column II.*

1. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that $DM = CM$. Point D is joined to point B.



Column I		Column II	
(A)	ΔAMC	(p)	congruent ΔBMD
(B)	$\angle DBC$	(q)	a right angle
(C)	ΔDBC	(r)	congruent ΔACB
(D)	CM	(s)	$(1/2)\, AB$

2. In figure, ABC and BDE are two equilateral triangles such that D is the mid-point of BC. If AE intersects BC at F, match them correctly.



Column I		Column II	
(A)	ar (ΔBDE)	(p)	$\frac{1}{4}$ ar (ΔABC)
(B)	ar (ΔBDE)	(q)	$\frac{1}{2}$ ar (ΔBAE)
(C)	ar (ΔABC)	(r)	2 ar (ΔBEC)
(D)	ar (ΔBFE)	(s)	ar (ΔAFD)

VSAQ Very Short Answer Questions

DIRECTIONS: *Give answer in one word or one sentence.*

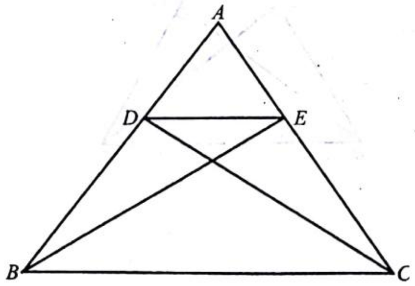
1. ABC is a triangle. Locate a point in the interior of ΔABC which is equidistant from all the vertices of ΔABC.

2. In a triangle locate a point in its interior which is equidistant from all the sides of the triangle.

3. Find all the other three angles of a parallelogram, if one of its angles is given to be

(i) 70° (ii) 135° (iii) 95°.

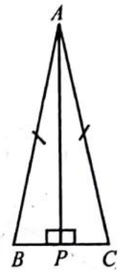
4. D and E are points on sides AB and AC respectively of ΔABC such that ar(ΔDBC) = ar(ΔEBC). Prove that $DE \parallel BC$.



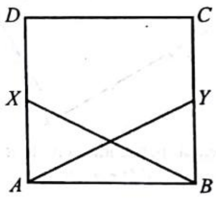
SAQ Short Answer Questions

DIRECTIONS: *Give answer in 2-3 sentences.*

1. ABC is an isosceles triangle with $AB = AC$, $AP \perp BC$ to Show that $\angle B = \angle C$.

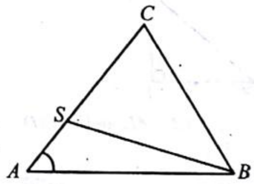


2. $ABCD$ is a square. X and Y are points on sides AD and BC respectively such that $AY = BX$. Prove that



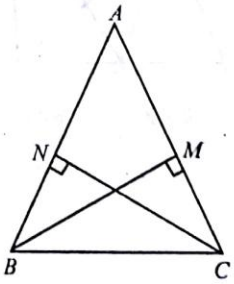
$BY = AX$ and $\angle BAY = \angle ABX$.

3. In figure, $AB = AC$ and S is any point on side AC. Prove that $CS < BS$.

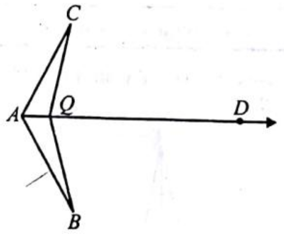


4. In the adjoining figure, $AB = AC$. $BM \perp AC$ and $CN \perp AB$

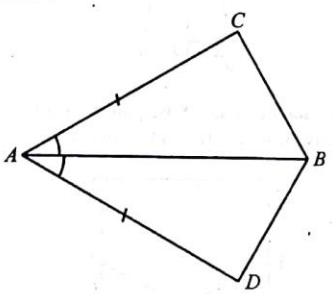
Prove that $BM = CN$



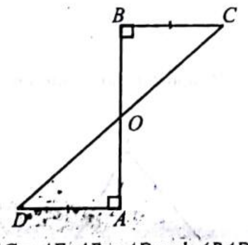
5. In figure, $\angle CQD = \angle BQD$ and AD is the bisector of $\angle BAC$. Prove that $\Delta CAQ \cong \Delta BAQ$ and hence $CQ = BQ$.



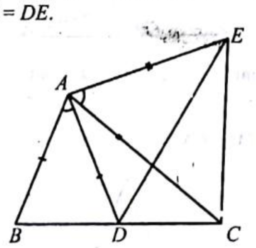
6. In quadrilateral $ACBD$, $AC = AD$ and AB bisects $\angle A$ (see figure). Show that $\Delta ABC \cong \Delta ABD$. What can you say about BC and BD ?



7. AD and BC are equal perpendiculars to a line segment AB (See figure). Show that CD bisects AB.



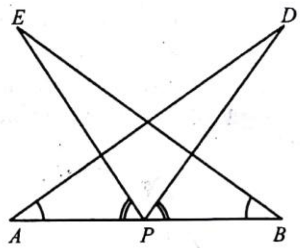
8. In figure, $AC = AE$, $AB = AD$ and $\angle BAD = \angle EAC$. Show that $BC = DE$.



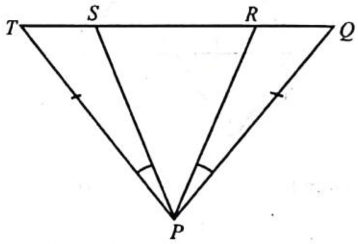
9. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$. (See figure) Show that

(i) $\Delta DAP \cong \Delta EBP$

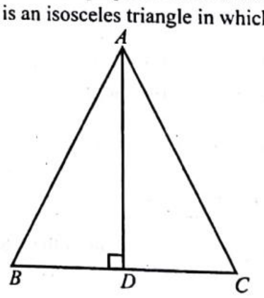
(ii) $AD = BE$



10. If in figure, $PQ = PT$ and $\angle TPS = \angle QPR$, prove that triangle PRS is isosceles.



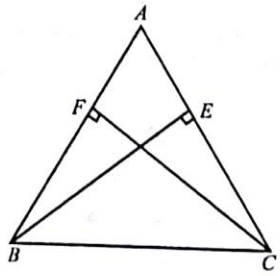
11. In ΔABC, AD is the perpendicular bisector of BC. Show that ΔABC is an isosceles triangle in which $AB = AC$.



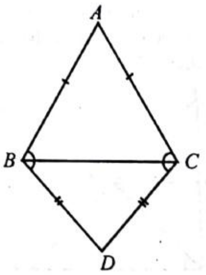
12. ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal. (See figure) Show that

(i) $\Delta ABE \cong \Delta ACF$

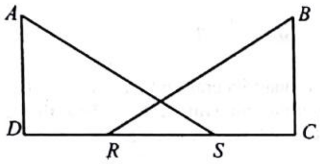
(ii) $AB = AC$, i.e., ΔABC is an isosceles triangle.



13. ABC and DBC are two isosceles triangles on the same base BC. (See figure) Show that $\angle ABD = \angle ACD$.

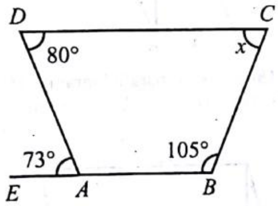


14. In figure, $AD \perp CD$ and $BC \perp CD$. If $AS = BR$ and $DR = CS$, prove that $\angle DAS = \angle CBR$.

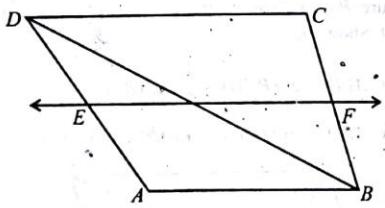


15. Show that in a right angled triangle, the hypotenuse is the longest side.

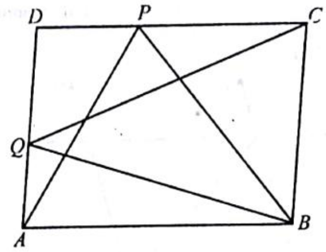
16. Use the information given in figure to calculate the value of x.



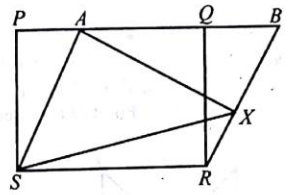
17. $ABCD$ is a trapezium in which $AB \parallel DC$, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F. Show that F is the mid-point of BC.



18. P and Q are any two points lying on the sides DC and AD respectively of a parallelogram $ABCD$. Show that : ar (ΔAPB) = ar (ΔBQC).



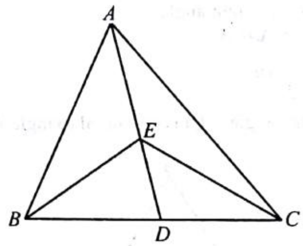
19. In figure $PQRS$ and $ABRS$ are parallelograms and X is any point on side BR. Show that :



(i) ar (parallelogram $PQRS$) = ar (parallelogram $ABRS$)

(ii) ar (ΔAXS) = $\frac{1}{2}$ ar (parallelogram $PQRS$).

20. In figure, E is any point on median AD of a ΔABC. Show that ar (ΔABE) = ar (ΔACE).

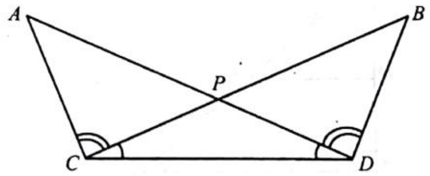


21. The angle of a quadrilateral are in the ratio 2 : 3 : 7 : 6. Find the measure of each angle of the quadrilateral.

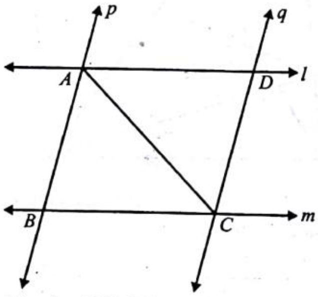
LAQ *Long Answer Questions*

DIRECTIONS: *Give answer in four to five sentences.*

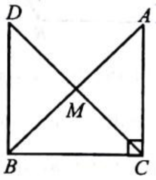
1. In figure, $\angle BCD = \angle ADC$ and $\angle ACB = \angle BDA$. Prove that $AD = BC$ and $\angle A = \angle B$.



2. l and m are two parallel lines intersected by another pair of parallel lines p and q. Show that $\Delta ABC \cong \Delta CDA$.

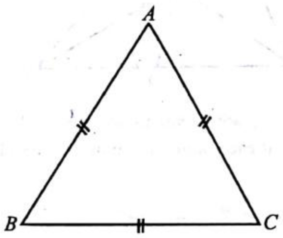


3. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that $DM = CM$. Point D is joined to point B. Show that:

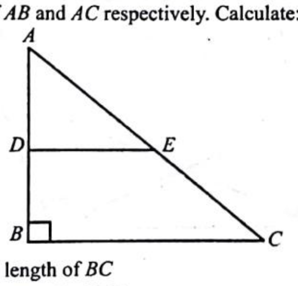


(i) $\Delta AMC \cong \Delta BMD$
(ii) $\angle DBC$ is a right angle.
(iii) $\Delta DBC \cong \Delta ACB$
(iv) $CM = \frac{1}{2} AB$

4. Show that the angles of an equilateral triangle ABC are 60° each.



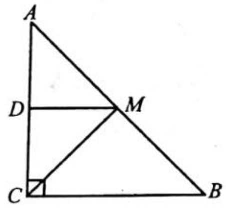
5. In the following figure, triangle ABC is right-angled at B. Given that $AB = 9$ cm, $AC = 15$ cm and D, E are the mid-points of AB and AC respectively. Calculate:



(i) The length of BC
(ii) The area of ΔADE.

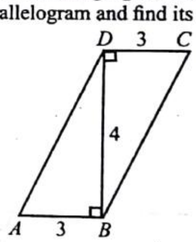
6. Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

7. ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

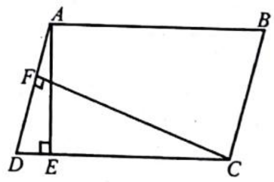


(i) D is the mid-point of AC
(ii) $MD \perp AC$
(iii) $CM = MA = \frac{1}{2} AB$

8. $ABCD$ is a quadrilateral and BD is one of its diagonals as shown in the following figure. Show that quadrilateral $ABCD$ is a parallelogram and find its area.



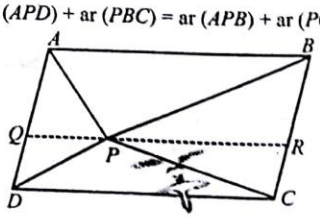
9. In figure, $ABCD$ is a parallelogram, $AE \perp DC$ and $CF \perp AD$. If $AB = 16$ cm, $AE = 8$ cm and $CF = 10$ cm, find AD.



10. In figure P is a point in the interior of a parallelogram $ABCD$. Show that

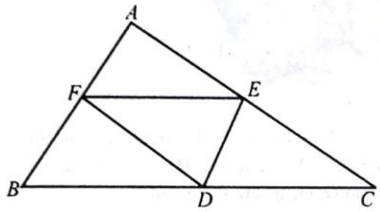
(i) $\text{ar}(APB) + \text{ar}(PCD) = \frac{1}{2} \text{ar}(ABCD)$

(ii) $\text{ar}(APD) + \text{ar}(PBC) = \text{ar}(APB) + \text{ar}(PCD)$



(Hint : Through P, draw a line parallel to AB)

11. D, E and F are respectively the mid-points of the sides BC, CA and AB of a ΔABC. Show that :



(i) $BDEF$ is a parallelogram

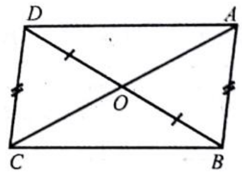
(ii) ar $(\Delta DEF) = \frac{1}{4}$ ar (ΔABC)

(iii) ar (BDEF) $= \frac{1}{2}$ ar (ΔABC).

12. In figure, diagonals AC and BD of quadrilateral $ABCD$ intersect at O such that $OB = OD$. If $AB = CD$, then show that :

(i) ar (ΔDOC) = ar (ΔAOB)

(ii) ar (ΔDCB) = ar (ΔACB)



(iii) $DA \parallel CB$ or $ABCD$ is a parallelogram.

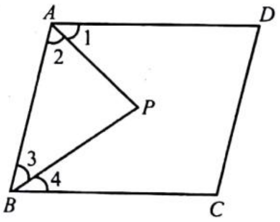
[Hint. From D and B, draw perpendiculars to AC.]

EXERCISE 2

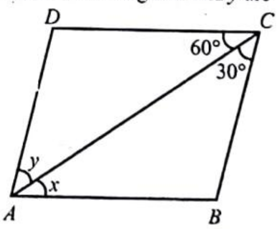
Multiple Choice Questions

DIRECTIONS: *This section contains multiple choice questions. Each questions has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. If the perpendiculars drawn from the mid-point on one side of a triangle to its other two sides are equal, then triangle is
   (a) Equilateral (b) Isosceles
   (c) Equi angular (d) Scalene
2. In a isosceles triangle $AB = AC$ and BA is produced to D, such that $AB = AD$ then $\angle BCD$ is
   (a) 70° (b) 90°
   (c) 60° (d) 45°
3. In ABC, $BD \perp AC$ and $CE \perp AB$. If BD and CE intersect at O, then $\angle BOC =$
   (a) $\angle A$ (b) $90 + A$
   (c) $180 + \angle A$ (d) $180 - \angle A$
4. If the three altitudes of a Δ are equal then triangle is
   (a) isosceles (b) equilateral
   (c) right angled (d) none
5. If D is any point on the side BC of a ΔABC, then
   (a) $AB + BC + CA > 2AD$
   (b) $AB + BC + CA < 2AD$
   (c) $AB + BC + CA > 3AD$
   (d) None
6. In a right angled triangle. One acute angle is double the other then the hypotenuse is
   (a) Equal to smallest side
   (b) Double the smallest side
   (c) Triple the smallest side
   (d) None of these
7. P and Q are the mid-points of the side AB and BC respectively of the triangle ABC, right angled at B. Then
   (a) $AQ^2 + CP^2 = AC^2$
   (b) $AQ^2 + CP^2 = (4/5)\,AC^2$
   (c) $AQ^2 + CP^2 = (4/3)\,AC^2$
   (d) $AQ^2 + CP^2 = (5/4)\,AC^2$
8. Each angle of an equilateral triangle is
   (a) 60° (b) 45°
   (c) 90° (d) 30°
9. In a triangle ABC, $\angle A + \angle B = 144°$ and $\angle A + \angle C = 124°$ then $\angle B =$
   (a) 56° (b) 60°
   (c) 65° (d) 45°
10. In a parallelogram $ABCD$ diagonals AC and BD intersects at O and $AC = 12.8$ cm and $BD = 7.6$ cm, then the measure of OC and OD respectively equal to
    (a) 1.9 cm and 6.4 cm (b) 3.8 cm and 3.2 cm
    (c) 3.8 cm, 3.2 cm (d) 6.4 cm and 3.8 cm
11. $ABCD$ is a rhombus with $\angle ABC = 56°$, then $\angle ACD$ will be
    (a) 56° (a) 124°
    (c) 62° (d) 34°
12. $LMNO$ is a trapezium with $LM \parallel NO$. If P and Q are the mid-points of LO and MN respectively and $LM = 5$ cm and ON = 10 cm then $PQ =$
    (a) 2.5 cm (b) 5 cm
    (c) 7.5 cm (d) 15 cm
13. In the adjoining figure, AP and BP are angle bisector of $\angle A$ and $\angle B$ which meets at P on the parallelogram $ABCD$. Then $2\angle APB =$



    (a) $\angle C + \angle D$ (b) $\angle A + \angle C$
    (c) $\angle B + \angle D$ (d) $2\angle C$
14. In the figure $ABCD$, the angles x and y are



    (a) 60°, 30° (b) 30°, 60°
    (c) 45°, 45° (d) 90°, 90°
15. Quadrilateral whose four sides are equal but angles are not equal is
    (a) square (b) quadrilateral
    (c) rectangle (d) parallelogram

16. $ABCD$ is quadrilateral. If AC and BD are its diagonals then the

(a) sum of the squares of the sides of the quadrilateral is equal to the sum of the squares of its diagonals.

(b) perimeter of the quadrilateral is equal to the sum of the diagonals

(c) perimeter of the quadrilateral is less than the sum of the diagonals

(d) perimeter of the quadrilateral is greater than the sum of the diagonals

17. In ΔABC if D is a point in BC and divides it in the ratio 3 : 5 then ar (ΔADC) : ar (ΔABC) =

(a) 3 : 5 (b) 3 : 8

(c) 5 : 8 (d) 8 : 3

18. Sides AB and AC of triangle ABC are trisected at D and E then ΔADE and trapezium $DECB$ have their areas in the ratio of

(a) 1 : 4 (b) 1 : 8

(c) 1 : 9 (d) 1 : 2

19. X and Y are respectively two points on the sides DC and AD of the parallelogram $ABCD$. The area of ΔABX is equal to

(a) 1/3 × area of ΔBYC

(b) area of ΔBYC

(c) 1/2 × area of ΔBYC

(d) 2 × area of ΔBYC

20. The base BC of triangle ABC is divided at D so that

$BD = \frac{1}{2}DC$. Area of ΔABD =

(a) 1/3 of the area of ΔABC

(b) 1/2 of the area of ΔABC

(c) 1/4 of the area of ΔABC

(d) 1/6 of the area of ΔABC

MTOC More than One Correct

DIRECTIONS: *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

1. Which of the following is/are correct?

(a) If two sides of a triangle are unequal, the larger side has the greater angle opposite to it.

(b) The sum of any two sides of a triangle is greater than its third side.

(c) If all the line segments that can be drawn to a given line from an external point, the perpendicular line segment is the shortest.

(d) If all the three sides of a triangle are equal, it is called a scalene triangle.

2. Find the incorrect statements.

(a) A trapezium is a quadrilateral, having one pair of opposite sides parallel.

(b) A parallelogram is a quadrilateral with both pairs of opposite sides parallel.

(c) A rhombus is a quadrilateral, whose all the sides are equal and each angle is 90°.

(d) A square is a quadrilateral, whose all sides are equal

3. Which of the following is/are not false?

(a) A diagonal of a parallelogram divides it in two congruent triangles.

(b) The opposite sides of a parallelogram are equal.

(c) The diagonals of a parallelogram never bisect each other.

(d) The opposite angles of a parallelogram are complementary.

4. Choose the correct statements among the following given options.

(a) Area of a parallelogram is the product of any of its sides and the corresponding altitude.

(b) The area of a trianlge is half the product of any of its sides and the corresponding altitude.

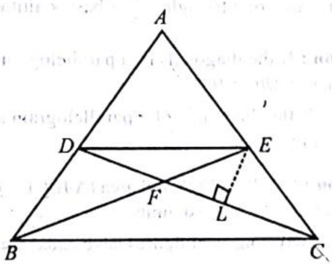
(c) The area of a trapezium is half the product of its height and the sum of the parallel sides.

(d) A diagonal of a parallelogram divides it into two triangles of distinct areas.

PBQ Passage Based Questions

DIRECTIONS: *Study the given paragraph(s) and answer the following questions.*

In the adjoining figure, $DE \parallel BC$ and $AD : DB = 4 : 3$



1. Value of $\frac{AD}{AB}$ is

(a) $\frac{3}{7}$ (b) $\frac{4}{7}$

(c) $\frac{4}{3}$ (d) none of these

2. Value of $\frac{DE}{BC}$ is

(a) $\frac{4}{7}$ (b) $\frac{3}{7}$

(c) $\frac{3}{4}$ (d) none of these

3. $\frac{\text{area }(\Delta DEF)}{\text{area }(\Delta DEC)} =$

(a) $\frac{11}{4}$ (b) $\frac{4}{7}$

(c) $\frac{4}{11}$ (d) none of these

A&R Assertion & Reason

DIRECTIONS: *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.

(b) If both **Assertion** and **Reason** are correct, but Reason **is not the correct explanation** of Assertion.

(c) If **Assertion** is **correct** but **Reason** is **incorrect**.

(d) If **Assertion** is **incorrect** but **Reason** is **correct**.

1. **Assertion :** If the angles of a quadrilateral are in the ratio 2 : 3 : 7 : 6, then the measure of angles are 40° 60°,140°,120° respectively.

Reason : The sum of the angles of a quadrilateral is 360°.

2. **Assertion :** If the base and the altitude of a triangle are 4 cm and 8 cm respectively, then its area is 36 cm^2.

Reason : Area of a triangle $= \frac{1}{2}$ × base × altitude.

3. **Assertion :** If the diagonals of a parallelogram $ABCD$ are equal, then $\angle ABC = 90°$.

Reason : If the diagonals of a parallelogram are equal, it becomes a rectangle.

4. **Assertion :** If $ABC \cong PQR$ and area (ΔABC) = 10 sq. units, then area (ΔPQR) = 20 sq. units.

Reason : Two congruent figures have equal areas.

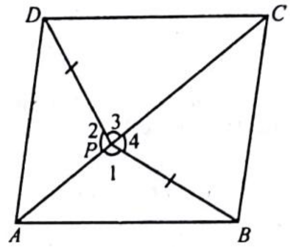
5. **Assertion :** A parallelogram consists of two congruent triangles.

Reason : Diagonal of a parallelogram divides it into two congruent triangles.

HOTS Hots Subjective Questions

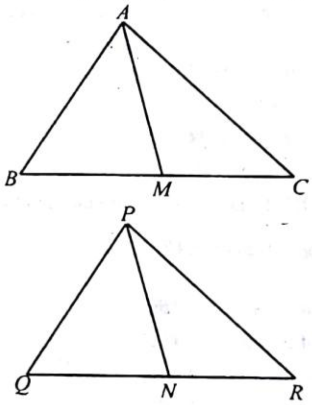
DIRECTIONS: *Answer the following questions.*

1. A point P is taken inside an equilateral four-sided figure $ABCD$ such that its distances from the vertices D and B are equal. Show that AP and PC are in one and the same straight line.

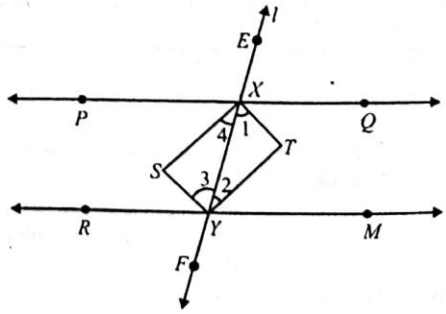


2. Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of ΔPQR (See figure). Show that :

(i) $\Delta ABM \cong \Delta PQN$ (ii) $\Delta ABC \cong \Delta PQR$



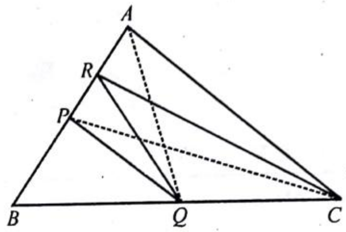
3. PQ and RM are two parallel lines and a transversal l intersects PQ at X and RM at Y. Bisector XS of $\angle PXY$, bisector YS of $\angle XYR$ meet at point S. Bisector XT of $\angle QXY$ and bisector YT of $\angle XYM$ meet at point T. Prove that the bisectors of the interior angles form a rectangle.



4. In triangle ABC, points M and N on sides AB and AC respectively are taken so that $AM = \frac{1}{4}AB$ and $AN = \frac{1}{4}AC$. Prove that $MN = \frac{1}{4}BC$

5. If E, F, G and H are respectively the mid-points of the sides of a parallelogram $ABCD$, show that ar $(EFGH)$ $= \frac{1}{2}$ ar $(ABCD)$.

6. P and Q are respectively the mid-points of sides AB and BC of a triangle ABC and R is the mid-point of AP. Show that :



(i) ar $(\Delta PRQ) = \frac{1}{2}$ ar(ΔARC)

(ii) ar $(\Delta RQC) = \frac{3}{8}$ ar (ΔABC)

SOLUTIONS

EXERCISE – I

FILL IN THE BLANKS

1. AB
2. AB
3. Equal
4. Congruent
5. larger
6. third
7. Isosceles
8. right angle triangle
9. parallelogram.
10. rectangle
11. Parallel
12. Areas
13. Equal
14. Corresponding altitude
15. Two

TRUE/FALSE

1. True 2. True 3. True 4. False
5. True 6. True 7. False 8. False
9. True 10. True 11. True 12. False
13. False 14. True 15. True 16. False
17. True 18. True

MATCH THE COLUMNS

1. (A) → p; (B) → q; (C) → r; (D) → s
2. (A) → p; (B) → q; (C) → r; (D) → s

VERY SHORT ANSWER QUESTIONS

1. Construct the perpendicular bisectors of two sides of ABC. Their point of intersection is the required point.

2. Draw the angle bisectors of any two angles of the triangle. Their point of intersection is the required point.

3. (i) 110°, 70°, 110° (ii) 45°, 135°, 45° (iii) 85°, 95°, 85°

4. Since, ΔDBC and ΔEBC are on the same base BC and have equal areas.
$\therefore$ Their altitudes must be the same.
$\therefore$ $DE \parallel BC$.

SHORT ANSWER QUESTIONS

1. In right triangle APB and right triangle APC,
Given Hyp. AB = Hyp. AC
$AP = AP$ (Common)
$\therefore$ $\Delta APB \cong \Delta APC$ (RHS Rule)
$\therefore$ $\angle ABP = \angle ACP$ (C.P.C.T.)
$\Rightarrow$ $\angle B = \angle C$.

2. In right triangles BAY and ABX,
Given that
Hyp. AY = Hyp. BX
$AB = AB$ (Common)
$\therefore$ $\Delta BAY \cong \Delta ABX$ (R.H.S. Axiom)
$\therefore$ $BY = AX$
and $\angle BAY = \angle ABX$. (C.P.C.T.)

3. In ΔABC,
$AB = AC$ (Given)
$\therefore$ $\angle ACB = \angle ABC$ ....(1)
(Angles opposite to equal sides of a triangle)
$\therefore$ $\angle ACB > \angle CBS$
$\therefore$ $BS > CS$ (Side opposite to greater angle is longer)
$\therefore$ $CS < BS$

4. In ΔABC, $AB = AC$ (given)
$\therefore$ $\angle ABC = \angle ACB$ (angles opposite equal sides are equal)
In Δs BCM and BCN, $\angle N = \angle M$ (each = 90°)
$\angle ABC = \angle ACB$ (from above)
$BC = BC$ (common)
$\therefore$ $\Delta BCM \cong \Delta CBN$ (A.A.S. axiom of congruency)
$\Rightarrow$ $BM = CN$ (C.P.C.T.)

5. Since, AD is the bisector of $\angle BAC$ therefore,
in ΔCAQ and ΔBAQ,
$\angle CAQ = \angle BAQ$
Given that $\angle CQD = \angle BQD$ (Given)
$\Rightarrow$ $180° - \angle CQD = 180° - \angle BQD$
$\Rightarrow$ $\angle AQC = \angle AQB$ (Common)
$AQ = AQ$ (ASA Axiom)
$\therefore$ $\Delta CAQ \cong \Delta BAQ$ (C.P.C.T.)
$\therefore$ $CQ = BQ$

6. In ΔABC and ΔABD,
Given $AC = AD$
$AB = AB$ (Common)
Since, AB bisects angle A
$\therefore$ $\angle CAB = \angle DAB$
Thus, $\Delta ABC \cong \Delta ABD$ (SAS Rule)
$BC = BD$ (CPCT)

7. From ΔOAD and ΔOBC we have given
$AD = BC$
$\angle OAD = \angle OBC$ (Each = 90°)
$\angle AOD = \angle BOC$ (Vertically Opposite Angles)
$\therefore$ By AAS rule $\Delta OAD \cong \Delta OBC$
$\therefore$ By CPCT, $OA = OB$
Thus CD bisects AB.

8. In ΔABC and ΔADE,
Given $AB = AD$,
$AC = AE$
and $\angle BAD = \angle EAC$
$\Rightarrow$ $\angle BAD + \angle DAC = \angle DAC + \angle EAC$
(Adding $\angle DAC$ to both sides)

$\Rightarrow$ $\angle BAC = \angle DAE$
$\therefore$ By SAS rule
$\Delta ABC \cong \Delta ADE$
$\Rightarrow$ $BC = DE.$ (CPCT)

9. (i) Since, P is the mid-point of the line segment AB
$\therefore$ from ΔDAP and ΔEBP,
$AP = BP$
Also, given $\angle DAP = \angle EBP$ and
$\angle EPA = \angle DPB$
Adding $\angle EPD$ to both sides
$\Rightarrow \angle EPA + \angle EPD = \angle EPD + \angle DPB$
$\Rightarrow \angle APD = \angle BPE$
Thus, by ASA rule
$\therefore \Delta DAP \cong \Delta EBP$

(ii) Since, $\Delta DAP \cong \Delta EBP$ (From above)
$\therefore AD = BE$ (CPCT)

10. From ΔPQT, we have given
$PQ = PT$
Since, Angles opposite to equal sides
$\therefore$ $\angle PTQ = \angle PQT$ ....(1)
Now, In ΔPST and ΔPRQ,
We have
$PT = PQ$ and $\angle TPS = \angle QPR$
$\therefore$ From (1)
$\angle PTQ = \angle PQT$
$\Rightarrow$ $\angle PTS = \angle PQR$
$\therefore$ $\Delta PST \cong \Delta PRQ$ (ASA Axiom)
and by C.P.C.T
$PS = PR$
$\Rightarrow$ ΔPRS is isosceles.

11. ΔADB and ΔADC gives us that
$\angle ADB = \angle ADC$ (Each = 90°)
($\because$ AD is the perpendicular bisector of BC)
$\therefore$ $DB = DC$
$AD = AD$ (Common)
$\therefore$ $\Delta ADB \cong \Delta ADC$ (By SAS Rule)
$\therefore$ $AB = AC$ (C.P.C.T)
Hence proved.

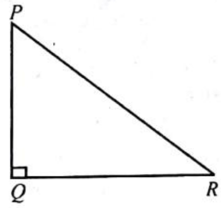
12. (i) In ΔABE and ΔACF, we have given
$BE = CF$
$\angle BAE = \angle CAF$ (Common)
$\angle AEB = \angle AFC$ (Each = 90°)
$\therefore \Delta ABE = \Delta ACF$. (By AAS Rule)

(ii) From (i) $\Delta ABE \cong \Delta ACF$
$\therefore AB = AC$ (C.P.C.T.)
$\therefore$ ΔABC is an isosceles triangle.

13. Since ΔABC is an isosceles triangle on the base BC
$\therefore$ $\angle ABC = \angle ACB$ ...(1)
Similarly, ΔDBC is an also isosceles triangle on the base BC
$\therefore$ $\angle DBC = \angle DCB$ ...(2)
Adding the corresponding sides of (1) and (2), we get
$\angle ABC + \angle DBC = \angle ACB + \angle DCB$
$\Rightarrow$ $\angle ABD = \angle ACD.$

14. $DR = CS$
$\Rightarrow$ $DR + RQ = CS + RS$ ....(1)
$\Rightarrow$ $DS = CR$
In right triangles ADS and BCR,
Hyp. AS = Hyp. BR
$DS = CR$
$\therefore$ $\Delta ADS \cong \Delta BCR$ (R.H.S. Axiom)
$\therefore$ $\angle DAS = \angle CBR.$ (C.P.C.T.)

15. Let PQR be a right angled triangle in which $\angle Q = 90°$
Then, $\angle P + \angle R = 90°$ (By angle sum property)
$\therefore$ $\angle Q = \angle P + \angle R$
$\Rightarrow$ $\angle Q > \angle P$



and $\angle Q > \angle R$
$\therefore$ $PR > QR$ ($\because$ Side opposite to greater angle is longer)
Similarly $PR > PQ$
$\therefore$ PR is the longest side, i.e., hypotenuse is the longest side.

16. Since, EAB is a straight line
$\therefore$ $\angle DAE + \angle DAB = 180°$
$\Rightarrow$ $73° + \angle DAB = 180°$
i.e., $\angle DAB = 180° - 73° = 107°$
Since, the sum of the angles of quadrilateral $ABCD$ is 360°
$\therefore$ $107° + 105° + x + 80° = 360°$
$\Rightarrow$ $292° + x = 360°$
and $x = 360° - 292° = 68°$

17. Let DB intersect EF at G. i.e. - G is the mid-point of BD. Since, E is the mid-point of DA and $EG \parallel AB$. Hence, G is the mid-point of DB. (By converse of mid-point theorem) Similarly, now G is the mid-point of BD and $GF \parallel AB \parallel DC$ by converse of mid-point theroem, F is the mid-point of BC.

18. Since, ΔAPB and $\parallel$ gm $ABCD$ are on the same base AB and between the same parallels line AB and DC.

$\therefore$ ar $(APB) = \frac{1}{2}$ ar $(ABCD)$ ....(i)

Similarly, ΔBCQ and $\parallel$ gm $ABCD$ have the same base and between same parallel lines.

$\therefore$ ar $(BCQ) = \frac{1}{2}$ ar $(ABCD)$ ....(ii)

From (i) and (ii), we get ar (APB) = ar (BCQ)

19. (i) ar $(PQRS)$ = ar $(ABRS)$ ....(1)
($\because$ Parallelograms $PQRS$ and $ABRS$ are on the same base SR and between the same parallels SR and PB.)

(ii) ar $(AXS) = \frac{1}{2}$ ar $(ABRS)$ ....(2)

($\because$ ΔAXS and ||gm $ABRS$ are on the same base AS and between the same parallels AS and RB)

From (1) and (2), we get, ar $(AXS) = \frac{1}{2}$ ar $(PQRS)$.

20. Since, AD is a median in ΔABC which divides it into two triangles of equal areas.

$\therefore$ ar (ΔABD) = ar (ΔACD) ....(1)

Similarly, ar (ΔEBD) = ar(ΔECD) ....(2)

($\because$ ED is a median in ΔEBC)

Subtracting (2) from (1), we get

ar (ΔABD) − ar(ΔEBD) = ar(ΔACD) − ar(ΔECD)

$\Rightarrow$ ar(ΔABE) = ar(ΔACE).

21. Let the measure of the angles of the given quadrilateral be $(2x)°, (3x)°, (7x)°, (6x)°$ respectively.

Then, $2x + 3x + 7x + 6x = 360$

[$\because$ The sum of the angles of a quadrilateral is 360°]

$\Rightarrow 18x = 360 \Rightarrow x = 20$

$\therefore$ First angle $= (2x)° = (2 \times 20)° = 40°$

Second angle $= (3x)° = (3 \times 20)° = 60°$

Third angle $= (7x)° = (7 \times 20)° = 140°$

Fourth angle $= (6x)° = (6 \times 20)° = 120°$

LONG ANSWER QUESTIONS

1. In ΔAPC and ΔBPD,

$\angle ACP = \angle BDP$ (Given)

$\angle APC = \angle BPD$ (Vertically Opposite Angles)

$\therefore$ Third $\angle A$ = Third $\angle B$

($\because$ The sum of the three angles of a Δ is 180°)

In ΔACD and ΔBDC,

$CD = CD$ (Common)

$\angle A = \angle B$ (Prove above)

$\angle BCD = \angle ADC$ (Given)

$\angle ACB = \angle BDA$ (Given)

Adding, we get

$\angle BCD + \angle ACB = \angle ADC + \angle BDA$

$\Rightarrow ACD = BDC$

$\therefore \Delta ACD \cong \Delta BDC$ (ASS. Axiom)

$\therefore AD = BC$ (C.P.C.T.)

2. Since, l and m and p and q are parallel lines therefore $AB \parallel DC$ and $AD \parallel BC$

$\therefore$ Quadrilateral $ABCD$ is a parallelogram.

($\because$ A quadrilateral is a parallelogram if both the pairs of opposite sides are parallel)

Since, $ABCD$ is ||gm

$\therefore BC = AD$ ....(1)

and $AB = CD$ ....(2)

(Opposite sides of a || gm are equal)

and $\angle ABC = \angle CDA$ ....(3)

(Opposite angles of a || gm are equal)

Thus, In ΔABC and ΔCDA we get

$AB = CD$ (From (2))

$BC = DA$ (From (1))

$\angle ABC = \angle CDA$ (From (3))

$\therefore \Delta ABC \cong \Delta CDA$ (SAS Rule)

3. (i) Sicne, M is the mid-point of the hypotenuse AB therefore, in ΔAMC and ΔBMD, we have

$AM = BM$

$CM = DM$ (Given)

$\angle AMC = \angle BMD$ (Vertically Opposite Angles)

$\therefore \Delta AMC \cong \Delta BMD.$ (SAS Rule)

(ii) Since $\Delta AMC \cong \Delta BMD$ (From (i) above)

$\therefore \angle ACM = \angle BDM$ (CPCT)

But these are alternate interior angles and they are equal

$\therefore AC \parallel BD$

Now, We have a transversal BC intersects AC and BD and $AC \parallel BD$

$\therefore \angle DBC + \angle ACB = 180°$

($\because$ The sum of the consecutive interior angles on the same side of the transversal is 180°)

$\Rightarrow \angle DBC + 90° = 180°$ ($\because ACB = 90°$ (given))

$\Rightarrow \angle DBC$ is a right angle.

(iii) Now, from ΔDBC and ΔACB, we have

$\angle DBC = \angle ACB$ (each = 90°) (From (ii))

$BC = CB$ (Common)

$\therefore \Delta AMC \cong \Delta BMD$ (From (i))

$\therefore AC = BD$ (CPCT)

So, from SAS rule,

$\Delta DBC \cong \Delta ACB$.

(iv) Since, $\Delta DBC \cong \Delta ACB$ (From (iii))

$\therefore DC = AB$ (CPCT)

Now, $DM = CM = \frac{1}{2} DC$

$\Rightarrow 2\,CM = AB$

$\Rightarrow CM = \frac{1}{2} AB.$

4. Since, ABC is an equilateral triangle.

$\therefore$ All the three sides are equal

i.e. $AB = BC = CA$ ....(1)

Consider $AB = BC$

$\Rightarrow \angle A = \angle C$ ....(2)

(Angles opposite to equal sides of a triangle are equal)

Consider $BC = CA$

$\therefore \angle A = \angle B$ ....(3)

(Angles opposite to equal sides of a triangles are equal)

From (2) and (3), we obtain

$\angle A = \angle B = \angle C$ ....(4)

Also, In ΔABC,

$\angle A + \angle B + \angle C = 180°$ ....(5)

(By angle sum property)

Let $\angle A = y°$. Then, $\angle B = \angle C = y°$ From (4)

$\therefore$ From (5),

$y° + y° + y° = 180°$

$3y° = 180° \Rightarrow y° = 60°$

$\Rightarrow \angle A = \angle B = \angle C = 60°.$

5. (i) In ΔABC, Which is right angled at B, we have

$BC^2 = AC^2 - AB^2$

$= (15)^2 - (9)^2 = 225 - 81$

$= 144$

$\Rightarrow \quad BC = \sqrt{144} = 12$cm.

(ii) Since, D and E are the mid-points of AB and AC respectively in ΔABC

$\therefore \ DE \parallel BC$

and $DE = \frac{1}{2}BC = \frac{1}{2}(12) = 6$ cm

$AD = BD = \frac{1}{2}AB = \frac{1}{2}(9) = \frac{9}{2}$cm

$\because \ DE \parallel BC$ and AB intersects them

$\therefore \ \angle ADE = \angle ABC = 90°$ (Corresponding $\angle$s)

$\Rightarrow \ \Delta ADE$ is a right angled triangle

$\therefore$ Area of ΔADE

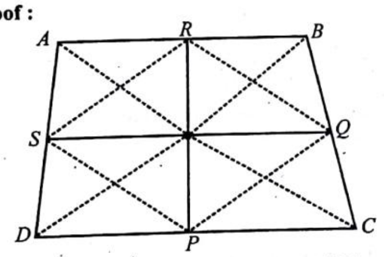
$= \frac{(AD)(DE)}{2} = \frac{9}{2}.\frac{6}{2} = \frac{27}{2} = 13.5\,\text{cm}^2$.

6. **Given :** P, Q, R and S are the mid-points of the sides DC, CB, BA and AD respectively in a quadrilateral $ABCD$.

To prove : PR and QS bisect each other.

Construction : Join PQ, QR, RS, SP, AC and BD.

Proof :



Since, R and Q are the mid-points of AB and BC respectively. in ΔABC

$\therefore \quad RQ \parallel AC$ and $RQ = \frac{1}{2}AC$

Similarly, P and S are the mid-points of AD and DC respectively $\therefore$ we have

$PS \parallel AC$ and $PS = \frac{1}{2}AC$

$\therefore \quad RQ \parallel PS$ and $RQ = PS$.

Thus a pair of opposite sides of a quadrilateral $PQRS$ are parallel and equal.

$\therefore$ Quadrilateral $PQRS$ is a parallelogram.

Since the diagonals of a parallelogram bisect each other.

$\therefore$ PR and QS bisect each other.

7. (i) In ΔACB, since, M is the mid-point of AB and $MD \parallel BC$.

$\therefore D$ is the mid-point of AC.

(By converse of mid-point theorem)

(ii) $\therefore \ \angle ADM = \angle ACB$ (Corresponding angles)

($\because$ $MD \parallel BC$ and AC intersects them)

But we have given $\angle ACB = 90°$

$\therefore \ \angle ADM = 90° \Rightarrow MD \perp AC$

(iii) Now, $\angle ADM + \angle CDM = 180°$

(Linear Pair Axiom)

$\therefore \ \angle ADM = \angle CDM = 90°$

In ΔADM and ΔCDM, in which

$AD = CD$ ($\because$ D is the mid-point of AC)

$\angle ADM = \angle CDM$ (Each measures 90°)

$DM = DM$ (Common)

$\therefore \ \Delta ADM \cong \Delta CDM$ (SAS Rule)

$\therefore \quad MA = MC$ (C.P.C.T.)

But we know M is the mid-point of AB

$\therefore \quad MA = MB = \frac{1}{2}AB$

$\therefore \quad MA = MC = \frac{1}{2}AB$

$\Rightarrow \quad CM = MA = \frac{1}{2}AB$.

8. From the figure it is clear that

$\angle CDB = \angle ABD = 90°$

But these angles form a pair of alternate equal angles.

$\therefore \quad DC \parallel AB$

Also, $DC = AB = 3$ units (From figure)

Now, we know that A quadrilateral is a parallelogram if a pair of opposite sides is parallel and is of equal length.

$\therefore$ Quadrilateral $ABCD$ is a parallelogram.

Now, area of the $\parallel$ gm $ABCD$

= Base × Corresponding altitude

= $AB \times BD$

= 3×4 square units ($\because$ $AB = 3$, $BD = 4$)

= 12 square units

9. Since, $ABCD$ is a parallelogram in which $AB \parallel DC$

$\therefore \ AB = DC$

$\Rightarrow$ Base $DC = 16$ cm

Height $AE = 8$ cm. (Given)

Area of $\parallel$ gm = Base × Corresponding Altitude

$\therefore$ Area of $\parallel$ gm $ABCD$

$= DC \times AE = 16 \times 8 = 128\ \text{cm}^2$ ...(1)

Again by considering AD as the base and height as $CF = 10$ cm.

$\therefore$ Area of $\parallel$ gm $ABCD = AD \times CF$

$= AD \times 10 = 10\,AD$. cm^2

Clearly $10.AD = 128$ (By using (1))

$\Rightarrow \quad AD = \frac{128}{10} = 12.8$ cm.

10. Draw a line through P parallel to DC which meets AD at Q and BC at R.

Since ΔAPB and parallelogram $ABRQ$ have the same base line AB and between the same parallels line AB and QR.

$\therefore \quad$ ar $(\Delta APB) = \frac{1}{2}$ ar ($\parallel$ gm $ABRQ$) ....(1)

Similarly,

ΔPCD and ||gm $DCRQ$ have the same base DC and between the same parallel lines DC and QR.

$\therefore$ ar (ΔPCD) = $\frac{1}{2}$ ar (|| gm $DCRQ$) ....(2)

From (1) and (2), we get, ar (ΔAPB) + ar (ΔPCD)

= $\frac{1}{2}$ ar (|| gm $ABRQ$) + $\frac{1}{2}$ ar (|| gm $DCRQ$)

Hence, ar (ΔAPB) + ar (ΔPCD) = $\frac{1}{2}$ ar (|| gm $ABCD$)

(ii) Similarly, ar (ΔAPD) + ar (ΔPBC)

= $\frac{1}{2}$ ar (|| gm $ABCD$)

Hence, ar (ΔAPB) + ar (ΔPCD)

= ar (ΔAPD) + ar (PBC)

11. (i) $EF \parallel BC$

($\because$ In a triangle ABC, the line segment FE joining the mid-points F and E of two sides AB and AC is parallel to the third side.)

$\Rightarrow$ $EF \parallel BD$ ($\because$ BD is the part of BC) ....(1)

Similarly, the line segment ED, joining the mid-points E and D of two sides CA and CB is || to the 3rd side.

$\therefore$ $ED \parallel BF$ ....(2)

From (1) and (2),

$BDEF$ is a parallelogram.

(ii) From (i), we can have

$AFDE$ and $FDCE$ are parallelograms.

Now, ar (ΔFBD) = ar (ΔDEF) ....(3)

($\because$ FD is a diagonal of ||gm $BDEF$.)

Similarly, ar (ΔDEF) = ar (ΔFAE) ....(4)

and, ar (ΔDEF) = ar (ΔDCE) ....(5)

From (3), (4) and (5), we have

ar (ΔFBD) = ar (ΔDEF) = ar (ΔFAE)

= ar (ΔDCE) ....(6)

Since, ΔABC is divided into four triangles ΔFBD, ΔDEF, ΔFAE and ΔDCE.

$\therefore$ ar (ΔABC) = ar (ΔFBD) + ar (ΔDEF) + ar (ΔFAE) + ar (ΔDCE) = 4 ar (ΔDEF) [From (6)]

$\Rightarrow$ ar (ΔDEF) = $\frac{1}{4}$ ar (ΔABC) ....(7)

(iii) Consider ar ($BDEF$)

= ar (ΔFBD) + ar (ΔDEF)

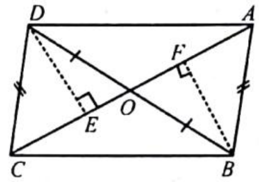
= ar (ΔDEF) + ar (ΔDEF) [From (3)]

= 2 ar (ΔDEF)

= $2.\frac{1}{4}$ ar (ΔABC) [From (7)]

= $\frac{1}{2}$ ar (ΔABC).

12.



(i) Draw $DE \perp AC$ and $BF \perp AC$

In ΔADB, AO is a median

$\therefore$ ar (ΔAOD) = ar (ΔAOB) ....(1)

($\because$ A median of a triangle divides it into two triangles of equal areas)

Similarly, in ΔCBD, CO is a median.

$\therefore$ ar (ΔCOD) = ar (ΔCOB) ....(2)

Adding (1) and (2), we get

ar (ΔAOD) + ar (ΔCOD) = ar (ΔAOB) + ar (ΔCOB)

$\Rightarrow$ ar (ΔACD) = ar (ΔACB)

$\Rightarrow$ $\frac{(AC)(DE)}{2} = \frac{(AC)(BF)}{2}$

$\left(\because \text{Area of a triangle} = \frac{\text{Base} \times \text{Corresponding altitude}}{2}\right)$

$\Rightarrow$ $DE = BF$ ....(3)

In right Δs DEC and BFA,

Hyp. DC = Hyp. BA

$DE = BF$ (From (3))

$\therefore$ $\Delta DEC \cong \Delta BFA$ (R.H.S. Rule)

$\therefore$ $\angle DCE = \angle BAF$ (C.P.C.T.)

But these angles form a pair of equal alternate interior angles.

$\therefore$ $DC \parallel AB$ ....(4)

Since, $DC = AB$ and $DC \parallel AB$.

$\therefore$ ABCD is a parallelogram

($\because$ A quadrilateral is a parallelogram if a pair of opposite sides is parallel and equal)

$\therefore$ $DA \parallel CB$

($\because$ Opposite sides of a ||gm are parallel)

(i) Since, $ABCD$ is a || gm

$\therefore$ $OC = OA$ ....(5)

ar (ΔDOC) = $\frac{OC \times DE}{2}$

ar (ΔAOB) = $\frac{OA \times BE}{2}$

ar (ΔDOC) = ar (ΔAOB).

($\because$ $DE = BF$ and $OC = OA$)

(ii) ar (ΔDOC) = ar (ΔAOB) [From (i)]

$\Rightarrow$ ar (ΔDOC) + ar (ΔOCB)

= ar (ΔAOB) + ar (ΔOCB)

(Adding same areas on both sides)

$\Rightarrow$ ar (ΔDCB) = ar (ΔACB).

EXERCISE – 2

MULTIPLE CHOICE QUESTIONS

1.	(b)	2.	(b)	3.	(d)	4.	(b)
5.	(a)	6.	(b)	7.	(d)	8.	(a)
9.	(a)	10.	(d)	11.	(c)	12.	(c)
13.	(a)	14.	(a)	15.	(b)	16.	(d)
17.	(c)	18.	(a)	19.	(b)	20.	(a)

MORE THAN ONE CORRECT

1. (a, b, c) 2. (c, d) 3. (a, b) 4. (a, b, c)

PASSAGE BASED QUESTIONS

1. (b) 2. (a) 3. (c)

ASSERTION & REASON

1. (a) 2. (d) 3. (a) 4. (d) 5. (a)

HOTS SUBJECTIVE QUESTIONS

1. In ΔAPD and ΔAPB, we have

$AD = AB$

$AP = AP$ (Common)

and given

$PD = PB$

$\therefore$ By SSS rule

$\Delta APD \cong \Delta APB$

$\therefore \angle 1 = \angle 2$ (C.P.C.T)

Similarly, $\Delta DPC \cong \Delta BPC$

$\therefore \angle 3 = \angle 4$ (C.P.C.T.)

But $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360°$

($\because$ Sum of all the angles round a point = 360°)

$\Rightarrow 2\angle 2 + 2\angle 3 = 360°$ ($\because \angle 1 = \angle 2$ and $\angle 3 = \angle 4$)

$\Rightarrow \angle 2 + \angle 3 = 180°$

$\Rightarrow$ AP and PC are in one and the same straight line (Linear Pair Axiom)

2. (i) In ΔABM and ΔPQN

$AB = PQ$ ....(1)

$AM = PN$ ....(2) (Given)

and $BC = QR$ (Given)

Since, M and N are the mid-points of BC and QR respectively

$\therefore 2BM = 2QN$

$\Rightarrow BM = QN$ ....(3)

From, (1), (2) and (3)

$\Delta ABM = \Delta PQN$ (SSS Rule)

(ii) From (i) part $\Delta ABM \cong \Delta PQN$

$\therefore$ By C.P.C.T.

$\angle ABM = \angle PQN$

$\Rightarrow \angle ABC = \angle PQR$ ....(4)

Now, in ΔABC and ΔPQR, we have given

$AB = PQ$

and $BC = QR$

$\therefore$ From (4), we have $\angle ABC = \angle PQR$

$\therefore \Delta ABC = \Delta PQR$. (SAS Rule)

3. Since $PQ \parallel RM$ and EF intersects them

$\therefore \angle QXY = \angle RYX$ (Alternate angles)

$\Rightarrow \frac{1}{2}\angle QXY = \frac{1}{2}\angle RYX$

$\Rightarrow \angle 1 = \angle 3$

But these angles form a pair of equal alternate angles for lines XT and SY and a transversal XY.

$\therefore XT = SY$ ....(1)

Similarly, we can have

$SX \parallel YT$ ....(2)

From (1) and (2), and fact that a quadrilateral is a parallelogram if both pairs of its opposite sides are parallel $SYTX$ is a parallelogram

Now, $\angle QXY + \angle MYX = 180°$

(Consecutive interior angles)

$\Rightarrow \frac{1}{2}\angle QXY + \frac{1}{2}\angle MYX = 90°$

$\Rightarrow \angle 1 + \angle 2 = 90°$

But $\angle 1 + \angle 2 + \angle XTY = 180°$

(Angle sum property of a Δ)

$\Rightarrow 90° + \angle XTY = 180°$

$\Rightarrow \angle XTY = 90°$

$\Rightarrow \angle YSX = 90°$

and $\angle SXT = 90$

($\because$ Consecutive interior angles are supplementary)

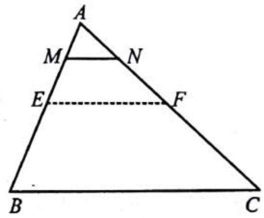
Now, $\angle SXT = 90°$

$\Rightarrow \angle SYT = 90°$

(Opposite angles of a ||gm are equal)

Thus each angle of the parallelogram $SYTX$ is 90°. Hence parallelogram $SYTX$ is a rectangle.

4.



We construct a line EF where E and F are the middle points of AB and AC respectively.

$EF \parallel BC$ and $EF = \frac{1}{2}BC$ ... ...(1)

($\because$ E and F are mid points)

Now, $AE = \frac{1}{2}AB$ and $AM = \frac{1}{4}AB$

$\therefore AM = \frac{1}{2}AE$

Similarly, $AN = \frac{1}{2}AF$

$\Rightarrow$ M and N are the mid-points of AE and AF respectively.

$\therefore$ $MN \parallel EF$ and

$MN \frac{1}{2} EF = \frac{1}{2}\left(\frac{1}{2}AE\right)$ [From (1)]

$= \frac{1}{4}BC$

5. Since, F and G are the mid-points of BC and DC respectively in $\Delta ABCD$.

$\therefore FG \parallel BD$ ...(1)

($\because$ In a triangle, the line segment joining the mid-points of any two sides is parallel to the third side.

Similarly, $EH \parallel BD$ ...(2)

(E and H are the mid-points of AB an AD respectively in ΔBAD of side)

From (1) and (2),

$EH \parallel BD$ ...(3)

Similarly, we can show that

$EF \parallel HG$ ...(4)

From (3) and (4),

Quadrilateral $EFGH$ is a parallelogram

Again $FO \parallel BA$

($\because$ F is the mid-point of CB and O is the mid-point of CA)

$\Rightarrow FO \parallel CG$ ...(5)

($\because$ $BA \parallel CD$) (opposite sides of a parallelogram are parallel)

$\therefore BA \parallel CG$ and $FO = \frac{1}{2}BA$

$= \frac{1}{2}CD$ (By Defn of || gm)

$= CG$ ...(6)

($\because$ G is the mid-point of CD)

From (5) and (6),

Quadrilateral $OFCG$ is a parallelogram

$\therefore OP = PC$

($\because$ Diagonals of a || gm bisect each other)

ar(ΔOPF) = ar(ΔCDF)

Similarly, ar(ΔOQF) = ar(ΔBQF)

[$\because$ ΔOPF and ΔCPF have equal bases ($\because$ $OP = PC$) and have a common vertex F $\because$ Their altitudes are also the same]

Adding, we get

ar (ΔOPF) + ar (ΔOQF) = ar (ΔCPF) + ar (ΔBQF)

ar (||gm $OQFP$) = ar (ΔCPF) + ar (ΔBQF) ...(7)

Similarly,

ar (||gm $OPGS$) = ar (ΔGPC) + ar (ΔDSG) ...(8)

ar (||gm $OSHR$) = ar (ΔDSH) + ar (ΔHAR) ...(9)

ar (||gm $OREQ$) = ar (ΔARE) + ar (ΔEQB) ..(10)

Adding (7), (8), (9) and (10), we get

ar (||gm $EFGH$) = {ar (ΔCPF) + ar (ΔGPC)} + {ar (ΔDSG) + ar (ΔDSH)} + {ar (ΔHAR) + ar (ΔBQF) + ar (ΔEQB)}

= ar (ΔECG) + ar (ΔGDH) + ar (ΔHAE) + ar (ΔEBF)

$= \frac{1}{2}$ar (|| gm $ABCD$)

6. We join AQ and CP.

(i) ar (ΔPRQ) = ar(ΔARQ)

$= \frac{1}{2}$ ar (ΔAPQ) $= \frac{1}{2}$ ar (ΔBPQ)

$= \frac{1}{2}$ ar (ΔCPQ) $= \frac{1}{2} \cdot \frac{1}{2}$ ar (ΔBPC)

$= \frac{1}{4}$ ar (ΔBPC) $= \frac{1}{4} \cdot \frac{1}{2}$ ar (ΔABC)

$= \frac{1}{8}$ ar (ΔABC) ....(1)

Also, $\frac{1}{2}$ ar(ΔARC) $= \frac{1}{2} \cdot \frac{1}{2}$ ar(ΔAPC)

$= \frac{1}{4}$ ar (ΔAPC) $= \frac{1}{4} \cdot \frac{1}{2}$ ar (ΔABC)

$= \frac{1}{8}$ ar (ΔABC) ....(2)

From (1) and (2), we have, ar(ΔPRQ)

$= \frac{1}{2}$ ar (ΔARC)

(ii) ar (ΔRQC) = ar (ΔRBQ)

= ar (ΔPRQ) + ar (ΔBPQ)

$= \frac{1}{8}$ ar (ΔABC) + $\frac{1}{2}$ ar (ΔPBC) [Using(1)]

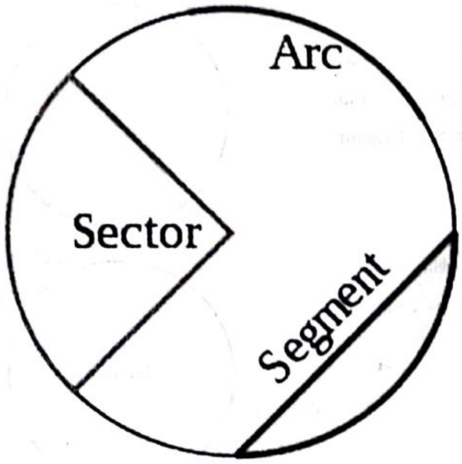
$= \frac{1}{8}$ ar (ΔABC) + $\frac{1}{2} \cdot \frac{1}{2}$ ar (ΔABC)

$= \frac{1}{8}$ ar (ΔABC) + $\frac{1}{4}$ ar (ΔABC) $= \frac{3}{8}$ ar (ΔABC)

(iii) ar (ΔPBQ) $= \frac{1}{4}$ ar (ΔABC) .....(3) [From (ii)]

ar (ΔARC) $= \frac{1}{4}$ ar (ΔABC)

From (3) and (4), ar (ΔPBQ) = ar (ΔARC).

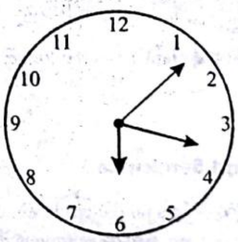


7
CHAPTER

Circles

INTRODUCTION

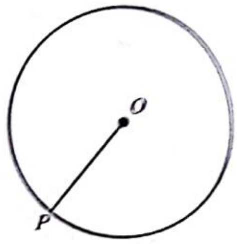
See the dial of a clock. You might have observed that the second's hand goes round the dial of the clock rapidly and its tip moves in a round path. This traced path by the tip of the second's hand is called a circle. In this chapter, you will study about circles, terms related to circles and properties of circles and some theorem related to circles.



Clock

CIRCLE

Circle is a set of point or collection of point in a plane which are at a fixed distance from a fixed point. The fixed point is called the centre of the circle. In the given diagram 'O' is the centre of the circle. The fixed distance is called the radius. In the diagram, OP is the radius of the circle.



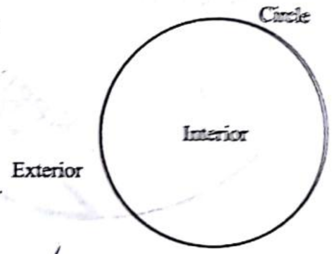
Edges of the round shaped objects such as wheel of a vehicle, five ₹ coin etc are some examples of circles. A circle divides the plane in which it lies into three parts.

These three parts are

(i) Inside the circle which is called interior of the circle.

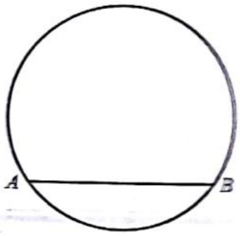
(ii) Circle

(iii) Outside the circle or the exterior of the circle.



Chord

A line segment whose end points lie on the circle is called a chord of the circle. In the given diagram AB is a chord.

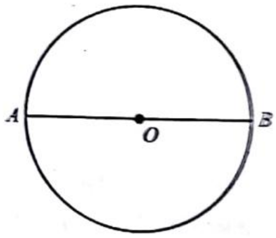


Diameter

A chord which passes through the centre of the circle is called the diameter of the circle.

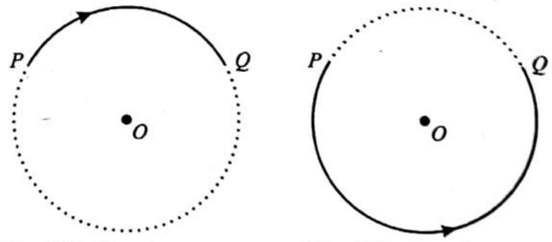
The length of the diameter is twice the length of the radius. In the given diagram, PQ is the diameter.

Note that the diameter is the longest chord of the circle.



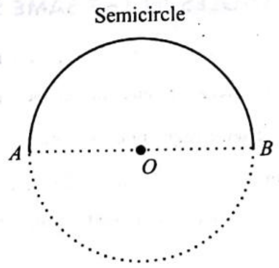
Arcs and Semicircle

If any two points on the circle divides the circle into two unequal parts. The smaller part is called minor arc and the larger part is called major arc. Arc of a circle is denoted by '⌒'. In the diagram PQ is arc.



(i) PQ is the minor arc (ii) PQ is the major arc

The arrow mark on the arcs show the direction in which we read the arcs. In the figure (i), PQ is the minor arc and QP is the major arc. In the figure (ii), PQ is the major arc and QP is the minor arc.



If two points on the circle divide the circle in two equal parts, then each part is called a semi circle.

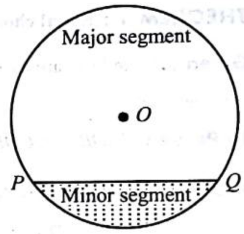
In other words,

A diameter of the circle divides the circle into two equal parts. Each part is called as semi circle.

Circumference

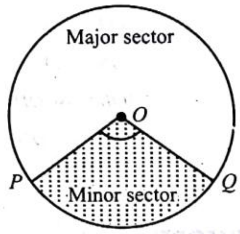
The length of the complete circle is called its circumference.

If a circular wire cut at a point and then re-open as a straight piece of wire. The length of this straight piece of wire is the circumference of the circular wire.



Segment

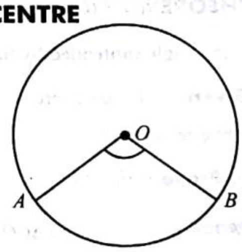
The region between a chord and either of its two arcs is called a segment of the circle. It is of two types major segment and minor segment if chord does not pass through the centre. Major segment is the segment in which centre of the circle lies. Minor segment is the segment in which centre does not lies.



Sector

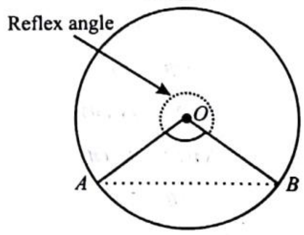
The region between an arc and the two radii, joining the centre to the end points of the arc is called a sector. Sector corresponds to the minor arc is called minor sector and the sector corresponding to the major arc is called major sector.

CENTRAL ANGLE – ANGLE SUBTENDED BY AN ARC OR CHORD AT THE CENTRE



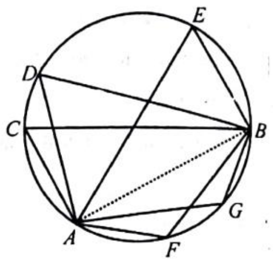
An angle formed, by an arc of a circle at the centre of the circle is called the central angle. In the given diagram $\angle AOB$ is the central angle made by the minor arc AB.

In the given figure the angle subtended by the minor arc AB at the centre O of the circle is $\angle AOB$ and the angle subtended by the major arc AB at the centre O of the circle is reflex $\angle AOB$.



Angle subtended by minor arc at the centre is always less than 180°, angle subtended by major arc is always greater than 180° and angle subtended by semicircle at the centre is always 180°. If a chord does not pass through the centre, then angle subtended by it at the centre is equal to the angle subtended by its corresponding arc.

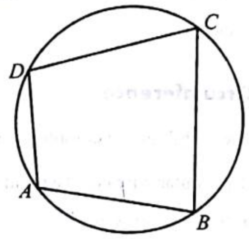
ANGLES IN THE SAME SEGMENT OF A CIRCLE



Any chord of a circle divides the circle in two segments. Angles made by any chord at different points (lie on the same side of the chord) on the circle are called angles in the same segment of the circle.

In the figure $\angle ACB$, $\angle ADB$ and $\angle AEB$ are the angles in the same segment of a circle. Also $\angle AFB$ and $\angle AGB$ are the angle in the same segment of a circle.

CYCLIC QUADRILATERAL

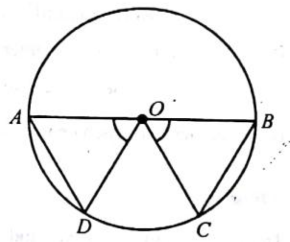


A quadrilateral is called cyclic, if all the four vertices of it lie on a circle. In the figure, $ABCD$ is a cyclic quadrilateral because its all four vertices are lie on a circle.

THEOREM 1 : Equal chord of a circle subtend equal angle at the centre.

Given : AB and CD are two equal chords of a circle with centre O. AB and CD subtends an angle $\angle AOB$ and $\angle COD$ at the centre O respectively.

To Prove : $\angle AOB = \angle COD$



PROOF : In ΔAOB and ΔCOD,

$AB = CD$ [Given]

$OA = OC$ [Radii of the same circle]

$OB = OD$

$\therefore$ $\Delta AOB \cong \Delta COD$ [By SSS congruent Rule i.e. side side side]

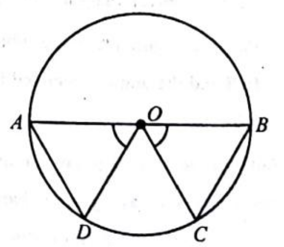
$\therefore$ $\angle AOB = \angle COD$ [Corresponding parts of congruent triangles are equal]

THEOREM 2 : (Converse of the Theorem 1)

If the angle subtended by the chords of a circle at the centre are equal, then the chords are equal.

Given : AB and CD are two chords of a circle with centre O. Chords AB and CD subtend equal angle $\angle AOB$ and $\angle COD$ at the centre respectively.

To Prove : $AB = CD$



PROOF : In ΔAOB and ΔCOD,

$OA = OC$ [Radii of the same circle]

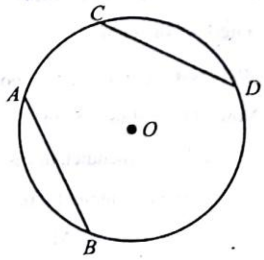
$OB = OD$ [Radii of the same circle]

$\angle AOB = \angle COD$ [Given]

$\therefore$ $\Delta AOB \cong \Delta COD$ [By Side-Angle-Side congruent rule]

$\therefore$ $AB = CD$ [Corresponding parts of congruent triangle are equal]

Illustration 1 : *AB* and *CD* are two chords each of length 10 cm of a circle whose centre is *O* and chord *AB* makes an angle of 65° at the centre. Find the angle which the chord *CD* will make at the centre.



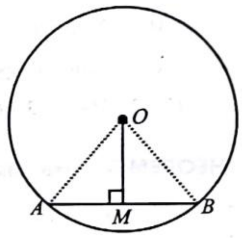
SOLUTION : As AB and CD are two equal chords so according to theorem-1, they will subtend equal angle at the centre. Now, AB subtend an angle of 65° at the centre so CD will also subtend the same angle of 65° at the centre.

THEOREM 3 : The perpendicular from the centre of a circle to a chord bisects the chord.

Given: AB is a chord of a circle with centre O. OM be the perpendicular from O to chord AB.

To Prove : OM bisects AB i.e. $AM = MB$

Construction : Join OA and OB



PROOF : In ΔAOB and ΔBOM

$OA = OB$ [Radii of the same circle]

$OM = OM$ [Common]

$\angle OMA = \angle OMB = 90°$

$\therefore \Delta OAM \cong \Delta OBM$ [By RHS congruent rule]

$\therefore AM = BM$ [Corresponding parts of two congruent triangles are equal]

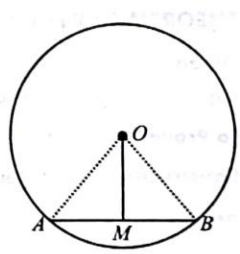
THEOREM 4 : (Converse of the Theorem 3)

The line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord.

Given : AB is a chord of a circle whose centre is O. M is the mid-point of the chord AB. i.e. $AM = MB$.

To Prove : OM is perpendicular to AB i.e. $OM \perp AB$

Construction : Join OA and OB.



PROOF : In $\Delta AOM \cong \Delta BOM$

$OA = OB$ [Radii of the same circle]

$OM = OM$ [Common side]

$AM = MB$ [Given]

$\therefore \Delta AOM \cong \Delta BOM$ [By SSS congruent rule]

$\therefore \angle AMO = \angle BMO$ ....(i) [Corresponding part of two congruent triangles are equal]

Now, $\angle AMO + \angle BMO = 180°$ [Linear pair angles]

But $\angle AMO + \angle BMO$ [from (i)]

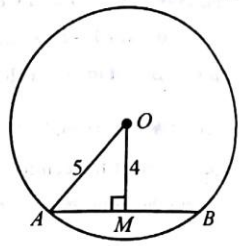
$\therefore \angle AMO + \angle AMO = 180°$

$2\angle AMO = 180°$

$\angle AMO = \frac{180°}{2} = 90°$

So, $OM \perp AB$

Illustration 2 : The radius of the circle is 5 cm and the perpendicular distance of a chord from the centre is 4 cm. Find the length of the chord.



SOLUTION : Let AB be the chord whose length is to be found.

Now, OA = radius = 5 cm

OM = perpendicular distance of AB from the centre is 4 cm.

In right angle triangle AOM,

$OA^2 = OM^2 + AM^2$ [By Pythagoras theorem]

$\therefore \quad AM^2 = OA^2 - OM^2$

$= 5^2 - 4^2$

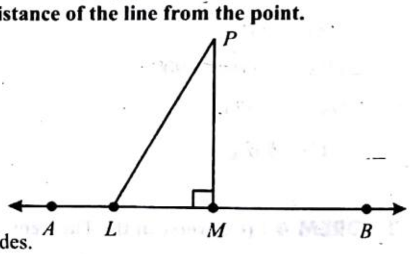
$= 25 - 16 = 9$

$\therefore \quad AM = \sqrt{9} = 3$ cm

Now, from theorem 3, the perpendicular from centre to a chord bisect the chord. So M is the mid-point of AB.

$\therefore \quad AB = 2\,AM = 2 \times 3 = 6$ cm.

THEOREM 5 : Perpendicular distance of a line from a point is the shortest distance of the line from the point.



Given : PM is perpendicular to a line AB from point P.

L is any point other than point M on the line AB.

To Prove : $PM < PL$

PROOF : PL is the hypotenuse of the triangle right angled at M.

We know that hypotenuse of a triangle is always greater any one of the other two sides.

$\therefore \quad PL > PM$

Or $\quad PM < PL$

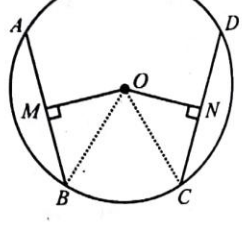
THEOREM 6 : Equal chords of a circle (or of congruent circles) are equidistance from the centre.

Given: AB and CD are two equal chords of a circle having centre O. OM and ON are the distance of AB and CD from centre O respectively i.e. $OM \perp AB$ and $ON \perp CD$.

To Prove : $OM = ON$

Construction : Join OB and OC.

PROOF : From theorem-3, perpendicular from centre to chord bisects the chord, so OM bisect AB and ON bisects CD.



$\therefore \quad AM = MB$ ...(i)

And $\quad CN = ND$ ...(ii)

But $\quad AB = CD$ [Given]

$AM + MB = CN + ND$

$MB + MB = CN + CN$ [from (i) and (ii)]

$\therefore \quad 2MB = 2CN$

$MB = CN$ ...(iii)

Now, In ΔOMB and ΔONC

$MB = CN$ [from (iii)]

$\therefore$ $OB = OC$ [Radii of the same circle]

$\therefore$ $\angle OMB = \angle ONC = 90°$ [$OM \perp AB$, $ON \perp CD$]

$\Delta OMB \cong \Delta ONC$ [By RHS congruent rule]

$\therefore$ $OM = ON$ [Corresponding parts of congruent triangles are equal]

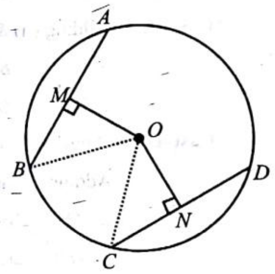
THEOREM 7 : (Converse of the Theorem 6)

Chords equidistance from the centre of a circle are equal in length.

Given : AB and CD are two chords of a circle having centre O. Whose distances of AB and CD from centre O are OM and ON respectively are equal in length i.e. $OM = ON$.

To Prove: $AB = CD$

Construction : Join OB and OC.



PROOF : In ΔOBM and ΔOCN

$OB = OC$ [Radii of the same circle]

$OM = ON$ [Given]

$\angle OMB = \angle ONC = 90°$

$\therefore$ $\Delta OMB \cong \Delta ONC$ [By RHS congruent rule]

$\therefore$ $MB = NC$ ...(i) [Corresponding parts of a congruent triangles are equal]

Now, from theorem-3, perpendicular from centre to a chord bisects the chord

$\therefore$ $AM = MB$ and $CN = ND$

Now, $MB = NC$ [from (i)]

$2MB = 2NC$ [Multiplying both sides by 2]

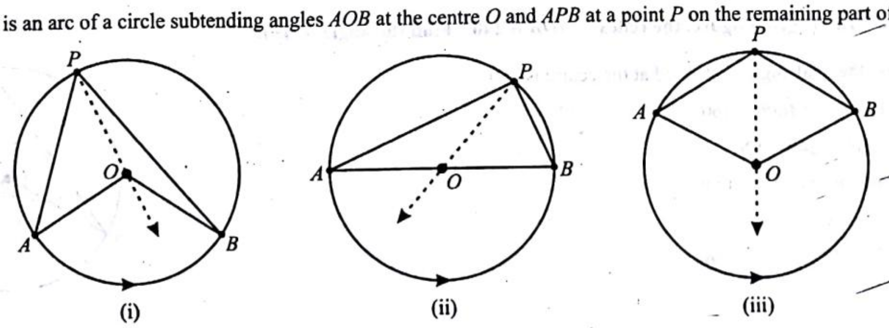
$MB + MB = NC + NC$

$AM + MB = CN + ND$ [$\because$ $AM = MB$ and $CN = ND$]

$\therefore$ $AB = CD$

THEOREM 8 : The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

Given : AB is an arc of a circle subtending angles AOB at the centre O and APB at a point P on the remaining part of the circle.



To Prove : $\angle AOB = 2\angle APB$ (if arc AB is minor)

$180° = 2\angle APB$ (if arc AB is semicircle)

Reflex $\angle AOB = 2\angle APB$ (if arc AB is major arc)

Construction : Join P to O and extends it to point C.

We consider the three cases as shown in the above figure (i) arc AB is minor (ii) arc AB is a semicircle and (iii) arc AB is major.

PROOF : In all the three cases $\angle AOC$ is the exterior angle of ΔAOP.

$\therefore$ $\angle AOC = \angle APO + \angle OAP$ ...(i) [Exterior angle of a triangle is equal to the sum of the two interior opposite angles]

Now, In ΔOAP

$OP = OA$ [Radii]

$\therefore$ $\angle APO = \angle OAP$ [Angles opp. equal sides in a triangle are equal]

$\therefore$ $\angle AOC = \angle APO + \angle APO$

$\angle AOC = 2\angle APO$ ...(ii)

Similarly, $\angle BOC = 2\angle OPB$ ...(iii)

Case-I : Adding (ii) & (iii), we get

$\angle AOC + \angle BOC = 2[\angle APO + \angle OPB]$

$\angle AOB = 2\angle APB$

Case-II : Angle subtended by semicircle at the centre of the circle is 180°.

Adding (ii) and (iii), we get

$\angle AOC + \angle BOC = 2\angle APO + 2\angle OPB$

$\Rightarrow 180° = 2(\angle APO + \angle OPB)$

$\Rightarrow 180° = 2\angle APB$

Case-III : Angle subtended by the major arc AB at the centre of the circle is the reflex angle $\angle AOB$.

Adding (ii) and (iii), we get

$\angle AOC + \angle BOC = 2\angle APO + 2\angle OPB$

$\Rightarrow$ Reflex $\angle AOB = 2(\angle APO + \angle OPB)$

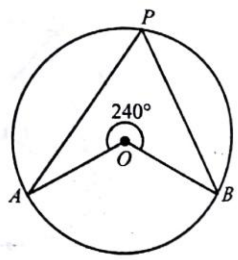
$\Rightarrow$ Reflex $\angle AOB = 2\angle APB$

Note : In case (ii) $\angle AOB = 2\angle APB$, but $\angle AOB = 180°$ [AB is the diameter]

$\therefore$ $\angle APB = \frac{180°}{2} = 90°$

This proves the property that the angle is the semicircle is a right angle.

Illustration 3 : In the given figure, the reflex $\angle AOB$ is 240°. Find the angle $\angle APB$.



SOLUTION : As the total angle subtended at the centre is 360°

$\therefore$ $\angle AOB$ + reflex $\angle AOB = 360°$

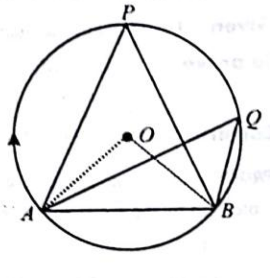
$\therefore$ $\angle AOB = 360 - 240 = 120°$

Now, from the above theorem

$\angle AOB = 2\angle APB$

$\angle APB = \frac{\angle AOB}{2} = \frac{120°}{2} = 60°$

THEOREM 9 : Angle in the same segment of a circle are equal.



Given : $\angle APB$ and $\angle AQB$ are two angles in the same segment $APQBA$ of a circle with centre O.

To Prove : $\angle APB = \angle AQB$

Construction : Join centre O to A and B.

PROOF : From theorem-8,

$\angle AOB = 2\angle APB$ ...(i)

and $\angle AOB = 2\angle AQB$ ...(ii)

$\therefore$ from (i) and (ii), we get

$\angle APB = \angle AQB$

THEOREM 10 : (Converse of the Theorem 9)

If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment. The four point lie on a circle (i.e. they are concyclic)

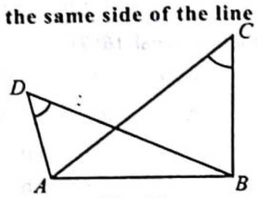


Fig. (i)

Given : A, B, C and D are four points such that point C and D lie on the same side of the line containing the line segment AB.

Also $\angle ACB = \angle ADB$. (Fig. (i))

To Prove : The four points A, B, C, D lie on a circle i.e. they are concyclic.

Construction : Draw a circle through the three non-collinear points A, B and D. Because one and only one circle passes through three non-collinear points.

PROOF : This is proved by contradiction.

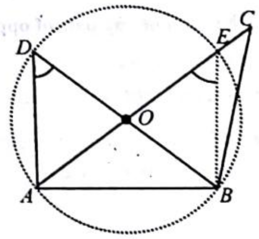


Fig. (ii)

Let the circle does not pass through the point C.

Then two cases arises:

(i) The circle intersect AC at E. see fig. (ii)

(ii) The circle intersect AC produced at E' see fig. (iii)

Case (i) When A, B, E and D lie on a circle [fig. (ii)], join EB.

$\angle ADB = \angle AEB$ ....(i) [Theorem-9 : Angle in the same segment of a circle are equal]

$\angle ACB = \angle ADB$ ...(ii) [Given]

From (i) and (ii), we get

$\angle AEB = \angle ACB$

This can be possible only when E and C coincides.

So, the circle passes through A, B, C and D.

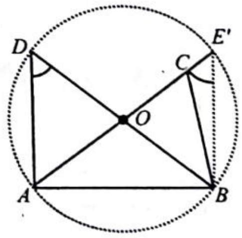


Fig. (iii)

Case (ii) When If A, B, E' and D lie on the circle [fig. (iii)], join $E'B$.

$\therefore$ $\angle ADB = \angle AE'B$ ....(iii)

[Theorem-8: Angle is the same segment of a circle are equal]

$\angle ACB = \angle ADB$ ...(iv) [Given]

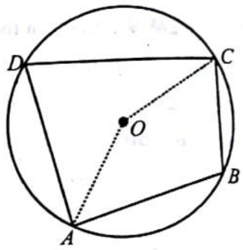
From (iii) and (iv),

$\angle ACB = \angle AE'B$

This can be possible only when E' and C coincides. So, the circle passes through A, B, C and D.

Hence, the four points A, B, C and D are concyclic.

THEOREM 11 : The sum of either pair of opposite angles of a cyclic quadrilateral is 180°.



Given : $ABCD$ is a cyclic quadrilateral whose vertices A, B, C and D lie on a circle having centre O.

To prove : (i) $\angle ABC + \angle ADC = 180°$

(ii) $\angle BAD + \angle BCD = 180°$

Construction : Join O to A and C.

PROOF : Since, the angle subtended by an arc at the centre of a circle is twice the angle subtended by the arc at any point on the remaining part of the circle.

$\therefore \quad \angle AOC = 2\angle ADC$

Similarly, Reflex $\angle AOC = 2\angle ABC$

Also, $\angle AOC +$ Reflex $\angle AOC = 360°$ [Sum of all the angles at a point is 360°]

$2\angle ADC + 2\angle ABC = 360°$

$\angle ABC + \angle ADC = \frac{360°}{2} = 180°$ ...(i)

In quadrilateral $ABCD$,

$\angle BAD + \angle ABC + \angle BCD + \angle ADC = 360°$ [Sum of all the angles of a quadrilateral is 360°]

$\Rightarrow \quad \angle BAD + \angle BCD + (\angle ABC + \angle ADC) = 360°$

$\Rightarrow \quad \angle BAD + \angle BCD + 180° = 360°$ [From equation (i), $\angle ABC + \angle ADC = 180°$]

$\Rightarrow \quad \angle BAD + \angle BCD = 180°$

THEOREM 12 : (Converse of the above Theorem 11)

If the sum of any pair of opposite angles of a quadrilateral is 180°. Then the quadrilateral is cyclic.

Given : A quadrilateral $ABCD$ in which $\angle B + \angle D = 180°$

To prove : $ABCD$ is a cyclic quadrilateral.

PROOF : This is proved by contradiction method. Let $ABCD$ be not a cyclic quadrilateral. Draw a circle passing through three non-collinear points A, B and C.

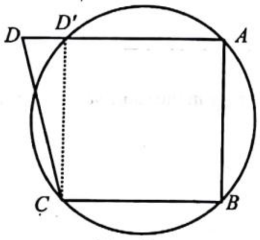


Fig. (i)

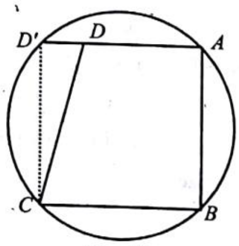


Fig. (ii)

Then two cases arises :

(i) The circle intersect AD at D', see fig (i)

(ii) The circle intersect AD produced at D', see fig. (ii)

Case-I Join CD'

$ABCD$ is a cyclic quadrilateral [fig. (i)]

$\therefore \quad \angle ABC + \angle AD'C = 180°$ ....(i)

[$\angle ABC$ and $\angle AD'C$ are opposite angles of a cyclic quadrilateral $ABCD'$]

But, $\angle B + \angle D = 180°$ [Given]

i.e., $\angle ABC + \angle AD'C = 180°$ ...(ii)

From (i) and (ii), we get

$\angle ABC + \angle AD'C = \angle ABC + \angle ADC$

$\angle AD'C = \angle ADC$

This can be possible only when D and D' coincides. So the circle passes through A, B, C and D.

Hence, $ABCD$ is a cyclic quadrilateral.

Case-II Join CD'

A, B, C and D' lie on the circle [fig (ii)]

$\therefore \quad \angle ABC + \angle AD'C = 180°$ ...(iii)

[$\angle ABC$ and $\angle AD'C$ are opposite angles of a cyclic quadrilateral $ABCD'$]

But $\angle B + \angle D = 180°$ [Given]

i.e. $\angle ABC + \angle ADC = 180°$ ...(iv)

From (iii) and (iv), we get

$\angle ABC + \angle AD'C = \angle ABC + \angle ADC$

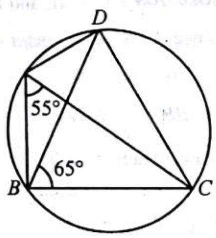
$\angle AD'C = \angle ADC$

This can be possible only when D and D' coincides. So, the circle passes through A, B, C and D.

Hence, $ABCD$ is a cyclic quadrilateral.

Illustration 4 : *ABCD* is a cyclic quadrilateral in which *AC* and *BD* are is diagonals.

If $\angle DBC = 65°$ and $\angle BAC = 55°$ Find $\angle BCD$



SOLUTION : $\angle BAC = \angle BDC = 55°$ [$\angle BAC$ and $\angle BDC$ are angles in the same segment]

In ΔBDC,

$\Delta BDC + \angle DBC + \angle BCD = 180°$

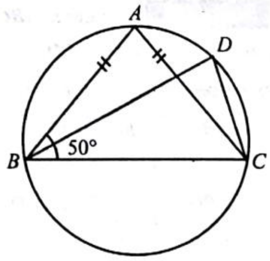
$55° + 65° + \angle BCD = 180°$

$\angle BCD = 180° - 120° = 60°$.

MISCELLANEOUS

Solved Examples

Example 1 : **In the given figure, *ABC* is an isosceles triangle in which $AB = AC$ and $\angle ABC = 50°$, find $\angle BDC$.**



SOLUTION :

$AB = AC, \therefore \angle ABC = \angle ACB = 50°$

$\therefore$ In ΔABC,

$\angle ABC + \angle ACB + \angle BAC = 180°$

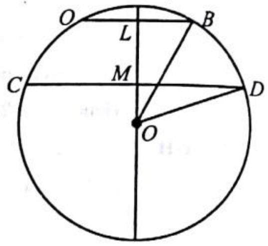
$50° + 50° + \angle BAC = 180°$

$\angle BAC = 180° - 100° = 80°$

Now, $\angle BAC = \angle BDC$ [Angle in the same segment of a circle are equal]

$\therefore \quad \angle BDC = 80°$

Example 2: **Two chords AB and CD of lengths 6 cm, 12 cm respectively of a circle are parallel. If the $\perp$ distance between AB and CD is 3 cm, find the radius of the circle.**



SOLUTION : Here $AB = 6\text{cm} \Rightarrow AL = LB = 3B$

$CD = 12$ cm $\Rightarrow CM = MD = 6$ cm

Also $LM = 3$cm. Let $OM = x$

In right triangle OLB,

$OL^2 + LB^2 = OB^2$ (By pythagoras theorem)

$\Rightarrow \quad (3+x)^2 + 3^2 = OB^2$ ...(1)

Now in right ΔOMD,

$OM^2 + MD^2 = OD^2$

$\Rightarrow \quad x^2 + 6^2 = OB^2$ .....(2) (since $OD = OB$ = radius)

Form (1) & (2), $(3+x)^2 + 3^2 = x^2 + 6^2$

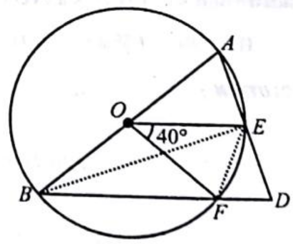
$\Rightarrow \quad 9 + 6x = 36 - 9$ or $x = 3.$

From (1), $OB^2 = (3+3)^2 + 3^2 = 36 + 9$

$OB = 3$ cm.

Hence radius = 3 cm

Example 3: **In the given figure, O is the centre of a circle and AB is a diameter. If $\angle EOF = 40°$, find $\angle EDF$.**



SOLUTION : Join BE and EF.

Since AB is the diameter of the circle

$\angle AEB = 90°$ [Angle in the semicircles is 90°]

$\angle EBF = \frac{1}{2}\angle EOF = \frac{40°}{2} = 20°$

[Angle made by a chord at any point on the circumference is half the angle made by the chord at the centre of the circle.]

Also $\angle AEB = 90°$

$\therefore \quad \angle BED = 180° - 90° = 90°$

In ΔBFD,

$\angle BED + \angle EBD + \angle EDB = 180°$

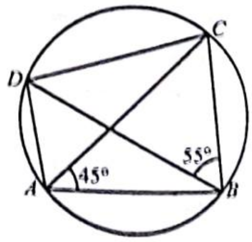
$90° + \angle EBF + \angle EDB = 180°$ [$\angle EBD = \angle EBF$]

$90° + 20° + \angle EDB = 180°$

$\angle EDB = 180° - 110° = 70°$

i.e. $\angle EDF = 90°$

Example 4: **In figure, $ABCD$ is a cyclic quadrilateral in which AC and BD are its diagonals. If $\angle DBC = 55°$ and $\angle BAC = 45°$, find $\angle BCD$.**



SOLUTION : Given $\angle DBC = 55°$ and $\angle BAC = 45°$.

$\angle BAC = \angle BDC = 45°$ ...(i) [Angle in the same segment of a circle are equal]

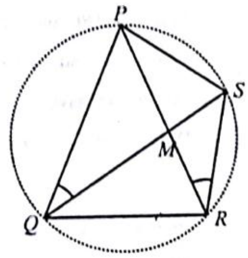
Now, in ΔBCD,

$\angle BDC + \angle DBC + \angle BCD = 180°$

$45° + 55° + \angle BCD = 180°$ [from i)]

$\angle BCD = 180° - 100° = 80°$

Example 5: **In the given figure, PQR is a triangle in which $PQ = PR$ and M is a point on PR. Through R a line is drawn to intersect QM produced at S such that $\angle PQS = \angle PRS$. Prove that $\angle PSR = 90° + \frac{1}{2}\angle QPR$.**



SOLUTION : Given $\angle PQS = \angle PRS$

Since angles $\angle PQS$ and $\angle PRS$ are equal, therefore P, Q, R, S are concyclic points.

In ΔPQR,

$\angle PQR + \angle PRQ + \angle QPR = 180°$

But $PQ = PR$ [Given]

$\therefore \angle PQR = \angle PRQ$

$\therefore \angle QPR + 2\angle PQR = 180°$

$\angle PQR = \frac{180° - \angle QPR}{2} = 90° - \frac{1}{2} < QPR$ ...(i)

Now, In ΔPSR

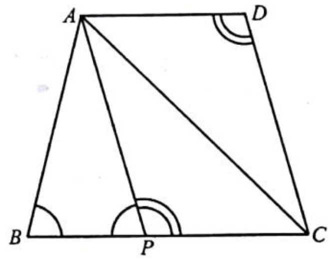
$\angle PSR + \angle PQR = 180°$ [$PQRS$ is a cyclic quadrilateral whose sum of opposite angles is 180°]

$\therefore \angle PSR = 180° - \angle PQR$

$= 180° - \left[90° - \frac{1}{2}\angle QPR\right]$ [from (i)]

$= 90° + \frac{1}{2}\angle QPR$

Example 6: **P is a point on the side BC of a triangle ABC such that $AB = AP$. Through A and C, lines are drawn parallel to BC and PA respectively, so as to intersect at D as shown in the figure. Show that $ABCD$ is a cyclic quadrilateral.**



SOLUTION : In ΔABP, we have

$AB = AP$

$\Rightarrow \angle ABP = \angle APB$

Since $AD \parallel BC$ and $AP \parallel DC$.

$APCD$ is a parallelogram.

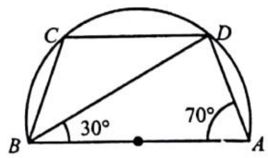
$\Rightarrow \angle APC = \angle ADC$

But $\angle APB + \angle APC = 180°$ [Linear pair axiom]

$\Rightarrow \angle ABP + \angle ADC = 180°$ [$\angle APB = \angle ABP$ and $\angle APC = \angle ADC$]

$\Rightarrow ABCD$ as a cyclic quadrilateral.

Example 7: **In the given figure AB is the diameter, $\angle BAD = 70°$ and $\angle DBC = 30°$. Find $\angle BDC$.**



SOLUTION : $ABCD$ is a cyclic quadrilateral

$\therefore \angle BAD + \angle BCD = 180°$ [Sum of opposite angles of a cyclic quadrilateral is 180°]

$\angle BCD = 180° - \angle BAD$

$= 180° - 70° = 110°$ [$\angle BAD = 70°$, Given] ...(i)

Now, In ΔBCD.

$\angle DBC + \angle BCD + \angle BDC = 180°$

$30° + 110° + \angle BDC = 180°$ [$\angle DBC = 30°$, $\angle BCD = 110°$, Given]

$\therefore \angle BDC = 180° - 140° = 40°$

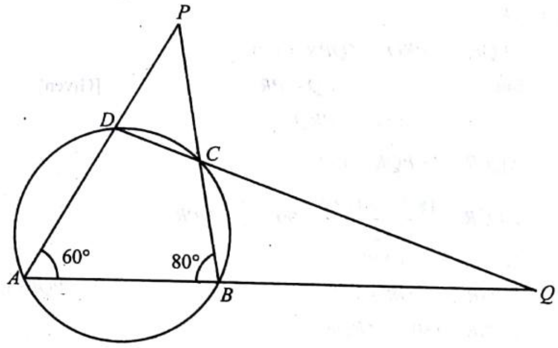
$\angle BDC = 40°$

Example 8: **In the given figure, $\angle A = 60°$ and $\angle ABC = 80°$, find $\angle DPC$ and $\angle BQC$.**

SOLUTION : In a cyclic quadrilateral, exterior angle is equal to opposite interior angle. So, in cyclic quadrilateral $ABCD$, we have

$\angle PDC = \angle ABC$ and $\angle DCP = \angle A$

$\Rightarrow$ $\angle PDC = 80°$ and $\angle DCP = 60°$ [$\because \angle ABC = 80°$ and $\angle A = 60°$]



In ΔDPC, we have

$\angle DPC = 180° - (\angle PDC + \angle DCP)$

$\Rightarrow$ $\angle DPC = 180° - (80° + 60°) = 40°$

Similarly, we have

$\angle QBC = \angle ADC$ and $\angle BCQ = \angle A$

$\Rightarrow$ $\angle QBC = 180° - \angle ABC$ and $\angle BCQ = 60°$

[$\because \angle ADC + \angle ABC = 180°$ and $\angle A = 60°$]

$\Rightarrow$ $\angle QBC = 180° - 80° = 100°$ and $\angle BCQ = 60°$

Now, in ΔBQC, we have

$\angle BQC = 180° - (\angle QBC + \angle BCQ) = 180° - (100° + 60°) = 20°$

EXERCISE 1

FIB Fill in the Blanks

DIRECTIONS: *Complete the following statements with an appropriate word/term to be filled in the blank space (s).*

1. A radius of a circle is a line segment with one end point at .... and the other end on ....
2. A diameter of a circle is a chord that passes through the .... of the circle.
3. The centre of a circle lies in the .... of the circle.
4. An arc is a .... when its ends are the ends of a diameter.
5. A point, whose distance from the centre of a circle is greater than its radius, lies on the .... of the circle.
6. The longest chord of a circle is a .... of the circle.
7. Segment of a circle is the region between an arc and the related .... of the circle.
8. Angles in the same segment of a circle are ....
9. The sum of either pair of opposite angles of a cyclic quadrilateral is ....
10. Congruent arcs of a circle subtend equal angles at the ....

T/F True/False

DIRECTIONS : *Read the following statements and write your answer as true or false.*

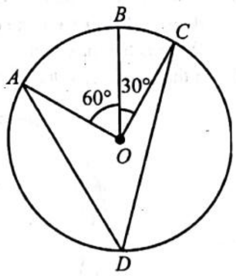
1. Diameter is the longest chord of the circle.
2. A diameter of a circle divides the circular region into two parts. Each part is called a semi-circular region.
3. Every circle has a unique centre and it lies inside the circle.
4. From a given point at the exterior of a circle, two tangents can be drawn to it and these two tangents are equal in length.
5. Line segment joining the centre to any point on the circle is a radius of the circle.
6. A circle is a plane figure.
7. If a circle is divided into three equal arcs, each is a major arc.
8. Sector is the region between the chord and its corresponding arc.
9. A circle has only a finite number of equal chords.
10. A chord of a circle which is twice as long as its radius is a diameter of the circle.

MTC Match the Columns

DIRECTIONS: *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column I have to be matched with statements (p, q, r, s, t) in column II.*

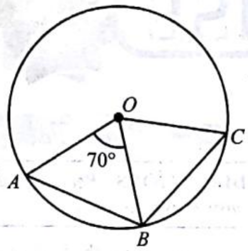
I. **Column-I** — **Column-II**

(A) In the given figure, $\angle ADC =$ — (p) $120°$

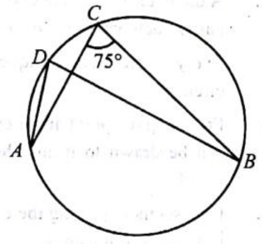


(B) Distance of a chord AB of a circle from the centre is 12 cm and length of the chord is 10cm. The diameter of the circle is .... cm. — (q) 75

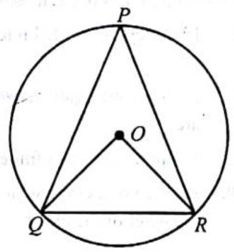
(C) In the figure given below, O is the centre of the circle. If $AB = BC$ and $\angle AOB = 70°$ then $\angle OBC$ is equal to — (r) 45°



(D) In the given figure, the points A, B, C and D lie on a circle. If $\angle ACB = 75°$ then $\angle ADB$ is equal to — (s) 55°



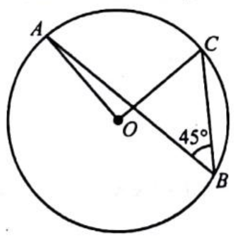
(E) If an equilateral triangle PQR is inscribed in a circle with centre O, then $\angle QOR$ is equal to — (t) 26°



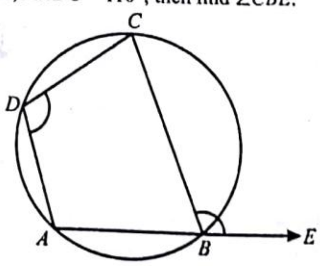
Very Short Answer Questions

DIRECTIONS: *Give answer in one word or one sentence.*

1. Define the term cyclic quadrilateral.
2. How many circles can be drawn passing through three given non-collinear points?
3. What is the sum of either pair of opposite angles of a cyclic quadrilateral?
4. In the given figure, $\angle ABC = 45°$ prove that $OA \perp OC$.



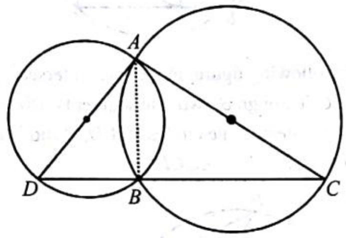
5. Define a term circle.
6. What is the diameter of the circle ?
7. Explain the word 'radius' of the circle.
8. What is the measure of angle in a semicircle?
9. How many circles can be drawn passing through (i) one point (ii) two points (iii) three collinear points.
10. Draw different pairs of circle in a plane. How many points does each pair have in common? What is the maximum number of common points?
11. In the figure, $\angle ADC = 110°$, then find $\angle CBE$.



SAQ Short Answer Questions

DIRECTIONS: *Give answer in 2-3 sentences.*

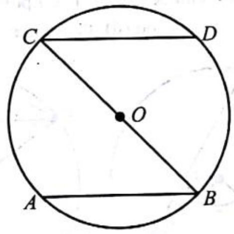
1. Two circles intersect at two points A and B. AD and AC are diameters to the two circles. Prove that B lies on the line segment DC.



2. A chord of a circle is equal to its radius. Find the angle subtended by this chord at a point in major segment.

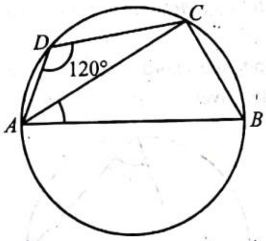
3. A quadrilateral $ABCD$ is inscribed in a circle such that AB is a diameter and $\angle ADC = 130°$. Find $\angle BAC$.

4. In the given figure chords AB and CD are parallel and BC is a diameter of the circle with centre O. Show that $AB = CD$

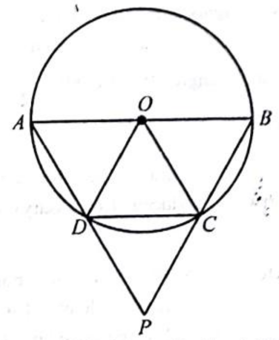


5. Prove that the perpendicular bisectors of the sides of a cyclic quadrilateral are concurrent.

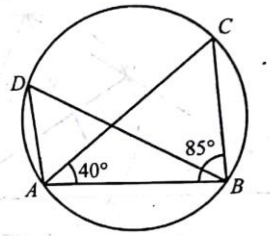
6. In the given figure, $ABCD$ is a cyclic quadrilateral whose side AB is a diameter of the circle through A, B, C, D. If $\angle ADC = 120°$, find $\angle BAC$.



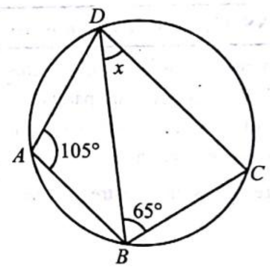
7. AB is a diameter of the circle with centre O and chord CD is equal to radius OC. AD and BC produced, which meet at P. Prove that $\angle CPD = 60°$.



8. In the given figure, points A, B, C and D lie on a circle. If $\angle CAB = 40°$ and $\angle ABC = 85°$ then find $\angle ADB$.

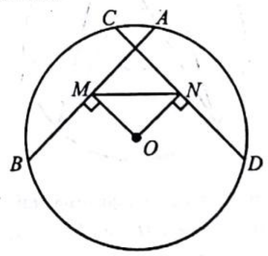


9. In the given figure, $ABCD$ is a cyclic quadrilateral. If $\angle BAD = 105°$ and $\angle CBD = 65°$ then find the value of x.



10. Suppose you are given a circle. Give a construction to find its centre.

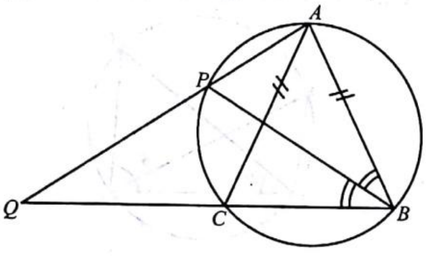
11. In figure, AB and CD are equal chords of a circle whose centre is O. If $OM \perp AB$ and $ON \perp CD$. Prove that $\angle OMN = \angle ONM$



12. Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the triangle DEF are $90° - \frac{1}{2}A$, $90° - \frac{1}{2}B$ and $90° - \frac{1}{2}C$.

13. $ABCD$ is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E. Prove that $AE = AD$.

14. The bisector of $\angle B$ of an isosceles triangle ABC with $AB = AC$ meets the circumcircle of ΔABC at P as shown in the figure. If AP and BC produced meet at Q prove that $CQ = CA$.

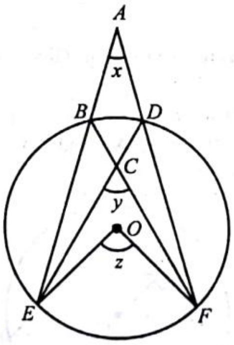


LAQ Long Answer Questions

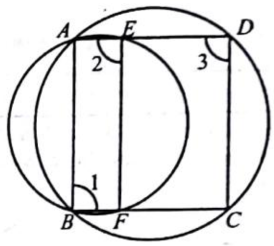
DIRECTIONS: *Give answer in four to five sentences.*

1. Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle.

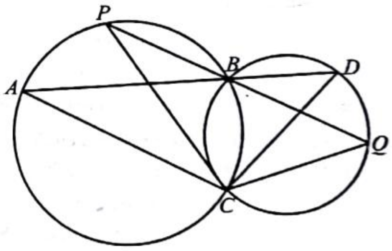
2. In figure, O is the centre of the circle. Prove that $x + y = z$.



3. In figure, $ABCD$ is a cyclic quadrilateral. A circle passing through A and B meets AD and BC at the points E and F respectively. Prove that $EF \| DC$.



4. In the following figure, two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A,D, P and Q respectively. Prove that $\angle ACP = \angle QCD$.



5. BC is a chord with centre O. A is a point on an arc BC as shown in figures (i) and (ii). Prove that

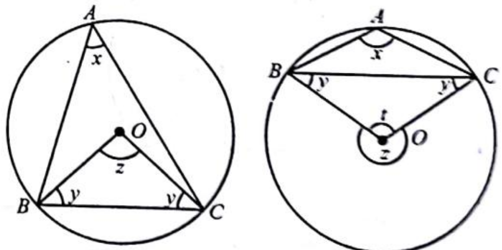
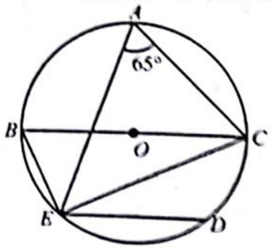


Fig. (i) Fig. (ii)

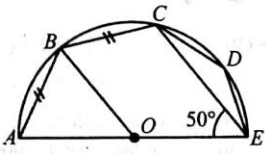
(i) $\angle BAC + \angle OBC = 90°$, if A is the point on the major arc.

(ii) $\angle BAC - \angle OBC = 90°$, if A is the point on the minor arc.

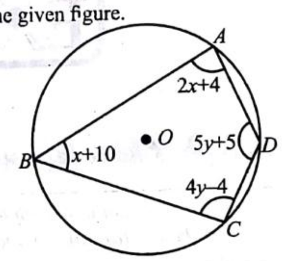
6. In the given figure, chord ED is parallel to the diameter AC of the circle. Given, $\angle CBE = 65°$, find $\angle DEC$.



7. In the given figure, O is the centre and AE is the diameter of the semicircle $ABCDE$. If $AB = BC$ and $\angle AEC = 50°$, then find $\angle CBE$, $\angle CDE$, $\angle AOB$ and prove $BO \parallel CE$.



8. The sum of either pair of opposite angles of a cyclic quadrilateral is 180°, prove it and hence find the values of x and y in the given figure.



EXERCISE 2

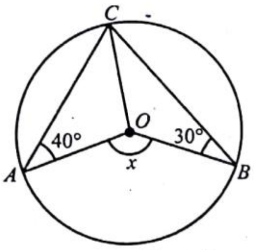
MCQ Multiple Choice Questions

DIRECTIONS: *This section contains multiple choice questions. Each questions has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.*

1. The region between a chord and either of the arcs is called
(a) an arc (b) a sector
(c) a segment (d) a semicircle

2. Diagonals of a cyclic quadrilateral are the diameters of that circle, then quadrilateral is a
(a) parallelogram (b) square
(c) rectangle (d) trapezium

3. In the given figure, O is the centre of the circle. The value of x is

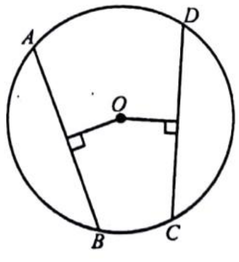


(a) 140° (b) 70°
(c) 290° (d) 210°

4. If P is a point in the interior of a circle with centre O and radius r, then
(a) $OP = r$ (b) $OP > r$
(c) $OP \geq r$ (d) $OP < r$

5. When two circles intersect at points A and B with AC and AD being the diameters of the first and second circle then the points B, C and D are
(a) concurrent (b) circumcentre
(c) orthocentre (d) collinear

6. In the adjoining figure, O is the centre of a circle. AB and CD are its two chords. If $OM \perp$ AB, ON $\perp$ CD and $OM = ON$, then

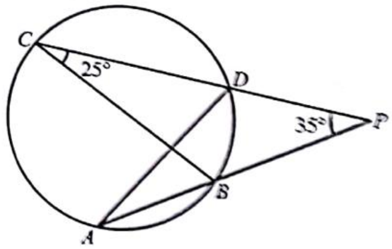


(a) $AB \neq CD$ (b) $AB < CD$
(c) $AB > CD$ (d) $AB = CD$

7. If PQ is a chord of a circle with radius r units and R is a point on the circle such that $\angle PRQ = 90°$, then the length of PQ is
(a) r units (b) $2r$ units
(c) $\frac{r}{2}$ units (d) $4r$ units

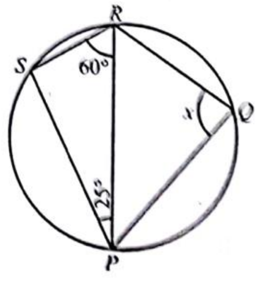
8. If an equilateral triangle PQR is inscribed in a circle with centre O, then $\angle QOR$ is equal to
(a) 60° (b) 30°
(c) 120° (d) 90°

9. In the adjoining figure, chords AB and CD of a circle when produced meet at P. If $\angle APD = 35°$ and $\angle BCD = 25°$, then $\angle ADC$ is equal to



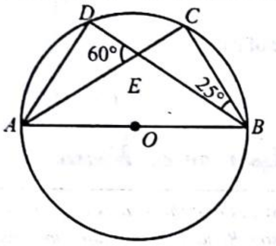
(a) 60° (b) 70°
(c) 50° (d) 120°

10. In the adjoining figure, $PQRS$ is a cyclic quadrilateral. If $\angle SPR = 25°$ and $\angle PRS = 60°$, then the value of x is



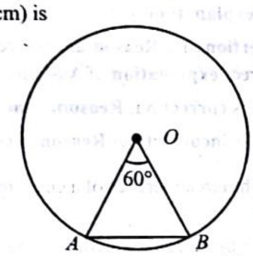
(a) 105° (b) 95°
(c) 115° (d) 85°

11. In the given figure, O is the centre of the circle, $\angle CBE = 25°$ and $\angle DEA = 60°$. The measure of $\angle ADB$ is



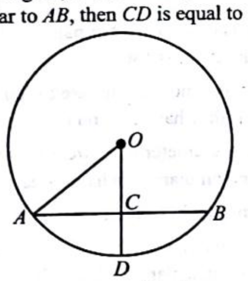
(a) 90° (b) 85°
(c) 95° (d) 120°

12. In the given figure $\angle AOB$ chord AB subtends angle equal to 60° at the centre of the circle. If $OA = 5$ cm, then length of AB (in cm) is



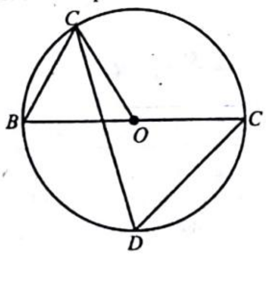
(a) $\frac{5}{2}$cm (b) $\frac{5\sqrt{3}}{2}$cm
(c) 5 cm (d) $\frac{5\sqrt{3}}{4}$cm

13. In the given figure, if $OA = 5$cm, $AB = 8$ cm and OD is perpendicular to AB, then CD is equal to



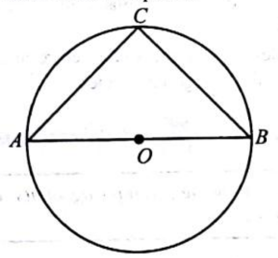
(a) 2 cm (b) 3 cm
(c) 4 cm (d) 5 cm

14. In the given figure, BC is a diameter of the circle and $\angle BAO = 60°$. Then $\angle ADC$ is equal to



(a) 30° (b) 45°
(c) 60° (d) 120°

15. In the given figure, if AOB is a diameter of the circle and $AC = BC$, then $\angle CAB$ is equal to



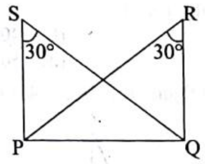
(a) 30° (b) 60°
(c) 90° (d) 45°

MTOC More than One Correct

DIRECTIONS: *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONE OR MORE may be correct.*

1. Two circles are drawn with sides PQ and PR of a triangle PQR as diameters. Circles intersect at a point S. Then
(a) $\angle PSQ$ and $\angle PSR$ form a linear pair angles.
(b) $\angle PSQ$ and $\angle PSR$ are complementary angles.
(c) $\angle PSQ$ and $\angle PSR$ are supplementary angles.
(d) Points Q, S, R are collinear points.

2. In the given figure, $\angle R = \angle S = 30°$, then the four points P, Q, R, S



(a) are concyclic (b) are collinear
(c) are not collinear (d) lie on the same circle.

3. Which of the following is/are correct?
(a) The perpendicular drawn from the centre of the circle to a chord bisects the chord.
(b) Congruent arcs of a circle subtend equal angles at the centre.
(c) Congruent arcs of a circle subtend right angles at the centre.
(d) Chords equidistant from the centre of a circle are equal in length.

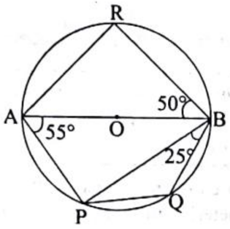
4. Which of the following is/are incorrect?
(a) A circle has only finite number of equal chords.
(b) A circle is not a plane figure.
(c) Sector is the region between the diameter and its arc.
(d) Achord of a circle, which is twice as long is its radius is a diameter of the circle.

5. Which of the following is/are incorrect ?
   (a) A continuous piece of a circle is arc of the circle.
   (b) Segment of a circle is the region between an arc and radius of the circle.
   (c) A circle divides the plane, on which it lies, in two parts.
   (d) Circles having the same centre and different radii are called congruent circles.

PBQ Passage Based Questions

DIRECTIONS: *Study the given paragraph(s) and answer the following questions.*

1. In the given figure, AB is the diameter of a circle with centre O. If $\angle PAB = 55°$, $\angle PBQ = 25°$ and $\angle ABR = 50°$ then

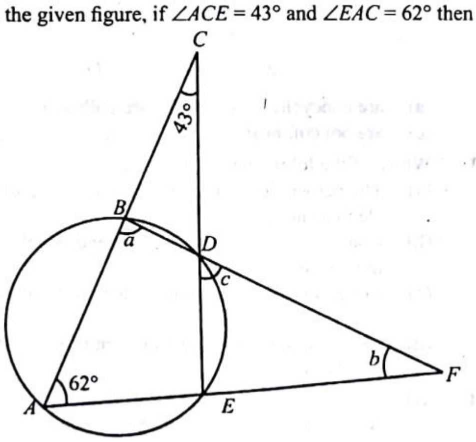


(i) The value of $\angle PBA$ is
   (a) 35° (b) 40°
   (c) 30° (d) 50°

(ii) The value of $\angle BPQ$ is
   (a) 35° (b) 40°
   (c) 30° (d) 50°

(iii) The value of $\angle BAR$ is
   (a) 35° (b) 40°
   (c) 30° (d) 50°

2. In the given figure, if $\angle ACE = 43°$ and $\angle EAC = 62°$ then



(i) The value of a is
   (a) 62 (b) 105
   (c) 13 (d) 118

(ii) The value of b is
   (a) 62 (b) 105
   (c) 13 (d) 118

(iii) The value of c is
   (a) 62 (b) 105
   (c) 13 (d) 118

A&R Assertion & Reason

DIRECTIONS: *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.
(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.
(c) If **Assertion** is **correct** but **Reason** is **incorrect**.
(d) If **Assertion** is **incorrect** but **Reason** is **correct**.

1. **Assertion:** The circumference of a circle must be a positive real number.

   Reason: If r (> 0) is the radius of the circle, then its circumference $2\pi r$ is a positive real number.

2. **Assertion:** If P and Q are any two points on a circle, then the line segment PQ is called a chord of the circle.

   Reason: Equal chords of a circle subtend equal angles at the centre.

3. **Assertion:** The sum of either pair of opposite angles of a cyclic quadrilateral is 180°

   Reason: Two or more circles are called concentric circles if and only if they have different centre and radii.

4. **Assertion:** A diameter of a circle is the longest chord of the circle and all diameters have equal length.

   Reason: length of a diameter = radius

5. **Assertion :** Given a circle of radius r and with centre O. A point P lies in a plane such that $OP > r$ then point P lies on the exterior of the circle

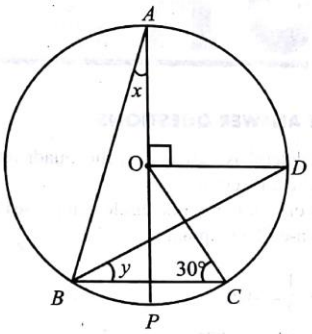
   Reason : The region between an arc and the two radii, joining the centre to the end points of the arc, is called a sector.

HOTS Hots Subjective Questions

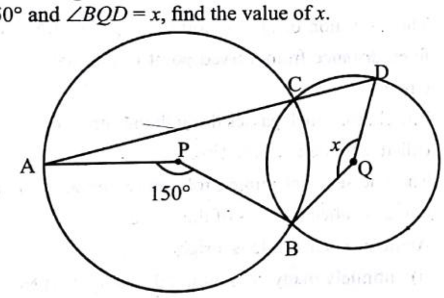
DIRECTIONS: *Answer the following questions.*

1. $ABCD$ is a parallelogram. The circle through A, B and C intersects CD produced at E. If $AB = 10$ cm, $BC = 8$ cm, $CE = 14$ cm. Find AE.

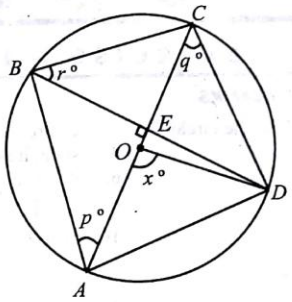
2. In figure, O is the centre of the circle, $\angle BCO = 30°$. Find x and y.



3. In the given figure, P and Q are centres of two circles, intersecting at B and C. ACD is a straight line. If $\angle APB = 150°$ and $\angle BQD = x$, find the value of x.



4. In the adjoining figure, AC is the diameter of a circle with centre O and chord $BD \perp AC$ intersecting each other at E. Find the values of p, q, r *in* terms of x.



5. $ABCD$ is a trapezium (with $AB \parallel DC$) inscribed in a circle with centre O. Diagonal AC is joined and also OA, OB, OC and OD are joined.
   (i) Is $\angle BAC = \angle DCA$? Why?
   (ii) Is $\angle DCA = \frac{1}{2}\angle DOA$? why?
   (iii) Is $\angle BOC = \angle DOA$? Why?
   (iv) Is BC = AD? Why?

6. Prove that the sum of the angles in the four segments exterior to a cyclic quadrilateral is equal to 6 right angles.

SOLUTIONS

EXERCISE – I

FILL IN THE BLANKS

1. the centre, the circle
2. centre
3. interior
4. semicircle
5. exterior
6. diameter
7. chord
8. equal
9. 180°
10. centre

TRUE/FALSE

1. True
2. True
3. True
4. True
5. True
6. True
7. False
8. False
9. False
10. True

MATCH THE COLUMNS

1. A-(r) ; B - (t) ; C - (s) ; D- (q) ; E - (p)

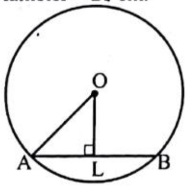
(A) $\angle ADC = \frac{1}{2}\angle AOC = \frac{1}{2}\times 90° = 45°$

($\because$ Angle subtended by an arc at the centre is double the angle formed by it on the remaining part of the circle)

(B) The perpendicular from the centre to the chord bisects the chord. AL = 5cm and OL = 12 cm,

$AO = \sqrt{(12)^2 + (5)^2} = \sqrt{169} = 13$ cm

Radius = 13cm, Diameter = 26 cm.



(C) $\angle BOC = 70°$ ($\because$ equal chords of a circle subtend equal angles at the centre).

$\Rightarrow \quad \angle OBC + \angle OCB = 180° - 70° = 110°$

$\therefore \quad \angle OBC = 55°$ ($\because OB = OC \Rightarrow \angle OBC = \angle OCB$)

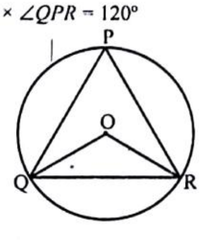
(D) Since angles in the same segment of a circle are equal

$\therefore \quad \angle ADB = \angle ACB = 75°$

(E) Since PQR is an equilateral triangle inscribed in a circle,

$\therefore \quad \angle QPR = 60°$

$\therefore \quad \angle QOR = 2 \times \angle QPR = 120°$



VERY SHORT ANSWER QUESTIONS

1. A quadrilateral is called a cyclic quadrilateral if its all vertices lie on a circle.
2. There is one and only one circle can passes through three given non-collinear points.
3. 180°
4. $\angle ABC = \frac{1}{2}\angle AOC$

   i.e., $\angle AOC = 2\angle ABC$
   $= 2 \times 45° = 90°$

   $\Rightarrow \quad OA \perp OC$
5. The collection of all the point in a plane, which are at a fixed distance from a fixed point in the plane, is called a circle.
6. The chord, which passes through the centre of the circle, is called a diameter of the circle.
7. The line segment joining the centre and any point on the circle is called a radius of the cricle.
8. Angle in a semicircle is a right angle.
9. (i) infinitely many (ii) infinitely many (iii) None
10. 0, 1, 2.

    Maximum number of common points is two.
11. $\angle CBE = \angle ADC = 110°$

SHORT ANSWER QUESTIONS

1. Join AB.

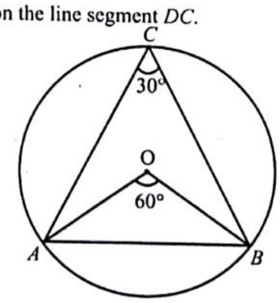
   $\angle ABD = 90°$ (Angle in a semicircle)

   $\angle ABC = 90°$ (Angle in a semicircle)

   So, $\angle ABD + \angle ABC = 90° + 90° = 180°$

   $\therefore$ DBC is a line.

   i.e., B lies on the line segment DC.

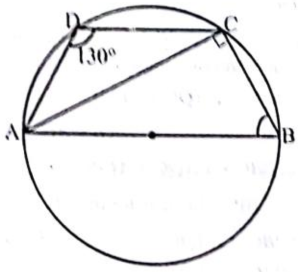


2. $AB = OA = OB$ (given)

   $\therefore \quad \angle AOB = 60°$ $\angle AOB = 2\angle ACB$

   $\therefore \quad \angle ACB = 30°$

3.



$\angle B = 180° - 130° = 50°$ ($\because$ $ABCD$ is a cyclic quadrilateral)

$\angle ACB = 90°$ (Angle in the semi-circle)

$\angle BAC = 180° - (90° + 50°) = 40$

4. From O, draw $OM \perp AB$ and $ON \perp CD$

In $\Delta\, OMB$ and ΔONC

$OB = OC$ (radii of same circle)

$\angle M = \angle N$ (each = 90°, by construction)

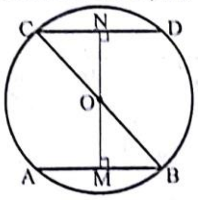
$\angle OBM = \angle OCN$

(Alternate angles)

$\therefore$ $\Delta\, OMB \cong \Delta\, ONC$ (By AAS rule)

$\therefore$ OM = ON (c.p.c.t)

$\therefore$ AB = CD ($\because$ Chords which are equidistant from the centre of a circle are equal).

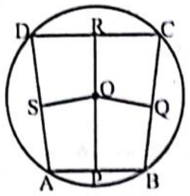


5. **Given:**

$ABCD$ is a cyclic quadrilateral of the circle with centre O.

To prove:

Perpendicular bisectors of the sides AB, BC, CD and DA are concurrent.



PROOF :

We know that the perpendicular bisector of a chord of a circle passes through the centre of the circle.

$\because$ The centre O of the circle will lie on each perpendicular bisector.

Hence, the perpendicular bisectors of the sides are concurrent and their point of concurrence is the centre of the circle.

6. ABCD is a cyclic quadrilateral,

$\therefore$ $\angle ABC + \angle ADC = 180°$

($\because$ Opposite angles of a cyclic quadrilateral are supplementary.)

$\Rightarrow$ $\angle ABC + 120° = 180°$

$\Rightarrow$ $\angle ABC = 180° - 120° = 60°$

Also, $\angle ACB = 90°$ ($\angle$ in a semi-circle)

Now, in ΔABC

$\angle BAC + \angle ABC + \angle ACB = 180°$

(By angle sum property)

$\Rightarrow$ $\angle BAC + 60° + 90° = 180°$

$\Rightarrow$ $\angle BAC = 180° - 150° = 30°$.

7. Join AD. In ΔOCD,

$OC = OD$ ... (i) (Radii of the same circle)

$OC = CD$ ... (ii) (Given)

From (i) and (ii),

$OC = OD = CD$

ΔOCD is an equilateral triangle

$\because$ $\angle COD = 60°$

$\because$ $\angle CAD = \frac{1}{2}\angle COD = \frac{1}{2}(60°) = 30°$

($\because$ Angle subtended by an arc of a circle at the centre is twice the angle subtended by it at any point of the remaining part of the circle.)

$\Rightarrow$ $\angle PAD = 30°$ ... (iii)

$\Rightarrow$ $\angle ADB = 90°$ ... (iv) (angle in a semi-circle)

$\angle ADB + \angle ADP = 180°$ (Linear Pair Axiom)

$\Rightarrow$ $90° + \angle ADP = 180°$ (From (iv))

$\Rightarrow$ $\angle ADP = 90°$ ... (v)

In ΔADP,

$\angle APD + \angle PAD + \angle ADP = 180°$

$\Rightarrow$ $\angle APD + 30° + 90° = 180°$ (From (iii) and (v))

$\Rightarrow$ $\angle APD + 120° = 180°$

$\Rightarrow$ $\angle APD = 180° - 120° = 60°$

$\Rightarrow$ $\angle CPD = 60°$

8. In $\Delta\, ABC$

$\angle ACB + 40° + 85° = 180°$

$\Rightarrow \angle ACB = 180° - 40° - 85° = 55°$

Now, $\angle ADB = \angle ACB = 55°$

($\because$ angles in the same segment of a circle)

9. $\angle BCD + \angle BAD = 180°$

(Sum of the opp. angles of a cyclic quad.)

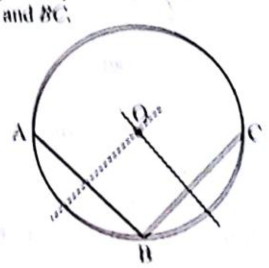
$\Rightarrow \angle BCD + 105° = 180° \Rightarrow \angle BCD = 75°$

In ΔBCD, $x + 65° + 75° = 180° \Rightarrow x = 40°$

10. Steps of construction :

(i) Take any three points A, B and C on the circle.

(ii) Join AB and BC.



(ii) Draw the perpendicular bisector of AB and BC. Let these intersect at O. Then, O is the centre of the circle.

11. Since Chord AB = Chord CD

$\therefore$ $OM = ON$ ... (i)

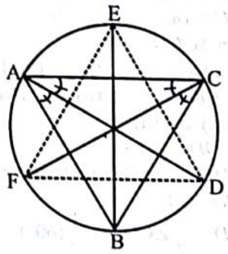
($\because$ Equal chords of a circle are equidistant from the centre of the circle)

In ΔOMN,

$OM = ON$ [From (i)]

$\therefore$ $\angle OMN = \angle ONM$ (Angles opp. to equal sides)

12.



Join DE, EF and FD. $\angle FDE = \angle FDA + \angle EDA = \angle FCA + \angle EBA$ ($\because$ Angles in the same segment are equal)

$= \frac{1}{2}\angle C + \frac{1}{2}\angle B$

$\Rightarrow$ $\angle D = \frac{\angle C + \angle B}{2} = \frac{180° - \angle A}{2}$ (By angle sum property of Δ)

$= 90° - \frac{\angle A}{2}$

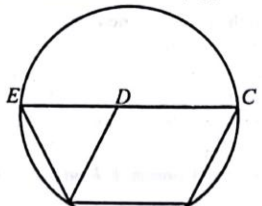
Similarly, $\angle E = 90° - \frac{\angle B}{2}$ and $\angle F = 90° - \frac{\angle C}{2}$

13. We have $\angle AED + \angle ABC = 180°$ ...(i)

($\because$ Opposite angles of a cyclic quadrilateral $ABCE$ are supplementary.)

Also, $\angle ADE + \angle ADC = 180°$ (Linear Pair Axiom)

But $\angle ADC = \angle ABC$ (Opposite angles of a $\parallel$ gm)



$\therefore$ $\angle ADE + \angle ABC = 180°$ ...(ii)

From (i) and (ii), we have

$\angle AED + \angle ABC = \angle ADE + \angle ABC$

$\Rightarrow$ $\angle AED = \angle ADE$

$\therefore$ In ΔADE, $AE = AD$

($\because$ Sides opposite to equal angles of a triangle are equal.)

14. In ΔAQC,

$\angle ACB = \angle AQC + \angle QAC$ (Exterior angle is equal to the sum of opp. interior angles.)

$\Rightarrow$ $\angle ABC = \angle AQC + \angle QAC$

($\because$ $AB = AC$, $\therefore$ $\angle ACB = \angle ABC$)

$\Rightarrow$ $2\angle PBC = \angle AQC + \angle PBC$

($\because$ BP is the bisector of $\angle B$, $\therefore$ $\angle ABC = 2\angle PBC$)

$\Rightarrow$ $\angle PBC = \angle AQC$ ($\because$ $\angle PBC = \angle PAC$)

$\Rightarrow$ $\angle PAC = \angle AQC$

$\Rightarrow$ $\angle QAC = \angle AQC$

$\Rightarrow$ $CQ = CA$

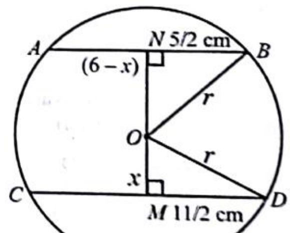
LONG ANSWER QUESTIONS

1. Let the radius of the circle be r cm. Let OM = x cm.

Then $ON = (6 - x)$ cm.

M is the mid-point of CD. ($\because$ OM $\perp$ CD)

$\therefore$ $MD = MC = \frac{1}{2}CD = \frac{1}{2}(11)$ cm $= \frac{11}{2}$ cm



Similarly, $NB = AN = \frac{1}{2}AB = \frac{1}{2}(5) = \frac{5}{2}$ cm

($\because$ N is the mid-point of AB and $ON \perp AB$)

In right ΔONB,

$OB^2 = ON^2 + NB^2$ (By Pythagoras theorem)

$\Rightarrow$ $r^2 = (6-x)^2 + \left(\frac{5}{2}\right)^2$ ... (i)

Similarly, in right ΔOMD, $OD^2 = OM^2 + MD^2$

(By Pythagoras theorem)

$\Rightarrow$ $r^2 = x^2 + \left(\frac{11}{2}\right)^2$ .. (ii)

From (i) and (ii), we get, $(6-x)^2 + \left(\frac{5}{2}\right)^2 = x^2 + \left(\frac{11}{2}\right)^2$

$\Rightarrow$ $36 - 12x + x^2 + \frac{25}{4} = x^2 + \frac{121}{4} \Rightarrow x = \frac{12}{12} = 1$

Putting x = 1 in (ii), we get ,

$r^2 = (1)^2 + \left(\frac{11}{2}\right)^2 = 1 + \frac{121}{4} = \frac{125}{4} \Rightarrow r = \frac{5\sqrt{5}}{2}$

Hence, the radius of the circle is $\frac{5\sqrt{5}}{2}$ cm.

2. Since, angle subtended by an arc of a circle at the centre is twice the angle subtended by it at any point of the remaining part of the circle therefore

$\angle EBF = \frac{1}{2}\angle EOF = \frac{1}{2}\angle z$

$\therefore \quad \angle ABF = \pi - \frac{1}{2}\angle z$ (Linear Pair Axiom) ... (i)

Similarly, $\angle EDF = \frac{1}{2}\angle EOF = \frac{1}{2}\angle z$

$\therefore \quad \angle ADE = \pi - \frac{1}{2}\angle z$ (Linear Pair Axiom) ... (ii)

$\angle BCD = \angle ECF = \angle y$ (Vert. Opp. Angles)

Now, in quadrilateral $ABCD$

$\angle ABC + \angle BCD + \angle CDA + \angle BAD = 2\pi$

(Angle sum property of a quadrilateral)

$\Rightarrow \quad \pi - \frac{1}{2}\angle z + \angle y + \pi - \frac{1}{2}\angle z + \angle x = 2\pi$

$\Rightarrow \angle x + \angle y = \angle z$

3. $\angle 1 + \angle 2 = 180°$ ... (i)

($\because$ Opposite angles of a cyclic quadrilateral $ABFE$ are supplementary.)

Similarly, $\angle 1 + \angle 3 = 180°$ ... (ii)

($\because$ $ABCD$ is a cyclic quadrilateral.)

From (i) and (ii),

$\angle 2 = \angle 3$

But these angles form a pair of equal corresponding angles.

$\therefore$ EF || DC.

4. $\angle ABP = \angle QBD$ ... (i) (vert. opp. $\angle s$)

$\angle ACP = \angle ABP$ ... (ii)

(angles in the same segment of a circle subtended by arc AP)

$\angle QCD = \angle QBD$ ... (iii)

(angles in the same segment of a circle subtended by arc QD)

From (i), (ii) and (iii), we get

$\angle ACP = \angle QCD$, as required.

5. The arc BC makes $\angle BOC = z$ at the centre and $\angle BAC = x$ at a point on circumference

$\therefore \; z = 2x$

(i) In $\Delta\, OBC$, $\angle OBC + \angle OCB + \angle BOC = 180°$

$\Rightarrow \quad z + 2y = 180°$

$\Rightarrow \quad x + y = 90°$ ($\because z = 2x$)

$\Rightarrow \quad \angle BAC + \angle OBC = 90°$

(ii) In ΔOBC, $\angle OBC + \angle OCB + \angle BOC = 180°$

$\Rightarrow \quad y + y + t = 180° \Rightarrow 180 - 2y = t$

Now, $z = 360° - t = 360° - (180 - 2y)$

$\Rightarrow \quad 2x - 2y = 180° \Rightarrow x - y = 90°$

$\Rightarrow \quad \angle BAC - \angle OBC = 90°$

6. $\angle CAE = \angle CBE$

($\because$ angles in the same segment of arc CDE)

$\Rightarrow \angle CAE = 65°$ ($\because \angle CBE = 65°$)

Since, AC is the diameter and the angle in a semi-circle is a right angle.

$\therefore \; \angle AEC = 90°$

Now, in ΔACE,

$\angle ACE + \angle AEC + \angle CAE = 180°$

$\Rightarrow \angle ACE = 180° - (90 + 65°) = 25°$

Since, AC || DE $\therefore$ $\angle DEC$ and $\angle ACE$ are alternate angles.

$\Rightarrow \angle DEC = \angle ACE = 25°$

7. Join OC

$2\angle AEC = \angle AOC \Rightarrow \angle AOC = 100°$

In $\Delta\, AOB$ and ΔBOC, $AB = BC$, $AO = OC$ and $OB = OB$

$\therefore \; \angle BOA = \angle BOC$

$\Rightarrow \angle BOA = 50°$ and $\angle BOC = 50°$

$\Rightarrow \angle BOA = \angle CEO$ (each 50°)

$\therefore BO \parallel CE$

Now, $\angle AOC + \angle COE = 180°$

$\therefore \quad \angle COE = 80°$ ($\because \angle AOC = 100°$)

Also, $\angle COE = 2\angle CBE$

$\Rightarrow \quad \angle CBE = 40°$,

8. II part :

$\angle A + \angle C = 180° \Rightarrow 2x + 4 + 4y - 4 = 180$

$\Rightarrow x + 2y = 90°$ ... (1)

$\angle B + \angle D = 180° \Rightarrow x + 10 + 5y + 5 = 180°$

$\Rightarrow x + 5y = 165°$ ... (2)

On subtracting (1) from (2), we get

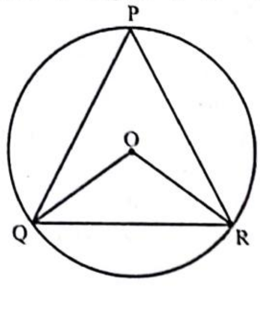
$3y = 75 \Rightarrow y = 25°$

Also, $x = 40°$

EXERCISE – 2

MULTIPLE CHOICE QUESTIONS

1. (c) a segment.
2. (c) rectangle
3. (a) $x = 140°$
4. (d) $OP < r$
5. (d) B, C and D are collinear
6. (d) Chords of a circle which are equidistant from the centre of the circle are equal.
7. (b) Since PQ is a chord of a circle and R is a point on the circle such that $\angle PRQ = 90°$, therefore, the arc PRQ is a semicircle $\Rightarrow PQ$ is a diameter.
   $\therefore$ Length of $PQ = 2 \times$ radius $= 2r$ units
8. (c) As PQR is an equilateral triangle inscribed in a circle,
   $\angle QPR = 60°$,
   $\therefore \angle QOR = 2 \times \angle QPR = 2 \times 60° = 120°$



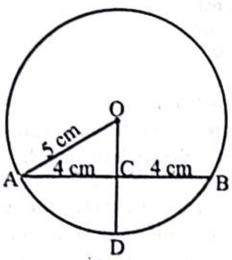
9. (a) $\angle BAD = \angle BCD$
(angles in the same segment of a circle)
$\Rightarrow$ $\angle PAD = \angle BAD = 25°$ ($\because$ $\angle BCD = 25°$, given)
Now, $\angle ADC = \angle PAD + \angle APD$ (ext. $\angle$ of a Δ = sum of two int. opp. $\angle$s)
$\Rightarrow$ $\angle ADC = 25° + 35° = 60°$.

10. (d) In ΔPRS, $\angle PSR + 25° + 60° = 180°$
$\Rightarrow$ $\angle PSR = 95°$
Now $\angle PQR + \angle PSR = 180°$
(sum of opp. $\angle$s of a cyclic quad. is 180°)
$\Rightarrow$ $x + 95° = 180° \Rightarrow x = 85°$.

11. (c) $\angle DEA = \angle BEC$ (Vertic. opp. angles)
$\Rightarrow$ $60° = \angle BEC$
Now, in ΔBEC,
$\angle E + \angle B + \angle C = 180°$
$\Rightarrow$ $60 + 25 + \angle C = 180°$
$\Rightarrow$ $\angle C = 95°$
Also, $\angle C = \angle D$
$\Rightarrow$ $\angle D = 95°$

12. (c) $OA = OB$ (radius of circle)
$\Rightarrow$ $5 = OB$
Thus, $\angle A = \angle B$ ($\because$ $OA = OB$)
Thus, in ΔOAB
$\angle A + \angle B + \angle O = 180° \Rightarrow 2\angle A + 60° = 180°$
$\Rightarrow$ $\angle A = 60°$
Hence, $\angle A = \angle B = \angle O = 60° \Rightarrow \Delta OAB$ is an equilateral triangle.

13.



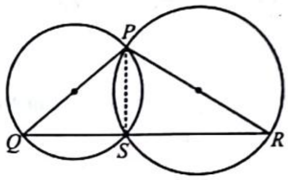
OA = 5 cm, AB = 8 cm
$OC = \sqrt{5^2 - 4^2}$
$OC = \sqrt{25-16} = \sqrt{9} = 3\text{cm}$
CD = OD – OC
CD = 5 – OC
CD = 5 – 3 = 2 cm

14. (c) $\angle BAO = 60°$ (given)
$OA = OB$ (Radius of the circle)
$\therefore$ $\angle BAO = \angle ABO$
$\angle ABO = \angle ADC = 60°$
(Angles in the same segment)

15. (d) 45°

MORE THAN ONE CORRECT

1. (a, c, d)



Since, PQ and PR are diameters of circle
$\therefore$ $\angle PSQ = 90°$ and $\angle PSR = 90°$
$\Rightarrow \angle PSQ + \angle PSR = 180°$
$\Rightarrow \angle PSQ$ and $\angle PSR$ form a linear pair angles.
$\Rightarrow Q, S, R$ lie on a line
$\Rightarrow S$ lies on QR.

2. (a, d)
3. (a, b,d)
4. (a,b,c)
5. (b,c,d)

PASSAGE BASED QUESTIONS

1. $\angle ARB = 90°$ (Angle in semi-circle is right angle)
Since, $ARBP$ is a cyclic quadrilateral
$\therefore$ $\angle ARB + \angle APB = 180°$
$\Rightarrow$ $\angle APB = 180° - 90° = 90°$
Now, in ΔAPB,
$\angle PAB + \angle ABP + \angle BPA = 180°$
$\Rightarrow 55 + \angle ABP + 90° = 180°$
$\Rightarrow \angle ABP = 180 - 145 = 35°$ ... (1)
Since, $ABQP$ is a cyclic quadrilateral
$\therefore$ $\angle PAB + \angle PQB = 180°$
$\Rightarrow \angle PQB = 180 - 55 = 125°$
Now, In ΔPQB,
$\angle BPQ + \angle PQB + \angle QBP = 180°$
$\Rightarrow \angle BPQ + 125° + 25° = 180°$
$\Rightarrow \angle BPQ = 180° - 150° = 30°$ ... (2)
In ΔBRA,
$\angle BRA + \angle RAB + \angle ABR = 180°$
$\Rightarrow 90° + \angle RAB + 50° = 180°$
$\Rightarrow \angle RAB = 180° - 140° = 40°$ ... (3)
From (1), (2) and (3)
(i) a (ii) c (iii) b

2. In ΔCAE, $\angle C + \angle A + \angle AEC = 180°$
$43° + 62° + \angle AEC = 180°$
$\Rightarrow \angle AEC = 75°$
Since, $ABDE$ is a cyclic quadrilateral.
$\therefore$ $a + \angle AEC = 180°$
$\Rightarrow a = 180° - 75° = 105°$ ... (1)
Now, $\angle A + \angle EDB = 180°$
$\Rightarrow$ $\angle EDB = 180° - 62° = 118°$
Now, c + $\angle EDB = 180°$ (Linear pair)
$\therefore$ c = 180° – 118° = 62° ... (2)

Also, $\angle AED + \angle DEF = 180°$ (linear pair)

$\Rightarrow \angle DEF = 180° - 75° = 105°$

In ΔDEF

$b + c + \angle DEF = 180°$

$b = 180° - (62° + 105°) = 13°$ ... (3)

So, from (1), (2) and (3)

(i) b (ii) c (iii) a

ASSERTION & REASON

1. (a) Both Assertion and Reason are correct and Reason is correct explanation for Assertion.
2. (b) Both Assertion and Reason are correct but reason is not correct explanation for Assertion.
3. (c) Assertion is correct. But Reason is not correct. Two or more circles are called concentric circles if and only if they have same centre but different radii.
4. (c) Assertion is correct but Reason is false.
5. (b) Both Assertion and Reason is correct.

HOTS SUBJECTIVE QUESTIONS

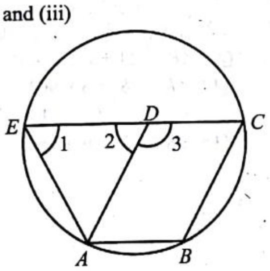
1. $\angle 4 = \angle 3$ [Opposite angles of a || gm] ... (i)

$\angle 4 + \angle 1 = 180°$ [Cyclic quadrilateral]

$\therefore \angle 3 + \angle 1 = 180°$ [Using (i)] ... (ii)

Also $\angle 2 + \angle 3 = 180°$ [Linear pair angles] ... (iii)

From eq. (ii) and (iii)



$\angle 3 + \angle 1 = \angle 2 + \angle 3$

$\Rightarrow \angle 1 = \angle 2$

$\Rightarrow AE = AD$

Also $AD = BC$

$BC = 8$ cm

$\therefore AE = 8$ cm

2. In ΔOCP,

$\angle OPC = 90°$

$\therefore$ AP $\perp$ BC

$\angle OCP = 30°$

$\therefore \angle POC = 90° - 30° = 60°$

DO $\perp$ AP,

$\therefore \angle DOP = 90°$

$\therefore \angle COD = 90° - 60° = 30°$

$\angle CBD = \frac{1}{2}\angle COD$ [Angle subtended theorem]

$\therefore y = \frac{1}{2} \times 30° = 15°$

$y = 15°$ $\angle AOD = 90°$

$\therefore \angle ABD = \frac{1}{2}\angle AOD$ [Angle subtended theorem]

$\angle ABD = \frac{1}{2} \times 90° = 45°$

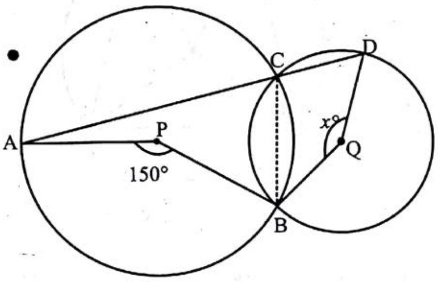
In ΔABP,

$x + 45° + y + 90° = 180°$

$x + 45° + 15° + 90 = 180°$

$x = 180° - 150° = 30°$

3. Join BC. We know that the angle made by an arc at the centre is twice the angle made by this arc at a point on the remaining part of the circle.



$\therefore \angle ACB = \frac{1}{2}\angle APB = \frac{1}{2} \times 150° = 75°$

Now, ACD being a straight line, we get

$\angle ACB + \angle BCD = 180°$ [Linear pair]

$\angle BCD = 180° - \angle ACB = 180° - 75° = 105°$

Similarly, for second circle.

$\angle BCD = \frac{1}{2}$ reflex $\angle BQD$

$\Rightarrow 105° = \frac{1}{2}(360° - x) \Rightarrow x = 150°$

Hence, $x = 150°$

4. Arc AD subtends $\angle AOD$ at the centre and $\angle ABD$ at a point B on the remaining part of the circle.

$\therefore \angle ABD = \frac{1}{2}\angle AOD = \frac{x°}{2}$

Now, $\angle AEB = 90°$ [$\therefore BD \perp AC$]

From ΔAEB, $p° = 180° - (\angle ABE + \angle AEB)$

$\therefore p° = 180° - (\angle ABD + \angle AEB)$

$= 180° - \left(\frac{x^0}{2} + 90°\right) = 90° - \frac{x°}{2}$

Again, arc AD subtends $\angle AOD$ at the centre and $\angle ACD$ at a point C of the circle.

$\therefore \angle ACD = \frac{1}{2}\angle AOD = \frac{x^\circ}{2} \Rightarrow q = \frac{x^\circ}{2}$

Now $\angle ABC = 90^\circ$ [Angle in a semicircle]

$\Rightarrow \angle ABE + \angle CBE = 90^\circ$

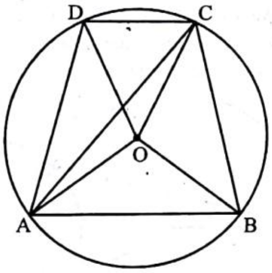
$\Rightarrow \angle ABD + \angle CBE = 90^\circ$

$\Rightarrow \frac{x^\circ}{2} + \angle CBE = 90^\circ \Rightarrow \angle CBE = 90^\circ - \frac{x^\circ}{2}$

$\Rightarrow r^\circ = 90^\circ - \frac{x^\circ}{2}$

Hence, $p^\circ = q^\circ = \frac{x^\circ}{2}$ and $r^\circ = 90^\circ - \frac{x^\circ}{2}$

5. Given ABCD is a trapezium in which AB || DC.



(i) $\angle BAC = \angle DCA$, because it is a pair of alternate angles.

(ii) $\angle DCA = \frac{1}{2}\angle DOA$ ($\because$ angle subtended by an arc at the centre is double the angle subtended by it on the remaining part of the cricle)

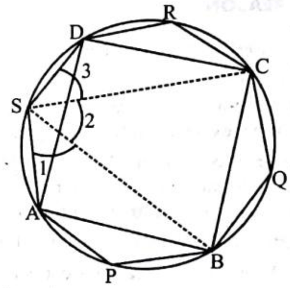
(iii) $\angle BOC = 2\angle BAC = 2\angle DCA = \angle DOA$.

(iv) $BC = AD$ since equal angles are subtended by equal chords.

6. **Given:** A cyclic quadrilateral $ABCD$ and angles $\angle P$, $\angle Q$, $\angle R$ and $\angle S$ are in the four external segments.

To Prove: $\angle P + \angle Q + \angle R + \angle S = 6$ right angles.

Construction: Join SB and SC.



PROOF : Since $APBS$ is a cyclic quadrilateral and sum of opposite pairs of angles in a cyclic quadrilateral is 180°

$\therefore \quad \angle 1 + \angle P = 180^\circ \quad \ldots$ (i)

Similarly, BQCS and CRDS are cyclic quadrilaterals

$\therefore \quad \angle 2 + \angle Q = 180^\circ \quad \ldots$ (ii)

and $\angle 3 + \angle R = 180^\circ \quad \ldots$ (iii)

Adding (i), (ii) and (iii), we get

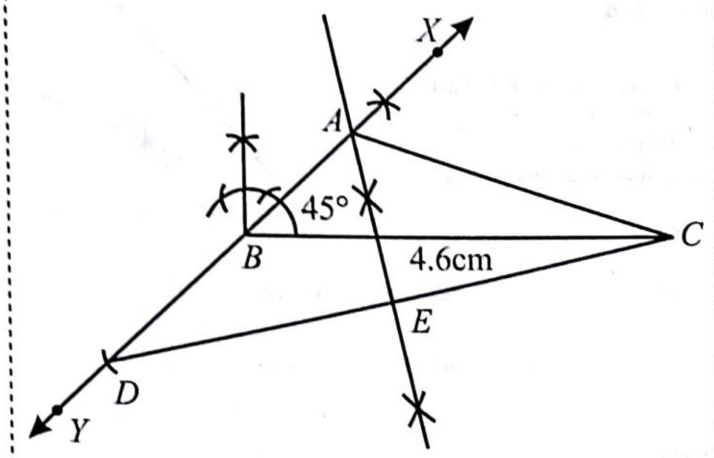
$\angle 1 + \angle P + \angle 2 + \angle Q + \angle 3 + \angle R = 180^\circ + 180^\circ + 180^\circ$

$\Rightarrow \angle P + \angle Q + \angle R + (\angle 1 + \angle 2 + \angle 3) = 3 \times 180^\circ$

$\Rightarrow \angle P + \angle Q + \angle R + \angle S = 6 \times 90^\circ$

$[\therefore \angle 1 + \angle 2 + \angle 3 = \angle S]$

$\Rightarrow \angle P + \angle Q + \angle R + \angle S = 6$ right angles.



8

CHAPTER

Constructions

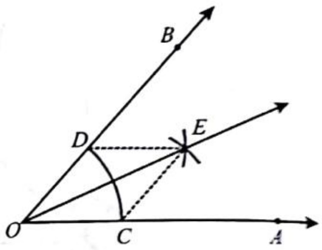
INTRODUCTION

The diagrams, which were necessary to prove theorems or solving the problems were not necessarily very accurate. They were drawn only to give you a feeling for the situation and as an add for proper reasoning. But many times, we need to draw accurate figures. For examples to draw road map of a city, to draw layout plan of a building to draw a design of a vehicle etc. To draw such accurate figures, we must have some basic knowledge of geometrical constructions, which is a process of drawing a geometrical figure using some geometrical instruments like graduated ruler, protector, compass, a pair of set-squares.

In earlier classes, you have learned some simple geometrical constructions like drawing a line segment of a given measurement, making an angle with the help of protector etc. In this chapter, you will learn some more basic geometrical constructions using ruler and compass with reasoning behind it, why these constructions are valid.

TO CONSTRUCT THE BISECTOR OF AN ANGLE

Given an $\angle AOB$. We want to construct bisector of $\angle AOB$.



STEPS OF CONSTRUCTION

(i) Taking O as centre and any suitable radius draw an arc as shown in the figure, which intersect the ray OA and OB at C and D respectively

(ii) Taking C as centre and any suitable radius ($> \frac{1}{2}CD$), draw an arc.

(iii) Taking D as centre and same radius as in step-(ii), draw an arc intersecting the arc drawn in step (ii) at point E.

(iv) Draw ray OE, which is required bisector of $\angle AOB$.

Justification

Join CE and DE

In ΔOCE and ΔODE,

$OC = OD$ (radii of the same arc)

$CE = DE$ (arcs of equal radii)

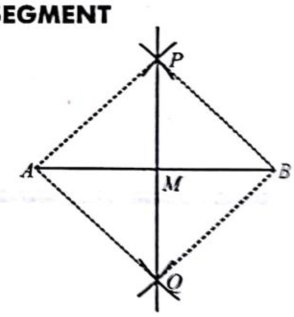
$OE = OE$ (common)

$\therefore \Delta OCE \cong \Delta ODE$ (by SSS rule of congruency)

$\therefore \angle COE = \angle DOE$ (CPCT)

TO CONSTRUCT THE PERPENDICULAR BISECTOR OF A GIVEN LINE SEGMENT

Given a line segment AB. We want to construct perpendicular bisector of AB.



STEPS OF CONSTRUCTION

(i) Taking A as centre and any suitable radius ($> \frac{1}{2} AB$), draw two arcs, one on each side of AB.

(ii) Taking B as centre and same radius as in step (i), draw two more arcs, one on each side of AB intersecting the previous arcs at P and Q.

(iii) Join PQ which intersects AB at M. Then the line PQ is the required perpendicular bisector of line segment AB.

Justification :

Join AP, BP, AQ and BQ.

In ΔAPQ and ΔBPQ

$AP = BP$ (arcs of equal radii)

$AQ = BQ$ (arcs of equal radii)

$PQ = PQ$ (common)

$\therefore \Delta APQ \cong \Delta BPQ$ (by SSS rule of congruency)

$\therefore \angle APM = \angle BPM$ (C.P.C.T.)

In ΔAMP and ΔBMP

$AP = BP$ (arcs of equal radii)

$MP = MP$ (common)

$\angle APM = \angle BPM$ (proved above)

$\therefore \Delta AMP \cong \Delta BMP$ (by SAS rule of congruency)

$\therefore AM = BM$ (C.P.C.T.)

And $\angle AMP = \angle BMP$ (C.P.C.T.)

Now $\angle AMP + \angle BMP = 180°$ (linear pair)

$2\angle AMP = 180°$

$\Rightarrow \angle AMP = 90°$

$\Rightarrow MP \perp AM \Rightarrow PQ \perp AB$.

Therefore, the line PQ is the perpendicular bisector of the line segment AB.

TO CONSTRUCT AN ANGLE OF 60° AT THE INITIAL POINT OF A GIVEN RAY

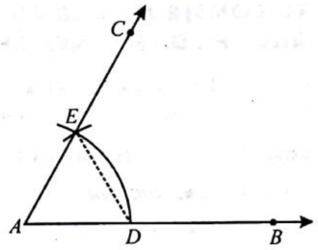
Given a ray AB, we want to construct another ray AC such that $\angle CAB = 60°$.

STEPS OF CONSTRUCTION

(i) Taking A as centre and any suitable radius, draw an arc to intersect the ray AB at D.

(ii) Taking D as centre and same radius as in step (i), draw another arc to intersect the previous arc at E.

(iii) Draw ray AC passi ng through E. Then $\angle CAB = 60°$.



Justification :

Join DE.

Then $AD = AE = DE$

$\therefore$ ΔADE is equilateral $\Rightarrow \angle EAD = 60°$

$\therefore$ $\angle CAB = 60°$

TO CONSTRUCT A TRIANGLE GIVEN ITS BASE, A BASE ANGLE AND SUM OF OTHER TWO SIDES

Given the base BC, a base angle, say $\angle B$ and the sum $(AB + AC)$ we want to construct ΔABC.

STEPS OF CONSTRUCTION :

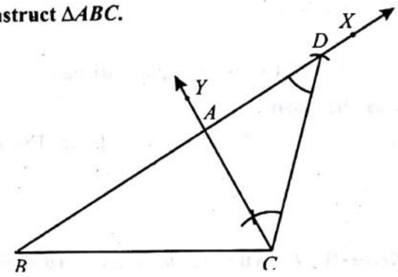
(i) Draw the base BC and draw a ray BX such that $\angle CBX$ equal to given angle.

(ii) Taking B as centre and radius equal to $(AB + AC)$, draw an arc intersecting the ray BX at D.

(iii) Join CD.

(iv) Draw ray CY, which intersects ray AX at A such that $\angle ACD = \angle ADC$

Then ΔABC is the required triangle



Justification :

Base BC and $\angle B$ are drawn as given

Now in ΔACD,

$\angle ACD = \angle ADC$ (By construction)

$\therefore$ $AC = AD$ (Sides opposite to equal angles in a triangle are equal)

Now $AB = BD - AD$

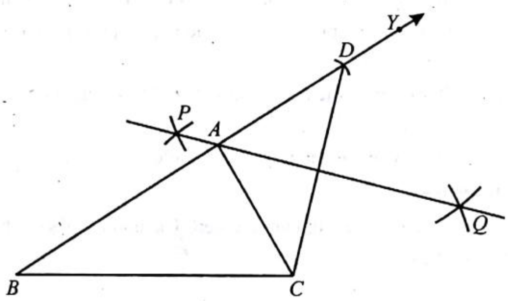
$= BD - AC$

$\Rightarrow$ $AB + AC = BD$

Alternative Method of Construction

(i), (ii), (iii) Follow the first three steps of construction given above. Then draw perpendicular bisector PQ of CD to intersect BD at a point A.

(iv) Join AC. Then ΔABC is the required triangle.



Justification :

Since A lies on the perpendicular bisector of CD,

Therefore $AC = AD$.

$\Rightarrow$ $AB + AC = AD$

> **Note :** The construction of any triangle is not possible if sum of its any two sides is equal or less than the third remaining side.

Illustration 1 : Construct ΔABC such that $BC = 3.5$ cm, $AB + AC = 5.2$ cm and $\angle B = 60°$.

STEPS OF CONSTRUCTION :

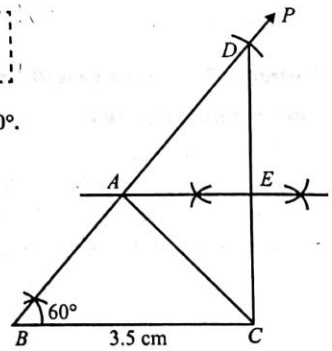
(i) Draw $BC = 3.5$cm

(ii) At B, construct $\angle CBP = 60°$

(iii) From ray BP, cut off $BD = 5.2$cm

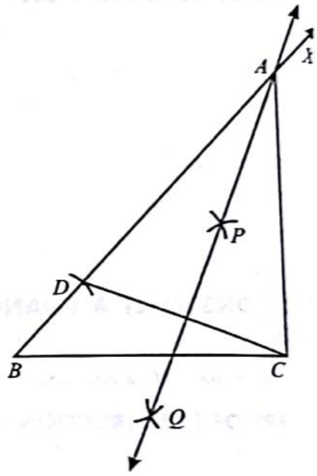
(iv) Join CD and draw its perpendicular bisector to intersect BD at A.

(v) Join AC. Then, ABC is the required triangle.



TO CONSTRUCT A TRIANGLE GIVEN ITS BASE, A BASE ANGLE AND THE DIFFERENCE OF THE OTHER TWO SIDES

Given the base BC, a base angle, say $\angle B$ and the difference of other two sides $AB - AC$ or $AC - AB$. We want to construct the ΔABC.



Case-I : *Let $AB > AC$ that is $AB - AC$ is given.*

STEPS OF CONSTRUCTION :

(i) Draw the base BC and a ray BX such that $\angle CBX$ equal to the given angle.

(ii) Taking B as centre and radius equal to $(AB - AC)$, draw an arc intersecting the ray BX at point D.

(iii) Join DC

(iv) Draw the perpendicular bisector PQ of line segment CD, which intersects the ray BX at A.

(v) Join AC.

Then ΔABC is the required triangle.

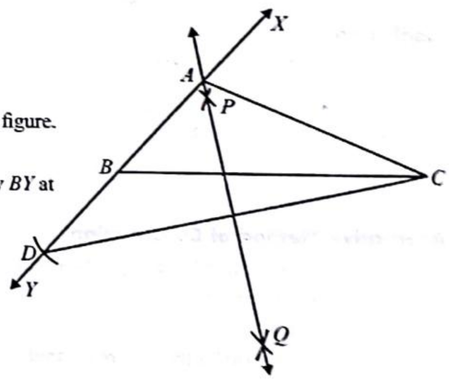
Justification :

Base BC and $\angle CBX$ are drawn as given. The point A lies on the perpendicular bisector of DC.

$\therefore \quad AC = AD$

Now $BD = AB - AD$

$\Rightarrow \quad BD = AB - AC \qquad [\because AC = AD]$



Case-II : *Let AB $<$ AC that is $AC - AB$ is given.*

STEPS OF CONSTRUCTION :

(i) Draw base BC and a line XY passing through the point B as shown in the figure. such that $\angle CBX$ is equal to the given angle.

(ii) Taking B as centre and radius $(AC - AB)$, draw an arc intersecting the ray BY at point D (on opposite side of point X with respect to BC).

(iii) Join DC.

(iv) Draw perpendicular bisector PQ of CD, which intersects BX at A.

(v) Join AC.

Then ΔABC is the required triangle.

Justification :

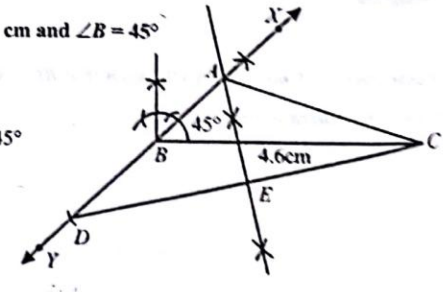
Base BC and $\angle B$ arc drawn as given. The point A lies on the perpendicular bisector of CD.

$\therefore \quad AC = AD$

Now $BD = AD - AB$

$\Rightarrow \quad BD = AC - AB$

Illustration 2 : Construct a ΔABC, given that $BC = 4.6$ cm, $AC - AB = 2.2$ cm and $\angle B = 45°$



STEPS OF CONSTRUCTION :

(i) Draw $BC = 4.6$ cm.

(ii) Draw ray XY, passing through the point B as shown, such that $\angle CBX = 45°$

(iii) With B as centre and radius 2.2 cm draw an arc to intersect ray BY at D.

(iv) Join DC and draw its perpendicular bisector to intersect ray BX at A.

(v) Join CA. Then, ABC is the required triangle.

TO CONSTRUCT A TRIANGLE, GIVEN ITS PERIMETER AND ITS TWO BASE ANGLES

Given the base angles, say $\angle B$ & $\angle C$ and $AB + BC + CA$, we want to construct the triangle ABC.

STEPS OF CONSTRUCTION

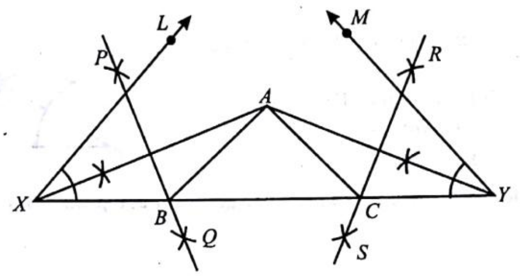
(i) Draw a line segment, say XY equal to $AB + BC + CA$.

(ii) Draw ray XL and YM such that $\angle LXY$ equal to $\angle B$ and $\angle MYX$ equal to $\angle C$.

(iii) Bisect $\angle LXY$ and $\angle MYX$. Let these bisectors intersect at a point A.

(iv) Draw perpendicular bisectors PQ of AX and RS of AY.

(v) Let PQ intersect XY at B and RS intersect XY at C. Join AB and AC.



Then ABC is the required triangle.

Justification :

B lies on the perpendicular bisector PQ of AX.

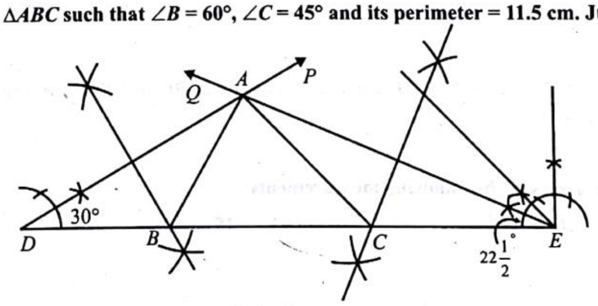
Therefore, $XB = AB$ and similarly, $CY = AC$.

This gives $BC + CA + AB = BC + XB + CY = XY$.

Again $\angle BAX = \angle AXB$ (As in ΔAXB, $AB = XB$) and $\angle ABC = \angle BAX + \angle AXB = 2\angle AXB = \angle LXY$

Similarly, $\angle ACB = \angle MYX$ as required.

Illustration 3 : Construct a ΔABC such that $\angle B = 60°$, $\angle C = 45°$ and its perimeter = 11.5 cm. Justify your construction.



STEPS OF CONSTRUCTION :

(i) Draw $DE = 11.5$ cm

(ii) At D, construct $\angle EDP = \frac{1}{2}$ of $60° = 30°$ and at E, construct $\angle DEQ = \frac{1}{2}$ of $45° = 22\frac{1}{2}°$.

(iii) Let rays DP and EQ meet at A.

(iv) Draw perpendicular bisector of AD to meet DE at B.

(v) Draw perpendicular bisector of AE to meet DE at C.

(vi) Join AB and AC. Then, ABC is the required triangle.

MISCELLANEOUS

Solved Examples

Example 1 : **Construct a triangle ABC, in which $BC = 3.5$ cm, $\angle B = 30°$ and $AB + AC = 6.4$ cm**

STEP OF CONSTRUCTION :

(i) Draw $BC = 3.5$ cm.

(ii) Draw $\angle CBX = 30°$

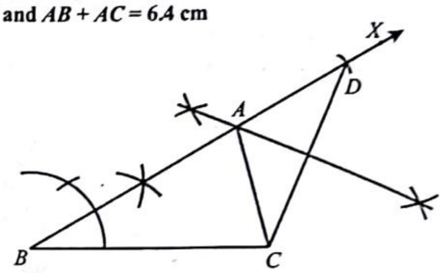
(iii) From ray BX, cut-off line segment BD equal to $AB + AC$ i.e. 6.4 cm.

(iv) Join CD.

(v) Draw the perpendicular bisector of CD meeting BD at A.

(vi) Join CA to obtain the required triangle ABC.

Hence, ΔABC is the required triangle.



Example 2 : **Construct an angle of 90° at the initial point of a given ray and justify your construction.**

STEPS OF CONSTRUCTION :

(i) Draw a ray AB.

(ii) Taking A as centre and any convenient radius, draw an arc as shown intersecting AB at C.

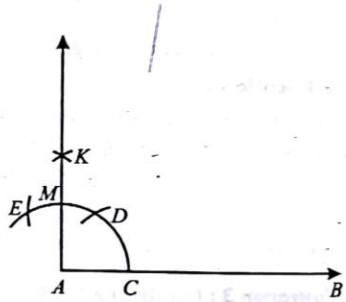
(iii) Taking C as centre and radius = AC, draw an arc intersecting the arc draw on step (ii) at D.

(iv) Again taking D as centre and same radius = AC, draw an arc intersecting the first arc at E.

(v) Again taking D and E as centres and radius > ½ DE, draw two arcs intersecting each other at K.

(vi) Join AK.

(vii) $\angle BAK = 90°$



Justification :

Clearly arc CD makes an angle of 60° at A and arc DE also makes an angle of 60° at A. Since M is the mid-point of ED.

$\therefore$ MD makes an angle of 30° at A.

Hence, $\angle BAK = 90°$.

Example 3 : **Construct the angles of the following measurements:**

(a) 30° **(b)** 22 ½° **(c)** 15°

STEPS OF CONSTRUCTION :

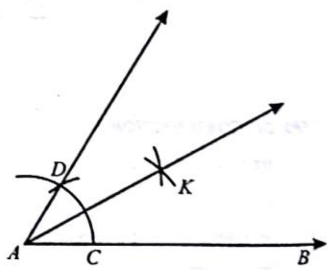
(a) (i) Draw A ray AB of any suitable length.

(ii) Taking A as centre and any suitable radius, draw an arc which intersects ray AB at C.

(iii) Taking C as centre and radius = AC, draw an arc which intersects the previous arc at D.

(iv) Taking C and D as centre and radius > ½ CD draw two arcs intersecting each other at K.

Now $\angle BAK = 30°$



(b) (i) Draw a ray AB.

(ii) Taking A as centre and with any suitable radius draw an arc intersecting AB at C.

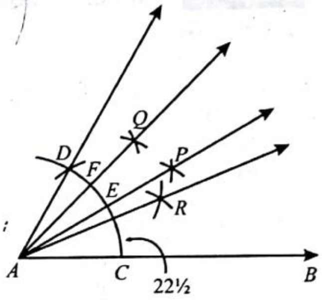
(iii) Taking C as centre and radius = AC, draw an arc such that it intersects the previous arc at D.

Now $\angle BAD = 60°$

(iv) Taking C and D as centres and radius > ½ CD draw two arcs intersecting each other at P. Join AP, which intersects the arc CD at E.
Now $\angle BAP = 30°$

(v) Taking D and E as centres and radius > ½ ED draw two arcs intersecting each other at Q. Join AQ, which intersects arc CD at F.

(vi) Taking C and F as centres and radius > ½ CF draw two arcs intersecting each other at R. Join AR.
Now $\angle BAR = 22½°$.



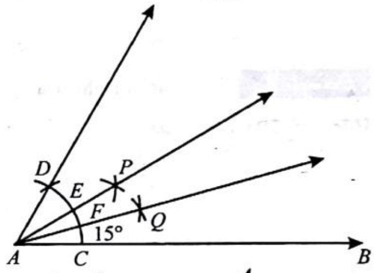
(c) (i) Draw a ray AB.

(ii) Taking A as centre and with any convenient radius draw an arc intersecting ray AB at C.

(iii) Taking C as centre and radius = AC, draw an arc such that it intersects the previous arc at D.
Now $\angle BAD = 60°$

(iv) Taking C and D as centres and radius > ½ CD draw two arcs intersecting each other at P. Join AP, which intersects the arc CD at E.
Now, $\angle BAP = 30°$

(v) With C and E as centres and radius > ½ CE draw two arcs intersecting each other at Q. Join AQ.
Now, $\angle BAQ = 15°$



Example 4 : **Construct an equilateral triangle, given its side 5 cm and justify the construction.**

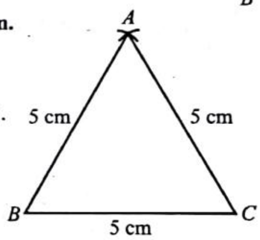
STEPS OF CONSTRUCTION :

(i) Draw $BC = 5$ cm

(ii) Taking B and C as centres and radius of length 5 cm, draw two arcs intersecting each other at A.

(iii) Join AB and AC

$\therefore$ ΔABC is the required equilateral triangle.



Justification :

Since the three sides

$AB = BC = CA = 5$ cm

Thus ΔABC is an equilateral triangle.

Example 5 : **Construct a right triangle whose base is 12 cm and sum of its hypotenuse and other side is 18 cm.**

STEPS OF CONSTRUCTIONS :

(i) Draw BC of length 12 cm.

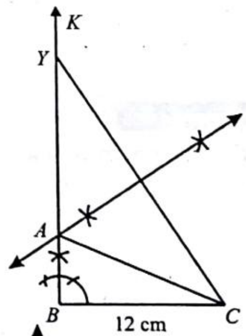
(ii) At B, draw $\angle CBK = 90°$

(iii) Along BK, cut off BY of length 18 cm. Join C to Y.

(iv) Draw the right bisectors of CY, which meet BY at A.

(v) Join A and C.

ΔABC is the required triangle.



Example 6 : **Construct a right triangle when one side is 3.5 cm and the sum of the other side and hypotenuse is 5.5. cm.**

STEPS OF CONSTRUCTION :

(i) Draw BC of length 3.5 cm.

(ii) At B, construct $\angle CBX = 90°$

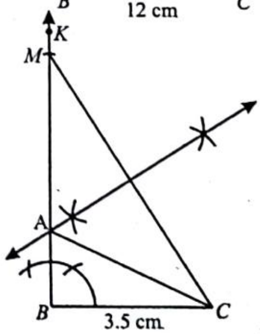
(iii) From BX, cut off BM of length 5.5 cm.

(iv) Join C to M

(v) D... perpendicular bisector of CM, let the bisector meet BM at A.

(vi) Join A to C.

Thus ΔABC is the required right triangle.



Example 7 : Construct a triangle, whose perimeter is 12 cm and the ratio between its sides are 3 : 4 : 5.

STEPS OF CONSTRUCTION :

(i) Draw a line-segment PQ of length 12 cm.

(ii) From P, draw a ray PX making an acute angle with PQ.

(iii) Sum of the ratio = 3 + 4 + 5 = 12.

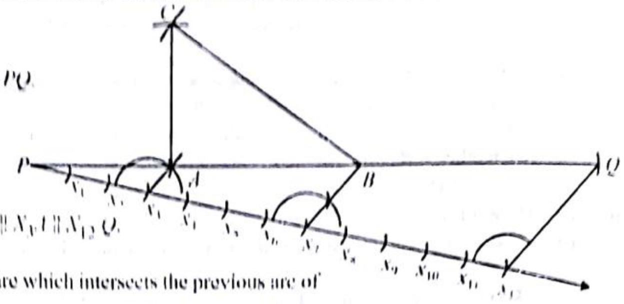
(iv) Mark points $X_1, X_2, X_3, \ldots\ldots, X_{12}$ on ray PX such that $PX_1 = X_1X_2 = X_2X_3 = \ldots\ldots = X_{11}X_{12}$

(v) Join X_{12} to Q. From the point X_7 and X_3 draw $X_7B \parallel X_3A \parallel X_{12}Q$.

(vi) Taking A as centre and AP as radius, draw an arc.

(vii) Taking B as centre and BQ as radius, draw another arc which intersects the previous arc of step (vi) at C.

(viii) Join AC and BC. Then, ΔABC is the required triangle



Example 8 : Construct a right triangle with perimeter 13 cm and one angle of 30°.

STEPS OF CONSTRUCTION :

(i) Draw XY of length 13 cm.

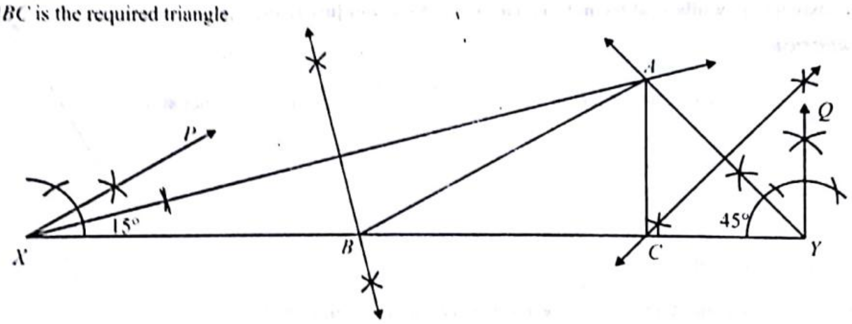
(ii) Draw $\angle PXY = 30°$ and $\angle XYQ = 90$

(iii) Draw bisectors of $\angle PXY$ and $\angle XYQ$ meeting each other at A

(iv) Draw right bisectors of AX and AY which meets XY at B and C respectively.

(v) Join AB and AC.

$\therefore$ ΔABC is the required triangle



Example 9 : Construct an equilateral triangle with perimeter 16 cm.

STEPS OF CONSTRUCTION :

(i) Draw XY of length 16 cm.

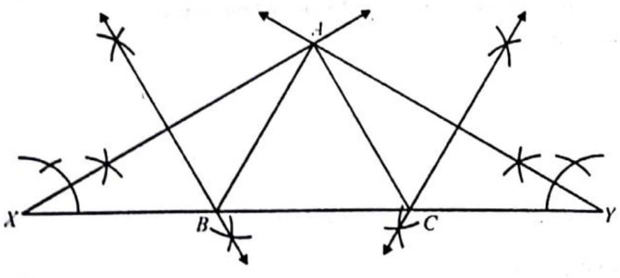
(ii) Draw $\angle PXY = 60°$ and $\angle XYQ = 60$

(iii) Draw bisectors of $\angle PXY$ and $\angle XYQ$ meeting each other at A.

(iv) Draw right bisectors of AX and AY meeting XY at B and C respectively.

(v) Join AB and AC

$\therefore$ ΔABC is the required equilateral triangle.

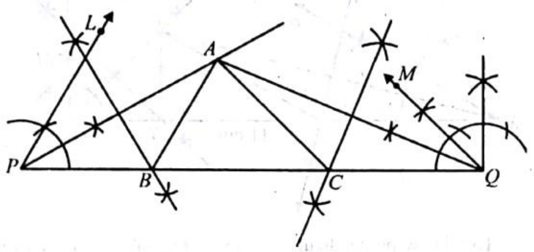


EXERCISE 1

FIB Fill in the Blanks

DIRECTIONS: *Study the diagram given below and steps of construction carefully. Compare them and fill suitable words or numbers in the blank boxes.*

Construction of a triangle ABC in which $\angle B = 60°$, $\angle C = 45°$ and perimeter = 11 cm.



STEPS OF CONSTRUCTION

1. Draw a line seqment PQ = (———)cm, (= $AB + BC + CA$).
2. At (———) construct an angle of 60° and at Q, an angle of (———).
3. Bisect these angles. Let the bisectors of these angles intersect at a point (———).
4. Draw perpendicular bisectors DE of (———) to intersect (———) at B and FG of AQ to intersect PQ at (———).
5. Join (———) and (———). Thus, (———) is the required triangle.

T/F True/False

DIRECTIONS: *Read the following statements and write your answer as true or false.*

1. An angle of 52.5° can be constructed.
2. An angle of 42.5° can be constructed.
3. A triangle ABC can be constructed in which $AB = 5$ cm, $\angle A = 45°$ and $BC + AC = 5$ cm.
4. A triangle ABC can be constructed in which $BC = 6$ cm, $\angle C = 30°$ and $AC - AB = 4$ cm.
5. A triangle ABC can be constructed in which $\angle B = 105°$, $\angle C = 90°$ and $AB + BC + AC = 10$ cm.
6. A triangle ABC can be constructed in which $\angle B = 60°$, $\angle C = 45°$ and $AB + BC + AC = 12$ cm.

SAQ Short Answer Questions

DIRECTIONS: *Give answer in 2-3 sentences.*

1. Construct an angle of 45° at the initial point of a given ray and justify the construction.
2. Construct a triangle XYZ in which $Y = 30°$, $Z = 90°$ and $XY + YZ + ZX = 11$ cm.
3. Construct $\Delta\, ABC$ where $BC = 4.5$ cm, $AB = 3.5$ cm and $\angle B = 45°$.

LAQ Long Answer Questions

DIRECTIONS: *Give answer in four to five sentences.*

1. Construct a perpendicular bisector of a line segment of length 6 cm. Write the steps of construction and also justify your construction.
2. Construct a triangle ABC where base $BC = 6$ cm, $\angle ABC = 60°$ and $AB + AC = 7$ cm. Justify your construction.
3. Construct a triangle ABC in which $AB = 5.8$ cm, $BC + CA = 8.4$ cm and $\angle B = 60°$. Justify your construction.
4. Construct a $\Delta\, ABC$ in which $BC = 5.6$ cm, $AC - AB = 1.6$ cm and $\angle B = 45°$. Justify your construction.
5. Construct an equilateral triangle if its altitude is 6 cm. Give justification for your construction.
6. Construct a right triangle in which one side is 3.5 cm and sum of the other side and hypotenuse is 5.5 cm.

SOLUTIONS

EXERCISE – I

FILL IN THE BLANKS

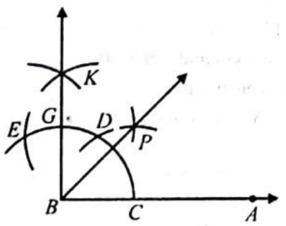
1. 11
2. P, 45°
3. A
4. AP, PQ, C
5. AB, AC, ABC

TRUE/FALSE

1. True. As $52.5° = \frac{210°}{4}$ and $210° = 180° + 30°$ which can be constructed.
2. False. As $42.5° = \frac{1}{2} \times 85°$ and 85° cannot be constructed.
3. False. As BC + AC must be greater than AB which is not so.
4. True. As $AC - AB < BC$, i.e., $AC < AB + BC$.
5. False. As $\angle B + \angle C = 105° + 90° = 195° > 180°$
6. True. As $\angle B + \angle C = 60° + 45° = 105° < 180°$

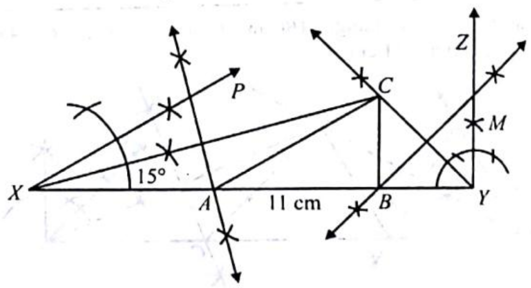
SHORT ANSWER QUESTIONS

1. ***Steps of Construction :***
   (i) Draw a ray BA
   (ii) Take B as centre and any radius, draw an arc which intersects AB at C.



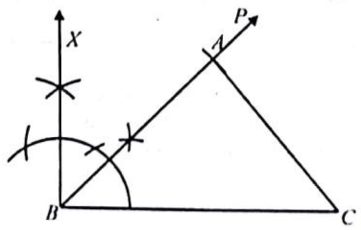
   (iii) Again take C as center and radius = BC, draw an arc which is intersects the arc drawn in step (ii) at D.
   (iv) Again take D as centre and radius as BC, draw an arc intersecting the arc drawn in step (ii) at E.
   (v) Now With D and E as centres and radius > ½ ED, draw two arcs which intersects each other at K . Join KB. Also let BK intersect the arc ED at G.
   Now, $\angle ABK = 90°$
   (vi) Take G and C as centre and radius > ½ CG draw two arcs, which intersects each other at P.
   Join BP.
   $\therefore \angle ABP = 45°$

2. ***Steps of Construction :***
   (i) Draw a line segment $AB = XY + YZ + ZX$ (= 11 cm).
   (ii) Draw $\angle LAB = 30°$ and $MBA = 90°$.
   (iii) Draw bisectors of the angle LAB and angle MBA, which meet at a point X.



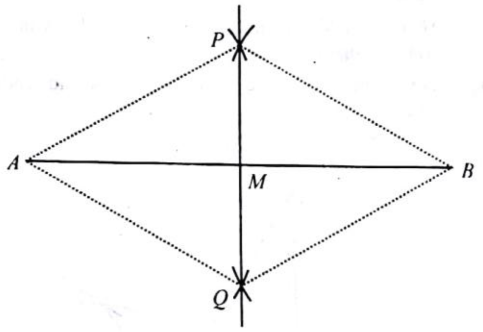
   (iv) Draw perpendicular bisectors DE of XA and FG of XB.
   (v) Let DE intersects AB at Y and FG intersects AB at Z. Join XY and XZ.
   Then, XYZ is the required triangle.

3. ***Steps of Construction***
   (i) Draw $BC = 4.5$ cm.
   (ii) At the point B, draw ray BX, such that $\angle CBX = 90°$
   (iii) Draw the bisector BD of $\angle CBX$.
   (iv) With B as centre and taking a radius of 3.5 cm, draw an arc, intersecting BD at A.
   (v) Join BA and AC. ABC is the required triangle.



LONG ANSWER QUESTIONS

1. ***Steps of Construction:***
   (i) Draw a line segment AB of length 6 cm.
   (ii) Taking A and B as centres and radius more than ½AB, draw arcs on both sides of the line segment AB (to intersect each other).



(iii) Let these arcs intersect each other at P and Q. Join PQ.

(iv) Let PQ intersect AB at the point M. Then line PMQ is the required perpendicular bisector of AB.

Justification:

Join A and B to both P and Q to form AP, AQ, BP and BQ.

In triangles PAQ and PBQ,

$AP = BP$ (Arcs of equal radii)

$AQ = BQ$ (Arcs of equal radii)

$PQ = PQ$ (common)

Therefore, $\Delta APQ \cong \Delta BPQ$ (SSS rule)

So, $\angle APQ = \angle BPQ$

or $\angle APM = \angle BPM$ (C.P.C.T.)

Now in triangles PMA and PMB,

$AP = BP$ (As before)

$PM = PM$ (Common)

$\angle APM = \angle BPM$ (Proved above)

Therefore, $\Delta PMA \cong \Delta PMB$ (SAS rule)

So, $AM = BM$ and $\angle AMP = \angle BMP$ (C.P.C.T.)

As $\angle AMP + \angle BMP = 180°$ (Linear pair axiom)

we get $\angle AMP + \angle AMP = 180°$

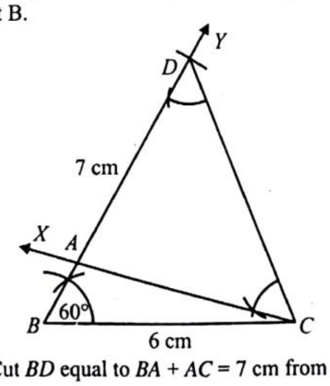
$\Rightarrow \angle AMP = 90°$

Therefore, PM, that is PMQ is the perpendicular bisector of AB.

2. ***Steps of construction :***

(i) Draw the base $BC = 6$ cm.

(ii) Using ruler and compass, draw an angle YBC of 60° at B.



(iii) Cut BD equal to $BA + AC = 7$ cm from the ray BY.

(iv) Join DC and make an angle DCX equal to BDC.

(v) Let CX intersect BY at A.

(vi) Thus ABC is the required triangle where $BA + AC = 7$ cm.

Justification:

First base BC and $\angle B$ are drawn as given in figure.

Now, in ΔACD

$\angle ACD = \angle ADC$ (By construction)

$\therefore \quad AC = AD$

Now, $AB = BD - AD = BD - AC$

$\Rightarrow \quad AB + AC = BD$

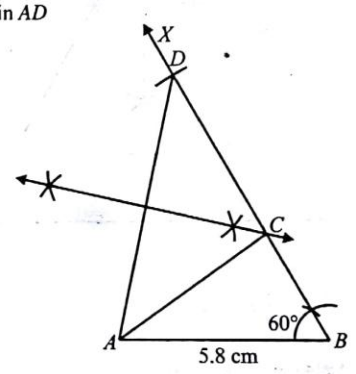
3. ***Steps of Construction :***

(i) Draw $AB = 5.8$ cm

(ii) Draw $\angle ABX = 60°$

(iii) From ray BX, cut off line segment $BD = BC + CA = 8.4$ cm.

(iv) Join AD



(v) Draw the perpendicular bisector of AD meeting BD at C.

(vi) Join AC to obtain the required triangle ABC.

Justification :

Clearly, C lies on the perpendicular bisector of AD.

$\therefore \quad CA = CD$

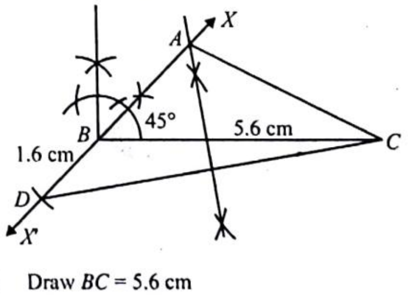
Now, $BD = 8.4$ cm

$\Rightarrow \quad BC + CD = 8.4$ cm

$\Rightarrow \quad BC + CA = 8.4$ cm

Hence, ΔABC is the required triangle.

4. ***Steps of Construction***



(i) Draw $BC = 5.6$ cm

(ii) At B, construct $\angle CBX = 45°$

(iii) Produce XB to X' to form line $X\,BX'$

(iv) From ray BX', cut-off line segment $BD = 1.6$ cm

(v) Join CD

5. (i) Draw a line XY.

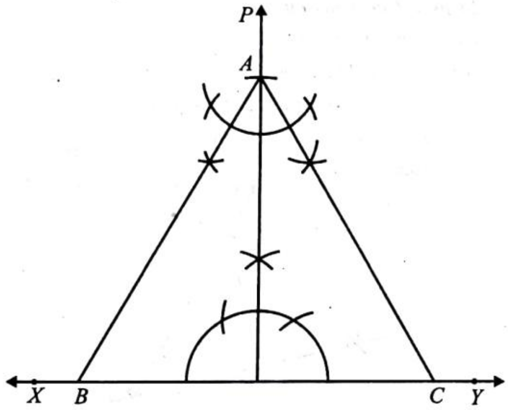
(ii) Take any point D on this line.

(iii) Construct perpendicular PD on XY.

(iv) Cut a line segment AD from D equal to 6 cm.

(v) Make angles equal to 30° at A on both sides of AD, say $\angle CAD$ and $\angle BAD$ where B and C lie on XY. Then ABC is the required triangle.

(vi) Draw perpendicular bisector of CD, which intersects XX' at point A. Join A to C.



Justification:

Since $\angle BAC = 30° + 30° = 60°$ and $AD \perp BC$. In ΔABD,

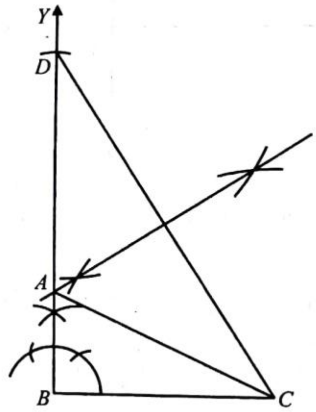
$\angle BAD = 30$ and $\angle ADB = 90°$

$\therefore \angle ABD = 60°$, i.e. $\angle ABC = 60°$

Similarly $\angle ACB = 60°$

Hence ΔABC is an equilateral triangle with altitude $AD = 6$ cm.

6. Given one side = 3.5 cm and sum of other side and hypotenuse = 5.5 cm.



Steps of Construction :

(i) Draw line segment $BC = 3.5$ cm.

(ii) Construct $\angle XBY = 90°$.

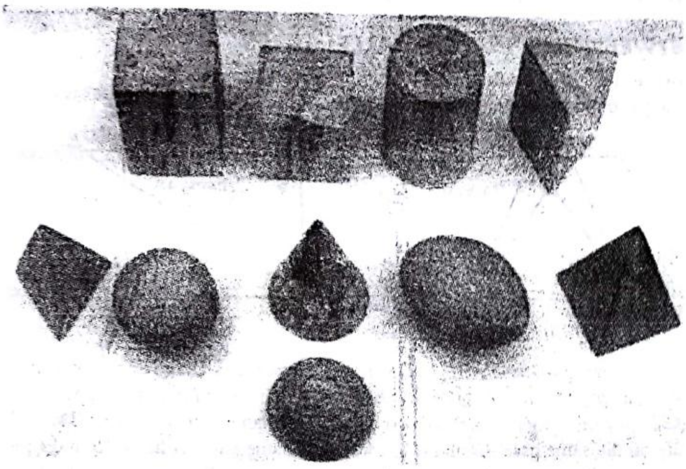
(iii) From BY cut off a line segment $BD = 5.5$ cm

(iv) Join CD.

(v) Draw the perpendicular bisector of CD intersecting BD at a point A.

(vi) Join AC.

So, ΔABC is the required triangle.



9
CHAPTER

Area of Parallelograms & Triangles, Heron's Formula, Surface Area and Volume of Solids

INTRODUCTION

You may recall that the part of the plane enclosed by a simple figure is called a planar region corresponding to that figure the magnitude or measure of this planar region is called its area. You are also familiar with the concept of congruent figures.

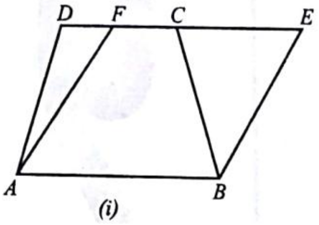
In this chapter, you will learn to find the areas of different plane figures, such as triangle, rectangle, parallelogram etc by studying some relationship between the areas of these figures under the condition when they lie on the same base and between the same parallels lines.

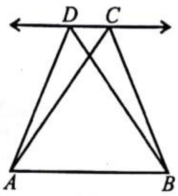
In this chapter, you will also study the Heron's Formula to find the area of any triangle whose all the three sides are given and formula to find the surface area & volume of some solid: cube, cuboid, cylinder, sphere and cone.

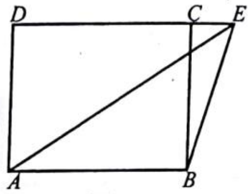
AREA OF PARALLELOGRAMS AND TRIANGLES, HERON'S FORMULA

FIGURES ON THE SAME BASE AND BETWEEN THE SAME PARALLEL LINES

Two figures are said to be on the same base and between the same parallel lines, if they have a common base (or side) and their vertices opposite to the common base lie on a same line parallel to the base. Look at the figures:

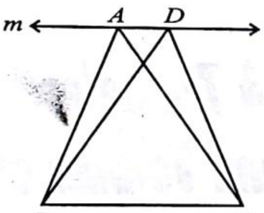

(i)

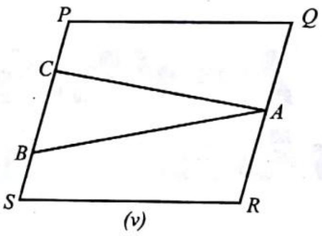

(ii)


(iii)

In fig (i) trapezium *ABCD* and parallelogram *ABEF* are on the same base *AB*. Besides these, the vertices *C* and *D* of trapezium *ABCD* opposite to base *AB* and the vertices *E* and *F* of parallelogram *ABEF* opposite to base *AB* lie on a line *DE* parallel to *AB*. So we can say that trapezium *ABCD* and parallelogram *ABEF* are on the same base *AB* and between the same pair of parallel lines *AB* and *DE*. Similarly in fig.(ii), triangle *ABC* and *ABD* are ou the same base *AB* and lie between the same parallel lines *AB* and *CD*. Also in fig. (iii) parallelogram *ABCD* and triangle *ABE* are on the same base *AB* and between the same parallel lines *AB* and *DE*.

Now ΔABC and ΔADB in fig (iv) have no common base. Also in fig. (v) ΔABC and parallelogram. *PQRS* are not on the same base.

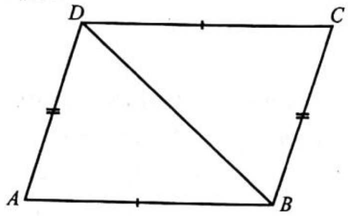

(iv)


(v)

THEOREM 1 : A diagonal of parallelogram divides it into two triangles of equal area.

Given : A parallelogram *ABCD* in which *BD* is one of its diagonals, side $AB \parallel CD$ and $BC \parallel AD$.

To Prove : Area of ΔABD = area of ΔBCD



PROOF : In ΔABD and ΔBCD,

$AB = CD$ (opp. sides of a || gm)

$AD = CB$ (opp. sides of a || gm)

$BD = BD$ (common side)

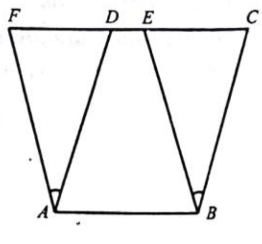
$\therefore$ $\Delta ABD \cong \Delta CDB$ [By SSS congruent Rule]

$\therefore$ area of ΔABD = area of ΔBCD [congruent area Axiom]

THEOREM 2 : Parallelograms on the same (or equal) base and between the same parallel lines are equal in area.

Given : Two parallelograms *ABCD* and *ABEF*, which have the same base *AB* and which are between the same parallel lines *AB* and *FC*.

To Prove : Area of || gm *ABCD* = are of || gm *ABEF*



PROOF : In ΔADF and ΔBCE

$AD = BC$ [opp. sides of a || gm]

$AF = BE$ [opp. sides of a ||gm]

Also, $AD \parallel BC$ and $AF \parallel BE$

So the angle between *AD* and *AF* is equal to the angle between *BC* and *BE*.

i.e. $\angle DAF = \angle CBE$

$\therefore$ $\Delta ADF \cong \Delta BCE$ [By SAS congruent Rule]

$\therefore$ Δ area ΔADF = area ΔBCE [By congruent area Axom] ...(i)

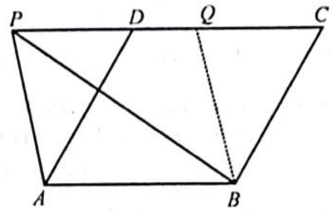
Now, area || gm $ABCD$ = area ($\square ABED$) + (ΔBCE)

= area ($\square ABED$) + area (ΔADF) [Using (i)]

area (|| gm $ABCD$) = area (|| gm $ABEF$)

Corollary : A parallelogram and a rectangle on the same base and between the same parallel lines are equal in area. Since a rectangle is also a parallelogram. So, the result is a direct consequence of the above theorem.

THEOREM-3 : If a triangle and a parallelogram are on the same (or equal) base and between the same parallels lines, then prove that area of the triangle is equal to half the area of the parallelogram.



Given : ΔABP and || gm $ABCD$ be on the same base AB and between the same parallel lines AB and PC (See fig.)

To Prove : Area of $\Delta PAB = \frac{1}{2}$ area of || gm $ABCD$

Construction : From B, draw $BQ \parallel AP$

PROOF : Since $AB \parallel PQ$ and $AP \parallel BQ$, therefore $ABQP$ is parallelogram.

Now parallelogram $ABQP$ and $ABCD$ are on the same base AB and between the same parallels AB and PC.n

$\therefore$ area of || gm $ABQP$ = area of || gm $ABCD$ ...(i)

Since diagonal PB divides || gm $ABQP$ in to two triangles of equal areas (Theorem-2).

$\therefore$ Area of ΔPAB = area of ΔBQP ...(ii)

$\therefore$ Now area of || gm $ABQP$ = area of ΔPAB + area of ΔBQP

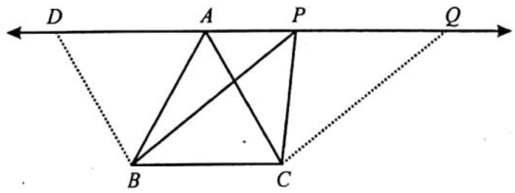
= 2 × area of ΔPAB [from (ii)]

$\therefore$ Area of $\Delta PAB = \frac{1}{2}$ area of || gm $ABQP$

$\Rightarrow$ area of $\Delta PAB = \frac{1}{2}$ area of || gm $ABCD$

THEOREM 4 : Two triangles on the same base (or equal bases) and between the same parallels are equal in area.

Given : ΔABC and ΔPBC are on the same base BC and between the same parallel lines BC and AP.



To Prove : Area of ΔABC = area of ΔPBC

Construction : Through B, draw $BD \parallel CA$ intersecting PA produced at D and through C, draw $CQ \parallel BP$ intersecting AP produced in Q.

PROOF : We have

$BD \parallel CA$ [By construction]

$BC \parallel DA$ [Given]

$\therefore$ $BCAD$ is a parallelogram.

Similar $BCQP$ is a parallelogram.

Now, || gm $BCQP$ and $BCAD$ are on the same base BC and lie between the same parallel lines BC and DQ,

$\therefore$ area of (|| gm $BCQP$) = area of (|| gm $BCAD$) ...(i)

We know that the diagonals of a || gm divides it into two triangles of equal area. [By theorem-1]

$\therefore$ Area of (ΔPBC) = $\frac{1}{2}$ area of (|| gm $BCQP$) ...(ii)

and area of (ΔABC) = $\frac{1}{2}$ area of (|| gm $BCAD$) ...(iii)

Now area of (|| gm $BCQP$) = area of (|| gm $BCAD$) [from (i)]

$\therefore \quad \frac{1}{2}$ area of (|| gm $BCQP$) = $\frac{1}{2}$ area of (|| gm $BCAD$)

$\therefore$ area of ΔABC = area of ΔPBC [From (ii) and (iii)]

HERON'S FORMULA

This is used to find the area of any type of triangle if we know the length of its all sides.

We know that the area of a triangle = $\frac{1}{2}$ × base × height (or altitude)

In some cases, length of each side of the triangle are given but height of the triangle is neither given nor we are able to find in any way, then to find the area of such type of triangle, we use Heron's formula which is given below.

$$\text{Area of a triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

Where a, b, c are length of the sides of a triangle and s is the semi-perimeter of the triangle

i.e. $s = \frac{a+b+c}{2}$

Illustration 1 : Find the area of a triangle, length of whose sides are 3cm, 4cm, and 5cm.

SOLUTION : We have a = 3cm, b = 4cm, c = 5cm

$$s = \frac{a+b+c}{2} = \frac{3+4+5}{2}\text{cm} = 6\text{cm}$$

$\therefore \quad ar(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{6\times(6-3)(6-4)(6-5)} = \sqrt{6\times3\times2\times1}\text{ cm}^2 = \sqrt{36} = 6\text{ cm}^2$$

Illustration 2 : Find the area of an equilateral triangle having each side's length 4 cm.

SOLUTION : $a = b = c = 4$ cm

$$s = \frac{a+b+c}{2} = \frac{4+4+4}{2} = 6$$

Area of a triangle by Heron's formula

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{6(6-4)(6-4)(6-4)} = \sqrt{6\times2\times2\times2}$$

$$= \sqrt{48} = \sqrt{4\times4\times3} = 4\sqrt{3}\text{ cm}^2$$

APPLICATION OF HERON'S FORMULA IN FINDING AREAS OF QUADRILATERALS

To find the area of a quadrilateral we divide it in triangular parts and then use the formula for area of the triangle. We add the area of divided triangle to get the area of whole quadrilateral.

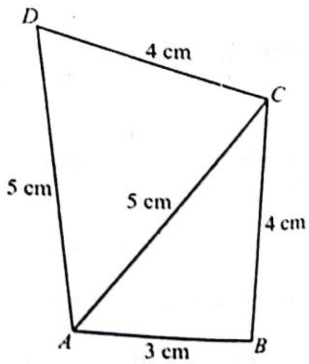


Illustration 3 : Find the area of a quadrilateral *ABCD* in which *AB* = 3 cm, *BC* = 4 cm, *CD* = 4 cm, *DA* = 5 cm and *AC* = 5 cm.

SOLUTION : We divide the quadrilateral $ABCD$ in two triangle ABC and ACD.

For ΔABC, $a = 3$, $b = 4$, $c = 5$

$$s = \frac{3+4+5}{2}\text{cm} = 6\text{cm}$$

Area (ΔABC) = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{6(6-3)(6-4)(6-5)} = \sqrt{6\times3\times2\times1} = 6 \text{ cm}^2$$

Similarly, for ΔACD,

$a = 5$ cm, $b = 5$ cm, $c = 4$ cm

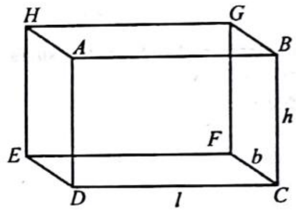
$$s = \frac{5+5+4}{2} \text{cm} = 7\text{cm}$$

Area (Δ ACD) = $\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{7(7-5)(7-5)(7-4)} = \sqrt{7\times2\times2\times3} = 2\sqrt{21}\text{ cm}^2$

$\therefore$ Area of quadrilateral $ABCD$ = Area of triangle ABC + Area of triangle ACD = 6 cm^2 + $2\sqrt{21}$ cm^2

SURFACE AREAS AND VOLUME OF SOLIDS

In this part of the chapter, you will study about some solids: cuboid, cube, cylinder, cone, sphere, hemisphere and the formula to the surface area and volume of these solids.



CUBOID

It is a solid bounded by a six rectangular plane regions (or surfaces).
A cuboid is also called a rectangular parallelopiped.

Faces : Cuboid $ABCDEFGH$ is bounded by six rectangular plane regions namely $ABCD$, $EFGH$, $ADEH$, $BCFG$, $ABGH$ and $CDEF$.

These six rectangular plane regions are six faces of the cuboid $ABGH$ and $CDEF$ are the top and bottom faces respectively. Hence a cuboid has six faces.

Edges : Any two adjacent faces of a cuboid meet in a line segment, which is called an edge of the cuboid. in the above fig. AB, AD, AH, HD, HE, HG, GF, BG, FE, BC, CF, DE and CD are 12 edges of the cuboid. Hence a cuboid has 12 edges.

Vertices : The point of intersection of three edges of a cuboid is called a vertex of the cuboid. In the fig. We have 8 vertices which are A, B, C, D, E, F, G and H.

Lateral Faces : All the faces except the top and bottom faces are called the lateral faces of a cuboid.

SURFACE AREA OF A CUBOID

Consider a cuboid of length l cm, breadth b cm and height h cm.

Total surface area of a cuboid, is equal to the sum of area of all the six faces of the cuboid.

Total surface area of a cuboid = Area of face $ABCD$ + Area of face $EFGH$ + area of face $DCFE$ + area of face $ABGH$ + area of face $FCBG$ + area of face $AHED$

$= (l \times h) + (l \times h) + (l \times b) + (l \times b) + (b \times h) + (b \times h)$

$= 2\,(lh + bh + lb) = 2\,(lb + bh + hl)$

$= 2$ [length × breadth + breadth × height + height × length]

Lateral surface of a cuboid = Area of face $ABCD$ + Area of face $EFGH$ + area of $FCBG$ + area of $AHED$

$= (l \times h) + (l \times h) + (b \times h) + (b \times h)$

$= 2\,(l \times h + b \times h) = 2\,(l + b) \times h$

$= 2$ (length + Breadth) × height

DIAGONAL OF THE CUBOID AND CUBE

AF, HC, BE, GD are the diagonals of the cuboid

$$AF^2 = AD^2 + DF^2$$
$$= h^2 + l^2 + b^2$$
$$AF = \sqrt{l^2 + b^2 + h^2}$$

Similarly length of each diagonal = $\sqrt{l^2 + b^2 + h^2}$

VOLUME OF A CUBOID

As we know that the volume of a solid object is the measure of the space occupied by it and the capacity of an object is the volume of the substance that can accomodate in its interior.

Volume of a cuboid = space occupied by the cuboid
= Area of the base × height
= $(l \times b) \times h = lbh$

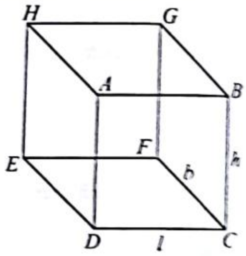
Volume of the cuboid = length × Breadth × Height

Note : Surface area of any object is measured in 'square units' Volume of any object is measured in 'cube units'

CUBE

A Cuboid whose edges are equal i.e. its length, breadth and height are equal is called a cube. Each face of cube is a square and area of these squares are equal.

For a cube, $l = b = h$



$\therefore$ Total surfaces area of a cube = $2\,(l \times l + l \times l + l \times l) = 6\,l^2 = 6 \times (\text{Edge})^2$

Lateral surface of a cube = $2\,(l \times l + l \times l) = 2\,(l^2 + l^2) = 4l^2 = 4\,(\text{Edge})^2$

Length of a diagonal of the cube = $\sqrt{l^2 + l^2 + l^2} = \sqrt{3}\,l$

Volume of the cube = $l \times l \times l = l^3 = (\text{Edge})^3$

Illustration 4 : Find the surface area of a chalk box whose length, breadth and height are 16 cm, 8cm and 6 cm, respectively

SOLUTION

Clearly, *a* chalk box is in the form of a cuboid.

Here, l = 16cm, b = 8 cm and h = 6 cm

Surface area of the cuboid = $2\,(lb + bh + lh) = 2\,(16 \times 8 + 8 \times 6 + 16 \times 6)\ \text{cm}^2$
$= 2\,(128 + 48 + 96)\ cm^2 = 544\ \text{cm}^2$

Illustration 5 : Find the surface area of a cube whose edge is 11 cm

SOLUTION

We know that the surface area of a cube = 6 $(\text{Edge})^2$

Here, edge = 11 cm

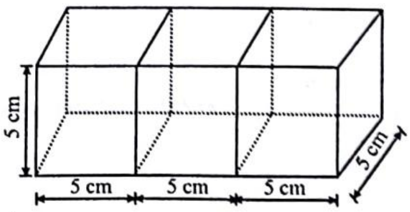
$\therefore$ Surface area of the given cube = $6 \times (11)^2\ \text{cm}^2 = (6 \times 121)\ \text{cm}^2 = 726\ \text{cm}^2$

Illustration 6 : Three cubes each of side 5 cm are joined end to end. Find the surface area of the resulting cuboid.

SOLUTION

The dimensions of the cuboid so formed are

l = Length = 15 cm, b = Breadth = 5cm, and h = Height = 5 cm.



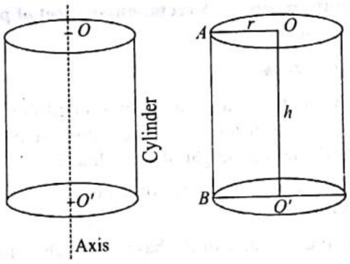
So, surface area of the cuboid = $2\,(15\times5 + 5\times 5 + 15 \times 5)\ \text{cm}^2 = 2\,(75 + 25 + 75)\ \text{cm}^2 = 350\ \text{cm}^2$

RIGHT CIRCULAR CYLINDER

A solid like measuring jars, circular pillars, circular pipes etc., whose cross section is uniform and circular, are examples of circular cylinder. Cylinders have a curved (lateral) surface with congruent circular ends. Some times lower circular end is called the base of the cylinder and upper circular end is called the top of the cylinder. The line joining the centres of the circular ends of a cylinder is called its axis. In figure, OO' is the axis of the cylinder. If the axis is perpendicular to the circular ends then cylinder is called right circular cylinder. In this chapter you will study about right circular cylinder only. Hence a right circular cylinder is simply called cylinder.

A right circular cylinder may also be considered as a solid generated by the revolution of a rectangle about one of its sides.

Thus, if a rectangle $OO'BA$ revolves about its side OO' and completes one revolution to arrive at its initial position, a right circular cylinder will be generated whose axis is OO' and radius $AO = BO' = r$ (say). The length of the axis OO' between the centres is called the length or the height (h) of the cylinder.

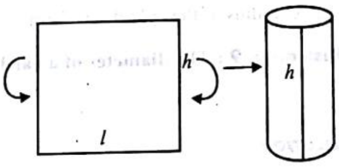


SURFACE AREA OF A RIGHT CIRCULAR CYLINDER

Consider a right circular cylinder of height 'h' and radius 'r'.

We can form curved surface of a right circular cylinder by folding a rectangular sheet as given below:

The area of the sheet gives us the curved surface area of the cylinder. Note that the length (l) of the sheet is equal to the circumference of the circular base which is equal to $2\pi r$.



So, curved surface area of the cylinder = Area of the rectangular sheet

$= l \times h =$ perimeter of the base of cylinder $\times h$

$= 2\pi r \times h = 2\pi rh$

Curved surface Area of a cylinder $= 2\pi rh$

Total surface Area of the cylinder = curved surface + Area of two circular ends

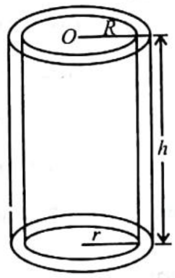
$= 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

SURFACE AREA OF A RIGHT CIRCULAR HOLLOW CYLINDER

A solid bounded by two coaxial cylinders of the same height and different radii is called a hollow cylinder.

Examples : Iron pipes, rubber tubes etc.

Let we have a hollow cylinder as shown in the figure having internal and external radii as r and R respectively. h is the height of this cylinder.



Base area $= \pi R^2 - \pi r^2 = \pi(R^2 - r^2)$

Curved (lateral) Surface area = external surface area + internal surface area

$= 2\pi Rh + 2\pi rh = 2\pi h(R + r)$

Total Surface Area = curved surface area +2 (area of base)

$= 2\pi Rh + 2\pi rh + 2\pi(R^2 - r^2)$

$= 2\pi (R + r)h + 2\pi(R - r)(R + r) = 2\pi (R + r)(h + R - r)$

VOLUME OF A RIGHT CIRCULAR CYLINDER

Just as a cuboid, of the right circular cylinder, volume = Area of the Base × height $= (\pi r^2) \times h = \pi r^2 h$

Volume $= \pi r^2 h$

VOLUME OF A HOLLOW CYLINDER

Volume of the material

= Exterior volume – Interior volume

$= \pi R^2 h - \pi r^2 h$

$= \pi(R^2 - r^2)h$

Illustration 7 : The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the diameter of the base of the cylinder.

SOLUTION

Let r be the radius and h be the height of the cylinder. Then,

$2\pi rh = 88$ and $h = 14$

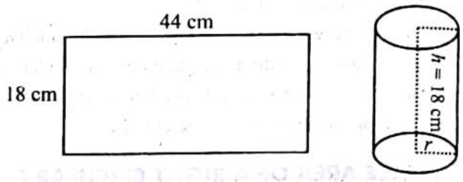
$\Rightarrow \quad 2 \times \frac{22}{7} \times r \times 14 = 88$

$\Rightarrow \quad 88\, r = 88$

$r = 1$

$\therefore$ Diameter of the base $= 2r = 2$ cm

Illustration 8 : A rectangular sheet of paper 44 cm × 18 cm is rolled along its length and a cylinder is formed. Find the radius of the cylinder.



SOLUTION

When the rectangular sheet is rolled along its length, we find that the length of the sheet forms the circumference of its base and breadth of the sheet becomes the height of the cylinder.

Let r cm be the radius of the base and h cm be the height. Then, $h = 18$ cm

Now,

Circumference of the base = Length of the sheet

$\Rightarrow \quad 2\pi r = 44 \Rightarrow 2 \times \frac{22}{7} \times r = 44 \Rightarrow r = 7\text{cm}$

Hence, radius of the cylinder is 7 cm.

Illustration 9 : The diameter of a garden roller is 1.4 m and it is 2 m long. How much area will it cover in 5 revolutions ? $\left(\text{Use } \pi = \frac{22}{7}\right)$

SOLUTION

Clearly,

Area covered = Curved surface × Number of revolutions

Here, $r = \frac{1.4}{2}\text{m} = 0.7\text{m}$ and $h = 2\text{m}$.

$\therefore \quad$ Curved surface $= 2\pi rh \text{ m}^2 = 2 \times \frac{22}{7} \times 0.7 \times 2 = 8.8\text{m}^2$

Hence, Area covered = Curved surface × No. of revolutions $= (8.8 \times 5) \text{ m}^2 = 44\text{m}^2$

Illustration 10 : Find the volume of a right circular cylinder, if the radius (r) of its base and height (h) are 7 cm and 15 cm respectively

SOLUTION

We know that :

Volume of a cylinder $= \pi r^2 h$

$\therefore$ Volume of the cylinder $= \frac{22}{7} \times (7)^2 \times 15\text{cm}^3 \quad [\because \pi = \frac{22}{7}]$

$= 22 \times 7 \times 15 \text{ cm}^3 = 2310 \text{ cm}^3$

Illustration 11 : The thickness of a hollow wooden cylinder is 2 cm. It is 35 cm long and its inner radius is 12 cm. Find the volume of the wood required to make the cylinder, assuming it is open at either end.

SOLUTION

We have,

r = Inner radius of the cylinder = 12 cm

Thickness of the cylinder = 2 cm

$\therefore \quad R$ = Outer radius of the cylinder = (12 + 2) cm = 14 cm

h = Height of the cylinder = 35 cm

$\therefore \quad$ Volume of the wood $= \pi (R^2 - r^2) h$

$$= \frac{22}{7} \times \{(14)^2 - (12)^2\} \times 35\text{cm}^3$$

$$= \frac{22}{7} \times (14+12) \times (14-12) \times 35\text{cm}^3$$

$$= 22 \times 26 \times 2 \times 5 \text{ cm}^3 = 5720 \text{ cm}^3$$

llustration 12 : A cylindrical road roller made of iron is 1 m wide. Its inner diameter is 54 cm and thickness of the iron sheet rolled into the road roller is 9 cm. Find the weight of the roller if 1 c.c. of iron weighs 8 gm.

SOLUTION

The width of the road roller is 1m i.e., 100 cm

So, Height (length) of the cylinder = 100 cm

Inner radius of the cylinder $= r = \frac{54}{2}$ cm $=$ 27cm

Thickness of the iron sheet = 9 cm

Outer radius of the cylinder $= R = (27 + 9)$ cm $= 36$ cm

Thus, Volume of the iron sheet used $= (\pi R^2 h - \pi r^2 h)$ cm³

$$= \pi (R^2 - r^2)\, h \text{ cm}^3$$

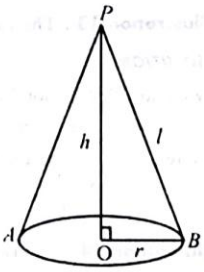
$$= [\pi (R + r)(R - r)h] \text{ cm}^3$$

$$= [3.14 \times (36 + 27)(36 - 27) \times 100] \text{ cm}^3$$

$$\frac{314}{100} \times 63 \times 9 \times 100 \text{cm}^3 = 178038 \text{cm}^3$$

$\therefore$ Weight of the roller $= 178038 \times 8$ gms $= 178038 \times \frac{8}{1000}$ kgs $= 1424.304$kgs

RIGHT CIRCULAR CONE



It is solid generated by revolving a right triangular sheet about its one of the perpendicular side.
In the figure, the right circular cone is formed by revolving the ΔAOP right angled at O about its one of perpendicular side, say OP.

Vertex : The top point P is called the vertex of the cone.

Base : A cone has a plane circular end, called the base of the cone.

Axis : The line joining the centre O of the base to the vertex P is called the axis of the cone. In the right circular cone, axis is perpendicular to the base

Radius (r) : It is the radius of the base of the cone i.e. OA or OB.

Height (h) : The distance of the vertex P from the centre O of the base is called height of the right circular cylinder. Hence length OP is the height in the above figure.

Slant Height (l) : The length of the line segment joining the vertex to any point on the circular edge of the base is called slant height in fig. PB is the slant height.

The slant height can be find out as

In rt. angle triangle POA

$$PA^2 = PO^2 + OA^2$$

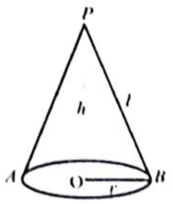
$$l^2 = h^2 + r^2$$

$$l = \sqrt{h^2 + r^2}$$

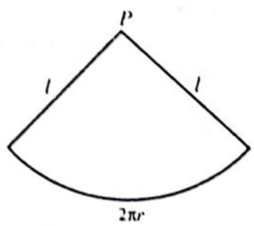
Note : The angle OPA or OPB is called the semi-vertical angle of the cone.

SURFACE AREA OF A RIGHT CIRCULAR CONE

For the given cone of base radius r as shown in fig. (i) the length of the circular edge is $2\pi r$ and area of the plane end (or base) is $2\pi r^2$. Now suppose, the given cone is hollow and cut the cone along the slant height PA and On spreading out it on a plane surface. You will find that the spread out figure as shown in (ii).



(i) Hollow cone



(ii) Spread out figure of the hollow cone

The spread out cone is a sector of a circle of radius equal to the slant height 'l' of the cone and whose arc is equal to the circumference $2\pi r$ of the base of the cone.

$\therefore$ Curved (or lateral) surface area of the cone $= \pi l^2 \times \frac{2\pi r}{2\pi l}$ [Since area of a sector of radius r and arc length l is $\pi r^2 \times \frac{l}{2\pi r}$]

$= \pi r l$

Total surface area of the cone = curved surface area + Area of the base

$= \pi r l + \pi r^2 = \pi r(l + r)$

VOLUME OF RIGHT CIRCULAR CONE

For this we consider the experiment. Take a conical cup of radius r and height h. Also take a cylindrical jar of radius r and height h. Fill the conical cup with water to the brim and transfer the water to the jar. Repeat the process two times more. You will find that 3 cup of water fill the cylindrical jar completely. Thus we find that

3(volume of cone of radius r and height h)

= (volume of a cylinder or radius r and height h) $= \pi r^2 h$

$\therefore$ Volume of a cone of radius r and height $h = \frac{1}{3}\pi r^2 h$

Also, volume of the cone of radius r and height h

$= \frac{1}{3}(\pi r^2) \times h = \frac{1}{3} \times$ (Area of the base) $\times$ height

Illustration 13 : The radius of the base of the cone is 12m and its slant height is 9m. Find its total surface area.

SOLUTION

We know that the total surface area S of a right circular cylinder of radius r and slant height l is given by

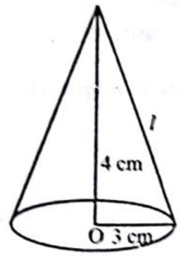
$S = \pi r^2 + \pi r l = \pi r(r + l)$

Here, $r = 12$ m and $l = 9$ m

$\therefore$ $S = \left[\frac{22}{7} \times 12 \times 12 + 9\right] m^2 = 792 m^2$

Illustration 14 : The radius of a cone is 3 cm and vertical height is 4 cm. Find its curved surface area

SOLUTION



Given = 3 cm and $h = 4$ cm.

Let l cm be the slant height of the cone.

$\therefore$ $l^2 = r^2 + h^2$

$\Rightarrow$ $l^2 = 3^2 + 4^2$

$\Rightarrow$ $l^2 = 25$

$\Rightarrow$ $l = \sqrt{25}$cm = 5cm

Hence area of the curved surface $= \pi r l$

$= \left(\frac{22}{7} \times 4 \times 5\right) cm^2 = 62.85 cm^2$

Illustration 15 : What is the height of the cone if the diameter is 8 cm and volume is 48 π cm^3.

SOLUTION

Let h cm be the height of the cone

D = Diameter of the cone = 8 cm

$\therefore$ r = Radius of the cone = 4cm

Now, Volume of the cone = $48\pi cm^3$ [Given]

As we know $V = \frac{1}{3}\pi r^2 h$

$\Rightarrow$ $\frac{1}{3} \times \pi \times 4 \times 4 \times h = 48\pi$

$\Rightarrow$ $h = \frac{48\pi \times 3}{16\pi}$ cm = 9cm

Hence, the height of the cone is 9 cm.

Illustration 16 : The volume of a cone is 18480 cm^3. If the height of the cone is 40 cm. Find the radius of its base.

SOLUTION

Let the radius of the cone be r cm.

We have, h = Height of the cone = 40 cm and, V = volume of the cone = 18480 cm^3

$$\therefore \quad \frac{1}{3}\times\frac{22}{7}\times r^2 \times 40 = 18480$$

$$\Rightarrow \quad r^2 = \frac{18480\times 3\times 7}{22\times 40} = 441$$

$$\Rightarrow \quad r = \sqrt{441}\text{cm} = 21\text{cm}$$

Illustration 17 : The base radii of two right circular cones of the same height are in the ratio 3 : 5. Find the ratio of their volumes.

SOLUTION

Let r_1 and r_2 be the radii of two cones and V_1 and V_2 be their volumes.

Let h be the height of the two cones. Then,

$$V_1 = \frac{1}{3}\pi r_1^2 \text{ and } V_2 = \frac{1}{3}\pi r_2^2 h$$

$$\therefore \quad \frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h}{\frac{1}{3}\pi r_2^2 h} = \frac{r_1^2}{r_2^2} = \frac{9}{25} \qquad \left[\because \frac{r_1}{r_2} = \frac{3}{5}\text{(Given)}, \ \therefore \ \frac{r_1^2}{r_2^2} = \frac{9}{25}\right]$$

So, the required ratio of the volumes of two cones is 9 : 25.

Illustration 18 : A cone and cylinder are having the same base. Find the ratio of their heights if their volumes are equal.

SOLUTION

Let the radius of the common base be r. Let h_1 and h_2 be the heights of the cone and cylinder respectively. Then,

Volume of the cone = $\frac{1}{3}\pi r^2 h_1$, Volume of the cylinder = $\pi r^2 h_2$

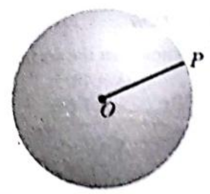
It is given that the cone and the cylinder are of the same volume.

$$\therefore \quad \frac{1}{3}\pi r^2 h_1 = \pi r^2 h_2$$

$$\Rightarrow \quad \frac{1}{3}h_1 = h_2$$

$$\Rightarrow \quad \frac{h_1}{h_2} = \frac{3}{1} \quad \Rightarrow \quad h_1 : h_2 = 3 : 1$$

Hence, the ratio of the height of the cone and cylinder is 3:1

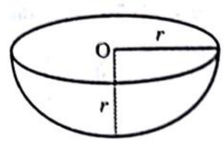


SPHERE

It is the set of all points is space which are at a constant distance from a fixed point. The fixed point is called the centre of the sphere and the fixed distance is called the radius of the sphere.

In the given figure, O is the centre and OP is radius of the sphere.

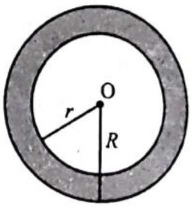
Diameter : A line segment through the centre of a sphere whose end-points are on the sphere is called the diameter of the sphere. In the above figure PP' is the diameter of the sphere.



HEMI-SPHERE : A plane through the centre of a sphere divides the sphere into two equal parts, each of which is called a hemi-sphere. It is shown in the figure.

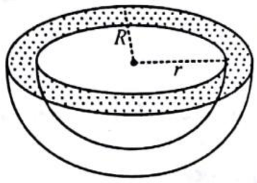
SPHERICAL SHELL

The difference of two solid concentric spheres is called a spherical shell. This is shown in the fig. Here r and R is the radius of internal and external spherical surfaces respectively.



HEMISPHERICAL SHELL : A plane through the centre of a spherical shell divides the spherical shell into two parts, each of which is called a hemispherical shell.

SURFACE AREA OF A SPHERE, HEMISPHERE, SPHERICAL SHELL AND HEMI-SPHERICAL CELL



(i) Surface area of a sphere of radius $r = 4\pi r^2$ = curved surface Area

(ii) Curved surface area of Hemi-sphere $= 2\pi r^2$

(iii) Total surface Area of Hemi-sphere = curved surface Area + Area of plane circular surface
$= 2\pi r^2 + \pi r^2 = 3\pi r^2$

(iv) If R and r be the outer and inner radii of a hemi spherical shell then
Outer curved surface area $= 2\pi R^2$
Inner curved surface area $= 2\ \pi r^2$
Surface area of plane ring $= \pi R^2 - \pi r^2 = \pi (R^2 - r^2)$
Total surface area = (outer curved surface area) + (inner curved area) + (surface area of plane ring)
$= 2\pi R^2 + 2\pi r^2 + \pi (R^2 - r^2) = \pi (3R^2 - r^2)$
Volume of a sphere, Hemisphere and a spherical shell

VOLUME OF A SPHERE, HEMISPHERE AND A SPHERICAL SHELL

(i) Volume of a sphere, $V = \frac{4}{3}\pi r^3$

(ii) Volume of a hemi-sphere, $V = \frac{2}{3}\pi r^3$

(iii) Volume of a spherical shell,
V = Outer volume – Inner volume
$= \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi (R^3 - r^3)$ cubic units

(iv) Volume of a hemispherical shell
= (Outer volume) – (Inner volume)
$\frac{2}{3}\pi R^3 - \frac{2}{3}\pi r^3$
$= \frac{2}{3}\pi(R^3 - r^3)$

Ilustration 19 : The radius of the sphere is 7 cm. Find its surface area.

SOLUTION

We know that the surface area S of a sphere of radius r is given by
$S = 4\pi r^2$
Given $r = 7$ cm
$\therefore$ $S = 4 \times \frac{22}{7} \times 7 \times 7\text{cm}^2 = 616\ \text{cm}^2$

Ilustration 20 : Find the surface area and total surface area of a hemisphere of radius 21 cm.

SOLUTION

We know that the surface area S and total surfaces area S_1 of a hemisphere of radius r are given by

Here, $r = 21$ cm

$\therefore \quad S = 2 \times \frac{22}{7} \times 21 \times 21 \text{ cm}^2$ and $S_1 = 3 \times \frac{22}{7} \times 21 \times 21 \text{ cm}^2$

$\Rightarrow \quad S = 2772 \text{ cm}^2$ and, $S_1 = 4158 \text{ cm}^2$

Illustration 21 : The radius of a sphere is 7 cm. Find its volume

SOLUTION

We know that the volume V of a sphere of radius r is given by

$$V = \frac{4}{3}\pi r^3 \text{ cubic units}$$

Here, $r = 7$ cm

$$\therefore \quad V = \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 \text{cm}^3 = 1437.33 \text{cm}^3$$

Illustration 22 : Find the volume of hemisphere of radius 3.5 cm

SOLUTION

$$V = \frac{2}{3}\pi r^3 \text{ cubic units}$$

Here, $r = 3.5$ cm

$$\therefore \quad V = \frac{2}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} \text{cm}^3 = \frac{11 \times 49}{3 \times 2} \text{cm}^3 = 89.83 \text{cm}^3$$

Illustration 23 : Find the volume of a sphere whose surface area is 154 square cm.

SOLUTION

Let the radius of the sphere be r cm. Then,

surface area = 154 cm^2

$\Rightarrow \quad 4\pi r^2 = 154$

$\Rightarrow \quad 4 \times \frac{22}{7} \times r^2 = 154$

$\Rightarrow \quad r^2 = \frac{154 \times 7}{4 \times 22} = \frac{49}{4}$

$\Rightarrow \quad r = \frac{7}{2} \text{cm}$

Let V be the volume of the sphere. Then,

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} \text{cm}^3 = \frac{1}{3} \times 11 \times 7 \times 7 \text{cm}^3 = 179.66 \text{cm}^3$$

MISCELLANEOUS

Solved Examples

Example 1 : **Find the area of a triangle, two of its sides are of length 6 cm and 12 cm and the perimeter is = 26 cm.**

SOLUTION

Here, $a = 6$ cm

$b = 12$ cm

and Perimeter = 26 cm

Let c be the length of third side

Now, Perimeter = $a + b + c$

$$26 = 6 + 12 + c$$

$$c = 26 - 18 = 8 \text{ cm}$$

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{6+12+8}{2} = 13 \text{ cm}$$

$$\therefore \quad \text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{13(13-6)(13-8)(13-12)} = \sqrt{13 \times 7 \times 5 \times 1}$$

$$= \sqrt{13 \times 35} = \sqrt{455} \text{ cm}^2$$

Example 2 : **The sides of a triangular plot are in the ratio 3 : 5 : 7 and its perimeter is 300 m. Find its area.**

SOLUTION

The sides are in the ratio 3 : 5 : 7. So let the sides are $3x$, $5x$ and $7x$ respectively.

Now perimeter = 300 m

$$\Rightarrow \quad 3x + 5x + 7x = 300$$

$$15x = 300$$

$$x = \frac{300}{15} = 20$$

So the sides are 60m, 100m and 140m

$a = 60$ m

$b = 100$m

$c = 140$ m

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{300}{2} = 150\text{m}$$

$$\therefore \quad \text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{150(150-60)(150-100)(150-140)}$$

$$= \sqrt{150 \times 90 \times 50 \times 10} = 1500\sqrt{3} \text{ m}^2$$

Example 3 : **A traffic signal board, indicating 'SCHOOL AHEAD' is an equilateral triangle with side's length a. Find the area of the signal board, using Heron's Formula. If its perimeter is 180 cm.**

SOLUTION

The traffic signal is in the form of an equilateral triangle having side's length 'a' cm.

$$\therefore \; s = \frac{a+a+a}{2} = \frac{3}{2}a$$

$\therefore$ Area of triangle using Heron's Formula

$$\therefore = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{\frac{3}{2}a\left(\frac{3}{2}a-a\right)\left(\frac{3}{2}a-a\right)\left(\frac{3}{2}a-a\right)}$$

$$= \sqrt{\frac{3}{2}a\times\frac{a}{2}\times\frac{a}{2}\times\frac{a}{2}} = \frac{\sqrt{3}}{4}a^2$$

If perimeter of triangle is 180 cm i.e. $a + a + a = 180$

$$\therefore \quad a = \frac{180}{3} = 60 \text{ cm}$$

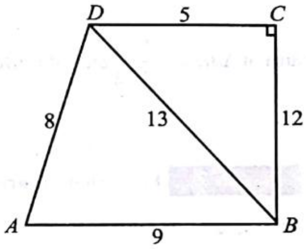
$$\therefore \quad \text{Area of triangle} = \frac{\sqrt{3}}{4}\times 60\times 60$$

$$= \sqrt{3}\times 15\times 60 = 900\sqrt{3} \text{ cm}^2$$

Example 4 : A park, in the shape of a quadrilateral $ABCD$, has $\angle C = 90°$, $AB = 9$ m, $BC = 12$ m, $CD = 5$ m, and $AD = 8$ m. How much area does it occupy?

SOLUTION

Given, a quadrilateral $ABCD$ in which $AB = 9$m, $BC = 12$ m, $CD = 5$m, and $AD = 8$m.
We divide the quadrilateral in two triangular region ABD and BCD.
Now, In ΔBCD it angle at C. We have



$$BD^2 = BC^2 + CD^2 \quad \text{[By pythagorous theorem]}$$

$$= 5^2 + 12^2 = 25 + 144 = 169 \text{ m}^2$$

$$BD = \sqrt{169} = 13\text{m}$$

$$\therefore \quad \text{Area of } \Delta BCD = \frac{1}{2}\times \text{base}\times\text{height} = \frac{1}{2}\times 5\times 12 = 30 \text{ m}^2$$

Note : This area can also be find out using Heron's formula.

Now, for triangle ABD, we have

$a = 9$ m, $b = 13$m, $c = 8$m

$$s = \frac{a+b+c}{2} = \frac{9+13+8}{2} = 15m$$

$$\therefore \quad \text{Area of } \Delta ABD = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{15\times 6\times 2\times 7} = \sqrt{3\times 5\times 2\times 3\times 2\times 7} = 3\times 2\sqrt{5\times 7} = 6\sqrt{35} \text{ m}^2$$

$$\therefore \quad \text{Area of quadrilateral } ABCD = \text{Area of } \Delta BCD + \text{area of } \Delta ABD = (30 + 6\sqrt{35}) \text{ m}^2$$

Example 5 : In figure, $ABCD$ is a parallelogram $AE \perp DC$ and $CF \perp AD$. If $AB = 16$ cm, $AE = 8$ cm and $CF = 10$ cm find AD.

SOLUTION

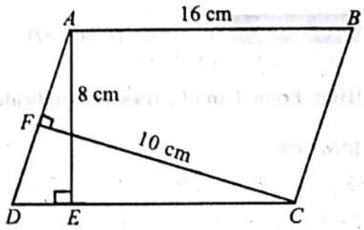
We have $AB = 16$ cm, $AE = 8$ cm, $CF = 10$ cm
We know that, Area of parallelogram = base × height

$$ABCD = CD\times AE = 16\times 8 = 128 \text{ cm}^2$$

Again Area of parallelogram = base × height = $AD \times CF$

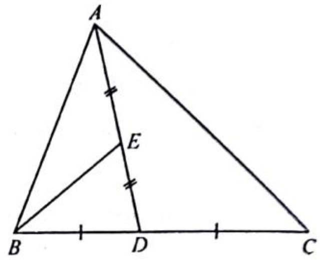
$$128 = AD\times 10$$

$$AD = \frac{128}{10} = 12.8 \text{ cm.}$$



Example 6 : ***ABC* is a triangle in which *D* is the mid-point of *BC* and *E* is the mid-point of *AD*. Prove that the area of $\Delta BED = \frac{1}{4}$ area of ΔABC.**

SOLUTION



Given : A ΔABC in which D is the mid-point of BC and E is the mid-point of AD.

To prove : ar $(\Delta BED) = \frac{1}{4}$ ar (ΔABC)

Proof : AD is a median of ΔABC

ar (ΔABD) = ar (ΔADC)

($\because$ Median of a Δ divides it into two Δs of equal area)

$= \frac{1}{2}$ ar (ΔABC)

Again, BE is a median of ΔABD,

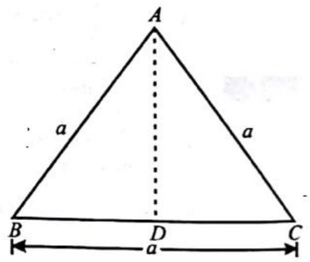
$\therefore$ ar (ΔBEA) = ar (ΔBED)

($\because$ Median of a Δ divides it into two Δs of equal area)

area of $\Delta BED = \frac{1}{2}(\frac{1}{2}$ area of $\Delta ABC) = \frac{1}{4}$. area of ΔABC

Example 7 : **Prove that the area of an equilateral triangle is equal to $\frac{\sqrt{3}}{4}a^2$, where a is the side of the triangle.**

SOLUTION



Draw $AD \perp BC$

$\Rightarrow \Delta ABD \cong \Delta ACD$ (By R.H.S.)

$\therefore BD = DC$ (CPCT)

$\because BC = a$

$\therefore BD = DC = \frac{a}{2}$

In right angled ΔABD

$$AD^2 = AB^2 - BD^2 = a^2 - \left(\frac{a}{2}\right)^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4} \quad \Rightarrow \quad AD = \frac{\sqrt{3}a}{2}$$

$$\text{Area of } \Delta ABC = \frac{1}{2}BC \times AD = \frac{1}{2}a\frac{\sqrt{3}a}{2} = \frac{\sqrt{3}a^2}{4}$$

Example 8 : **Diagonals *AC* and *BD* of a quadrilateral *ABCD* intersect each other at *P*. Show that ar $(\Delta APB) \times$ ar(ΔCPD) = ar $(\Delta APD) \times$ ar (ΔBPC).**

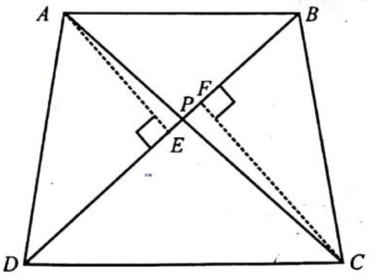
[Hint. From *A* and *C*, draw perpendiculars to *BD*.]

SOLUTION

Given : In a quadrilateral $ABCD$, diagonals AC and BD intersect each other at P.

To Prove : ar $(\Delta APB) \times$ ar(ΔCPD) = ar$(\Delta APD) \times$ ar (ΔBPC).

PROOF : From A and C, draw perpendicualrs AE and CF respectively to BD.



$$\text{ar}(\Delta APB) \times \text{ar}(\Delta CPD)\ \frac{(PB)\,(AE)}{2} \times \left(\frac{DP \times CF}{2}\right) = \frac{1}{4} = (PB)\,(AE)\,(DP)\,(CF) \quad ....(1)$$

$$\text{and ar}(\Delta APD) \times \text{ar}(\Delta BPC)\ \frac{(DP)(AE)}{2} \times \frac{(PB)(CF)}{2} = \frac{1}{4} = (PB)\,(AE)\,(DP)\,(CF) \quad ....(2)$$

From (1) and (2),

ar (ΔAPB) × ar (ΔCPD) = ar(ΔAPD) × ar (ΔBPC).

Hence proved.

Example 9 : **Find the area of the quadrilateral *ABCD in* which $AB = 12$ cm, $BC = 6$ cm, $CD = 7$ cm $BD = 9$ cm and $AD = 15$ cm.**

SOLUTION

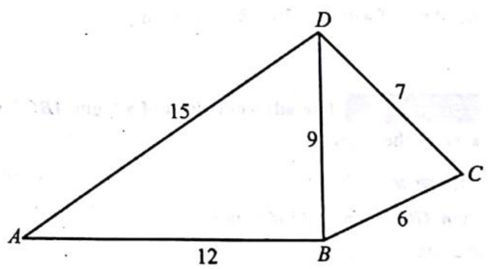
In the quadrilateral $ABCD$ diagonal BD divides it into two triangles ABD and BCD.

$\therefore$ Area of quadrilateral $ABCD$ = Area of $\Delta\, ABD$ + Area of $\Delta\, BCD$ ...(i)

For $\Delta\, ABD$,

$a = 12$ cm, $b = 9$ cm $c = 15$ cm

$$s = \frac{a+b+c}{2} = \frac{12+9+15}{2} = 18\text{cm}$$



Applying Heron's formula for $\Delta\, ABD$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{18(18-12)(18-9)(18-15)}$$
$$= \sqrt{18 \times 6 \times 9 \times 3}$$
$$= 18 \times 3 = 54 \text{ cm}^2$$

For ΔBCD,

$a = 6$ cm, $b = 7$ cm, $c = 9$ cm

$$s = \frac{6+7+9}{2} = 11\text{cm}$$

Applying Heron's formula for $\Delta\, BCD$.

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{11(11-6)(11-7)(11-9)} = \sqrt{11 \times 5 \times 4 \times 2}$$
$$= \sqrt{440} = 20.98 \text{ cm}^2$$

$\therefore$ Area of Quadrilateral $ABCD = 54 + 20.98$ [from (i)]

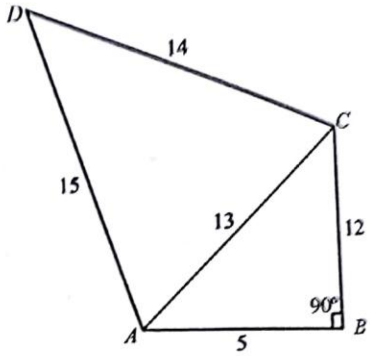
$= 74.98 \text{ cm}^2$

Example 10 : The sides of quadrilateral, takes in order are 5,12,14 and 15 meters respectively and the angle contained by the first two sides is 90° find the area of the quadrilateral.

SOLUTION

Let $ABCD$ be the quadrilateral

Such that $AB = 5$cm, $BC = 12$ cm, $CD = 14$cm and $DA = 15$ cm and $\angle ABC = 90°$



We divide the quadrilateral $ABCD$ into two triangle ABC and ACD

For ΔABC

$$\text{Area of } \Delta ABC = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 5 \times 12 = 30 \text{ cm}^2$$

Now, in ΔABC

$$AC^2 = AB^2 + BC^2 \quad \text{[Using Pythagorous theorem]}$$

$$= 5^2 + 12^2 = 25 + 144 = 169$$

$$AC = \sqrt{169} = 13 \text{ cm}$$

In ΔACD,

$$a = 13, b = 14, c = 15 \text{ cm}$$

$$s = \frac{13+14+15}{2} = \frac{42}{2} = 21\text{cm}$$

Applying Heron's formula

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{21(21-13)(21-14)(21-15)} = \sqrt{21 \times 8 \times 7 \times 6}$$

$$= \sqrt{\underline{7} \times \underline{3} \times 2 \times 2 \times 2 \times \underline{7} \times 2 \times \underline{3}}$$

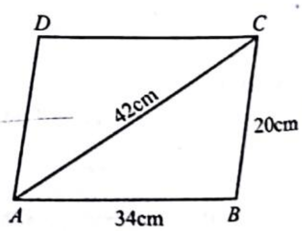
$$= 7 \times 3 \times 2 \times 2 = 21 \times 4 = 84 \text{ cm}^2$$

So, Area of $ABCD = 30 + 84 = 114 \text{ cm}^2$

Example 11 : The adjacent sides of a || gm $ABCD$ measure are 34 cm, and 20 cm and the diagonal AC is 42 cm. Find the area of the || gm.

SOLUTION

|| gm $ABCD$ can be divided into two triangles ABC and ACD.



Now, $ABCD$ is a || gm

So, $AB = CD = 34$ cm

$BC = DA = 20$ cm

In ΔABC,

$$a = 34 \text{ cm}, b = 20 \text{ cm}, c = 42 \text{ cm}$$

$$s = \frac{a+b+c}{2} = \frac{34+20+42}{2} = 43 \text{ cm}$$

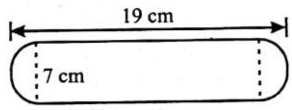
Applying Heron's formula

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{43(43-34)(43-20)(43-42)}$$

$$\text{Area of || gm } ABCD = 2 \times \text{Area of } \Delta ABC$$

Example 12 : A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 19 cm and diameter of the cylinder is 7 cm. Find the volume and total surface area of the solid.

SOLUTION



Total height of solid = 19 cm,

Diameter of cylinder = diameter of hemisphere = 7 cm

$\therefore$ Radius of cylinder = radius of hemisphere = 3.5 cm.

Height of cylindrical part = [19 – 2 (radius of hemisphere)] cm.

$= 19 - 2 \times 3.5 = 12$ cm.

Now volume of solid = Volume of cylinder + volume of two hemispheres

$$\Rightarrow \quad \text{Volume} = \pi r^2 h + 2 \times \frac{2}{3}\pi r^3 = \pi r^2\left(h + \frac{4r}{3}\right) = \frac{22}{7} \times (3.5)^2\left[12 + \frac{4 \times 3.5}{3}\right] = 641.67 \text{ cm}^3$$

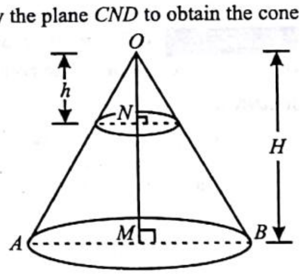
Now surface area of solid = Curved surface area of cylinder + curved surface area of two hemisphere

$$\Rightarrow \quad S = 2\pi r h + 2 \times 2\pi r^2 = 2\pi r\,(h + 2r) = 2 \times \frac{22}{7} \times 3.5\,(12 + 7) = 418 \text{ cm}^2$$

Example 13 : The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to its base. If its volume be 1/27 of the volume of the give cone, at which height above the base is the section cut ?

SOLUTION

Let OAB be the given cone of height, $H = 30$ cm, and base radius R cm. Let this cone be cut by the plane CND to obtain the cone OCD with height h cm and base radius r cm



Then, Δ OND ~ Δ OMB

So, $\dfrac{ND}{MB} = \dfrac{ON}{OM} \Rightarrow \dfrac{r}{R} = \dfrac{h}{30}$ ....(i)

Volume of $OCD = \dfrac{1}{27} \times$ Volume of cone OAB (Given)

$$\Rightarrow \quad \frac{1}{3}\pi r^2 h = \frac{1}{27} \times \frac{1}{3}\pi R^2 \times 30 \Rightarrow \left(\frac{r}{R}\right)^2 = \frac{10}{9h} \Rightarrow \left(\frac{h}{30}\right)^2 = \frac{10}{9h} \quad \text{[From (i)]}$$

$\Rightarrow \quad 9h^3 = 9000 \quad \Rightarrow \quad 1000 \quad \Rightarrow \quad h = 10$

$\therefore$ Height of the cone $OCD = 10$ cm.

Hence, the section is cut at the height of (30 – 10) cm, i.e. 20 cm from the base.

Example 14 : The surface area of a solid metallic sphere is 1256 cm². It is melted and recase into solid right circular cones of radius 2.5 cm and height 8 cm. Calculate (i) the radius of the solid sphere, (ii) the number of cones recase (Take $\pi = 3.14$)

SOLUTION

(i) Let the radius of the sphere be r cm.

Then, its surface area = $(4\pi r^2)$ cm^2

$\therefore \quad 4\pi r^2 = 1256 \Rightarrow 4 \times 3.14 \times r^2 = 1256$

$$\Rightarrow \quad r^2 = \frac{1256}{4 \times 3.14} = \frac{1256}{12.56} = 100 \Rightarrow r = 10$$

Hence, the radius of the sphere = 10 cm.

(ii) Volume of the sphere = $\dfrac{4}{3}\pi r^3 = \left[\dfrac{4}{3}\pi \times (10)^3\right]\text{cm}^3 = \left(\dfrac{4000}{3}\pi\right)\text{cm}^3$

Volume of a cone = $\dfrac{1}{3}\pi R^2 h = \left[\dfrac{1}{3}\pi \times (2.5)^2 \times 8\right]\text{cm}^3 = \left(\dfrac{50}{3}\pi\right)\text{cm}^3$

$\therefore$ Number of cones recase = $\dfrac{\text{Volume of the sphere}}{\text{Volume of 1 cone}} = \left(\dfrac{4000}{3}\pi \times \dfrac{3}{50\pi}\right) = 80$

Example 15 : **The volume of a rectangular block of stone is 10368 dm^3. Its length, breadth and height are in the ratio 3 : 2 : 1. Find its dimensions. Find also the cost of polishing its entire surface at 2 paise per dm^2.**

SOLUTION

Let the length of block = $3x$ dm
breadth of block = $2x$ dm
height of block = dx m

$\therefore$ Volume of the block of stone = $3x \times 2x \times x = 6x^3$ dm^3
But the given volume of the stone = 10368 dm^3

$$\therefore \quad 6x^3 = \frac{10368}{6} = 1728$$

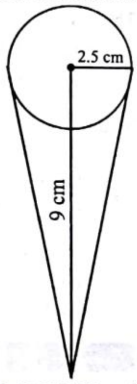
$$\Rightarrow \quad x = \sqrt[3]{1728} = (12\times12\times12)^{1/3} = (12^3)^{1/3} = 12$$

Hence, the length of the stone = 3 × 12 = 36 dm
breadth of the stone = 2 × 12 = 24 dm.
height of the stone = 1 × 12 = 12 dm.

Surface area of the block of stone = $2(\ell b + bh + h\ell)$
= 2 (36 × 24 + 24 × 12 + 12 × 36)
= 2 (864 + 288 + 432) = 2 × 1584 = 3168 dm^2

Total cost of polishing the entire surface at 2 paise per dm^2
= 3168 × 2 = 6336 = ₹ 63.36

Example 16 : **An ice-cream cone is the union of a right circular cone and a hemisphere that has the same circular base the cone. Find the volume of ice cream if the height of the cone is 9 cm and radius of its base is 2.5 cm.**



SOLUTION

Given height of the cone = 9 cm.

and radius of the base = 2.5 cm = $\frac{5}{2}$ cm.

$$\text{Volume of the cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\times\frac{22}{7}\times\frac{5}{2}\times\frac{5}{2}\times 9 = \frac{11\times25\times3}{7\times2}\text{cm}^3 = \frac{825}{14}\text{cm}^3$$

$$\text{Volume of the hemisphere} = \frac{2}{3}\pi r^3 = \frac{2}{3}\times\frac{22}{7}\times\frac{5}{2}\times\frac{5}{2}\times\frac{5}{2} = \frac{11\times125}{42}\text{cm}^3 = \frac{1375}{42}\text{cm}^3$$

$$\text{Volume of the ice cream cone} = \frac{825}{14}+\frac{1375}{42} = \frac{2475+1375}{42} = \frac{3850}{42}\text{cm}^3 = 91.66\text{ cm}^3$$

Example 17 : **A room is half as long again as it is broad. The cost of carpeting the room at ₹ 3.25 per m^2 is ₹ 175.50 and the cost of papering the walls at ₹ 1.40 per m^2 is ₹ 240.80. If 1 door and 2 windows occupy 8m^2, find the dimensions of the room.**

SOLUTION

Let the breadth of the room be x m. Then,

$$\text{Length} = x + \frac{x}{2}m = \frac{3x}{2}m$$

$$\therefore \quad \text{Area of the room} = x \times \frac{3x}{2}m^2 = \frac{3x^2}{2}m^2$$

$$\Rightarrow \quad \text{Cost of carpeting the room at the rate of ₹ 3.25 per m}^2 = ₹\left(\frac{3x^2}{2}\times3.25\right)$$

$$\therefore \quad \frac{3x^2}{2}\times3.25 = 175.50 \qquad [\because \text{ cost of carpeting} = ₹\ 175.50\text{ (given)}]$$

$$\Rightarrow \quad x^2 = \frac{175.50\times2}{3\times3.25} = \frac{351}{3\times3.25} = \frac{351}{6.5} = \frac{3510}{65} = 36$$

$$\Rightarrow \quad x = 6$$

Thus, breadth = 6m and length = $\left(6+\frac{6}{2}\right)$m = 9m

Let the height of the room be h metres. Then,

Area of 4 walls = 2 (length + breadth) × height = 2 (6 + 9) × h m^2 = 30 h m^2

Area of 1 door and 2 windows = 8 m^2

∴ Area to be papered = Area of 4 walls – Area of 1 door and 2 windows = (30 h – 8)m^2

∴ Cost of papering walls at ₹ 1.40 per m^2 = ₹ (30h – 8) × 1.40

$$\Rightarrow (30h - 8) \times 1.40 = 240.80$$

$$\Rightarrow (30h - 8) = \frac{240.80}{1.40}$$

$$\Rightarrow 30h - 8 = \frac{24080}{140}$$

$$\Rightarrow 30h - 8 = \frac{2408}{14} \Rightarrow 30h - 8 = 172 \Rightarrow 30h = 180 \Rightarrow h = 6$$

∴ Height = 6m

Hence, the dimensions of the room are :

Length = 9m, breadth = 6m and height = 6m

Example 18 : Water in a canal, 30 dm wide and 12 dm deep, is flowing with a velocity of 20 km per hour. How much area will it irrigate in 30 min, if 9 cm of standing water is desired?

SOLUTION

Water in the canal forms a cuboid of breadth = 12 dm = 12/10 m = 1.2m, height = 30 dm = 30/10 m = 3m

and, Length = Distance covered by water in 30 minutes

= velocity of water × time

$$= 20000 \times \frac{30}{60} \text{ m} = 10000 \text{ m}$$

∴ Volume of water flown in 30 min = lbh = (1.2 × 3 × 10000) m^3 = 36000m^3

Suppose area irrigated be Am^2. Then,

$$\Rightarrow A \times \frac{9}{100} = 36000 \Rightarrow A = 400000 \text{ m}^2$$

Hence, area irrigated = 400000 m^2

Example 19 : Water in a rectangular reservoir having base 80 m by 60 m is 6.5 m deep. In what time can the water be emptied by a pipe of which the cross-section is a square of side 20 cm, if the water runs through the pipe at the rate of 15 km/hr.

SOLUTION

Given : Dimension of the reservoir is 80 m, 60m and 6.5 m

So its volume will be

$$V = 60 \times 80 \times 6.5 = 48 \times 650 \text{ m}^3$$

Now, Dimension of pipe is 20 cm, 20 cm.

So its area of cross-section = $\frac{20}{100} \times \frac{20}{100}$ m^2

Rate of flow of water through the pipe = 15 km hr^{-1} = 15000 m hr^{-1}

In 1 hr volume of water that flow out through the pipe = $15000 \, \frac{20}{100} \times \frac{20}{100}$ m^3

Let after 't' hours the whole reservoir becomes emptied so the volume of water that flows out through the pipe after 't' hours

$$= 15000 \times \frac{20}{100} \times \frac{20}{100} \times t \text{ m}^3$$

$$\therefore \quad 15000 \times \frac{20}{100} \times \frac{20}{100} \times t = 48 \times 650$$

$$600\, t = 48 \times 650$$

$$t = \frac{48 \times 650}{600} = 52 \text{ hr}$$

So the required time will be 52 hours.

Example 20 : A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled into it. The diameter of the pencil is 7mm, the diameter of the graphite is 1 mm and the length of the pencil is 14 cm. Find the: (i) Volume of the graphite (ii) Volume of the wood (iii) The weight of the whole pencil, if the specific gravity of the wood is 0.7 gm/ cm^3 and that of the graphite is 2.1 gm /cm^3.

SOLUTION

(i) We have,

Diameter of the graphite cylinder = 1 mm = $\frac{1}{10}$cm

$\therefore$ Radius of the graphite cylinder = $\frac{1}{20}$cm

Length of the graphite cylinder = 14 cm

V_1 = Volume of the graphite cylinder = $\frac{22}{7} \times \frac{1}{20} \times \frac{1}{20} \times 14\text{cm}^3 = 0.11\text{cm}^3$

(ii) We have,

Diameter of pencil = 7 mm = $\frac{7}{10}$cm

Radius = $\frac{7}{20}$ cm

$\therefore$ V_2 = Volume of pencil = $\frac{22}{7} \times \frac{7}{20} \times \frac{7}{20} \times 14\text{cm}^3 = 5.39\text{cm}^3$

Volume of wood $V_2 - V_1 = (5.39 - 0.11)$ cm^3 = 5.28 cm^3

(iii) We have,

Specific gravity of wood = 0.7 gm/ cm^3

and, specific gravity of wood graphite = 2.1 gm/cm^3

$\therefore$ Weight of the pencil = Volume of wood × Specific gravity of wood + Volume of graphite × Specific gravity of wood graphite

$$= (5.28 \times 0.7 + 0.11 \times 2.1) \text{ gm} = 3.927 \text{ gm}$$

Example 21 : Water flows out through a circular pipe whose internal diameter is 2 cm, at the rate of 6 metres per second into a cylindrical tank. The radius of whose base is 60 cm. Find the rise in the level of water in 30 minutes?

SOLUTION

Internal diameter of the pipe = 2 cm

So its radius = 1 cm = $\frac{1}{100}$ m

Water that flows out through the pipe is 6 ms^{-1}

So volume of water that flows out through the pipe in 1 sec = $\pi \times \left(\frac{1}{100}\right)^2 \times 6$ m^3

$\therefore$ In 30 minutes, volume of water flow = $\pi \frac{1}{100 \times 100} \times 6 \times 30 \times 60$ m^3

This must be equal to the volume of water that rises in the cylindrical tank after 30 min and height up to which it rises say h.

$\therefore$ Radius of tank = 60 cm

$$= \frac{60}{100} \text{ m}$$

$$\text{volume} = \pi\left(\frac{60}{100}\right)^2 h$$

$$\pi\left(\frac{60}{100}\right)^2 h = \pi \times \frac{1}{100 \times 100} \times 6 \times 30 \times 60$$

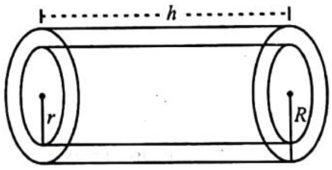
$$\frac{60 \times 60}{100 \times 100} h = \frac{6 \times 30 \times 60}{100 \times 100}$$

$$h = \frac{3 \times 36}{36} = 3 \text{ m}$$

So required height will be 3 m.

Example 22 : A cylindrical tube, open at both ends, is made of metal. The internal diameter of the tube is 10.4 cm and its length is 25 cm. The thickness of the metal is 8 mm everywhere. Calculate the volume of the metal.

SOLUTION



Internal diameter of the tube = 10.4 cm

Hence internal radius (r) = 5.2 cm

h = length of the tube = 25 cm

Let t = thickness = 8 mm = $\frac{8}{10}$ cm = 0.8 cm

So, external radius (R) = 5.2 + 0.8 = 6 cm

Now, volume of the metal which is the volume of the shaded protion

$$= \pi R^2 h - \pi r^2 h = \pi h(R^2 - r^2)$$

Putting the values, we get

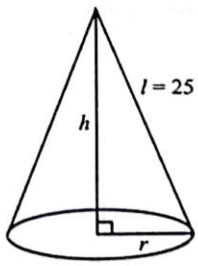
$$V = \frac{22}{7} \times 25 \times (6^2 - 5.2^2)$$

$$= \frac{22}{7} \times 25 \times (6 + 5.2)(6 - 5.2)$$

$$= \frac{22}{7} \times 25 \times 11.2 \times 0.8 = 704 \text{ cm}^3$$

Example 23 : Monica has piece of Canvas whose area is 551 m^2. She uses it to have a conical tent made, with a base radius of 7 m. Assuming that all the stitching margins and wastage incurred while cutting, amounts to approximately 1 m^2. Find the volume of the tent that can be made with it.

SOLUTION



Surface area of canvas = 551 m^2

Area which wastes while cutting is 1 m^2

so the area remaining = 551 − 1 = 500 m^2

Radius of the base r = 7 m

As the canvas in the form of a cone.

So its surface i.e. its curved surface area will be $\pi r l$

$$\pi r l = 550 \text{ m}^2$$

$$\frac{22}{7} \times 7 \times \ell = 550 \quad [l = \text{Slant height of the cone}]$$

$$l = \frac{550}{22} = \frac{55 \times 10}{22}$$

$$l = 25$$

$$h^2 + r^2 = l^2$$

$$h^2 = l^2 - r^2 = 28^2 - 7^2 = 625 - 49 = 576$$

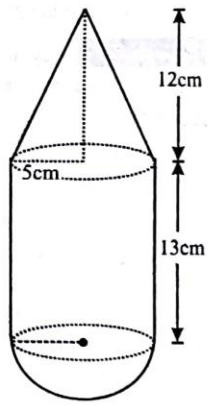
$$h = \sqrt{576} = 24 \text{ m}$$

So, the volume of the required tent will be $V = = \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24 = 22 \times 56 = 1232 \text{ m}^3$$

Example 24 : A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The height and radius of the cylindrical part are 13 cm and 5 cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Calculate the surface area of the toy if height of the conical part is 12 cm.

SOLUTION



Let r cm be the radius and h cm the height of the cylindrical part. Then,

$r = 5$ cm and $h = 13$ cm.

Clearly, radii of the spherical part and base of the conical part are also r cm. Let h_1 cm be the height, l cm be the slant height of the conical part. Then,

$$l^2 = r^2 + h_1^2$$

$$\Rightarrow \quad l = \sqrt{r^2 + h_1^2} \Rightarrow l = \sqrt{5^2 + 12^2} = 13 \text{ cm} \qquad [\because h_1 = 12 \text{ cm}, r = 5 \text{ cm}]$$

Now,

Surface area of the toy = Curved surface area of the cylindrical part

+ Curved surface area of hemispherical part

+ Curved surface area of conical part

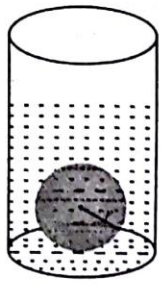
$$= (2\pi rh + 2\pi r^2 + \pi rl) \text{ cm}^2$$

$$= \frac{22}{7} \times 5 \times (2 \times 13 + 2 \times 5 + 13) \text{ cm}^2$$

$$= \frac{22}{7} \times 5 \times 49 \text{cm}^2 = 770 \text{cm}^2$$

Example 25 : A cylindrical tub of radius 12 cm contains water to a depth of 20 cm. A spherical ball is dropped into the tub and thus the level of water is raised by 6.75 cm. What is the radius of the ball ?

SOLUTION



Given : R^2 = radius of cylindrical tub = 12 cm

H = height of water level = 20 cm

h = level of water which raises = 6.75 cm

Let r = radius of spherical ball

Now, volume of spherical ball = volume of water level raised

$$\frac{4}{3}\pi r^3 = \pi R^2 h$$

$$r^3 = \frac{3}{4} \times 12 \times 12 \times 6.75 = 729$$

$$r = (729)^{1/3} = 9 \text{ cm}$$

EXERCISE 1

FIB Fill in the Blanks

DIRECTIONS: *Complete the following statements with an appropriate word/term to be filled in the blank space(s).*

1. __________ of a solid is the amount of space enclosed by the bounding surface.
2. The volume of a rectangular solid measuring 1 m by 50 cm by 0.5 m is __________ cm^3.
3. The sum of the areas of the plane and curved surfaces (faces) of a solid is called its __________ surface area.
4. The word solid sphere is used for the solid whose surface is a __________.
5. The curved surface area of a right circular cone whose slant height is 10 cm and base radius is 7 cm is __________.
6. A sphere has only __________ surface and that is curved.
7. Cube is a special form of __________
8. A right circular cone is generated by revolving a right angled triangle about one of the sides containing the __________
9. When a right angled triangular lamina is revolved about one of its sides (other than hypotenuse), then the solid so formed is called a __________
10. The solid bounded by two concentric spherical surfaces is called a __________.
11. Volume of a cylinder is three times the volume of a __________ on the same base and of the same height.
12. The volume of a sphere is __________ to two-thirds the volume of a cylinder of the same height and diameter.
13. In a cuboid, any three coterminal edges are mutually __________.

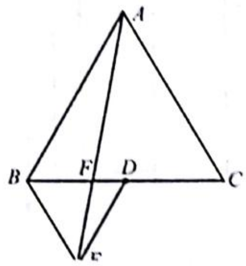
T/F True / False

DIRECTIONS: *Read the following statements and write your answer as true or false.*

1. Area of a quadrilateral whose sides and one diagonal are given, can be calculated by dividing the quadrilateral into two triangles and using the Heron's formula.
2. Sides of a triangle are in the ratio of 12 : 17 : 25 and its perimeter is 540 cm. Its area is 8000 cm^2.
3. Area of a triangle whose sides are 13 cm, 14 cm and 15 cm is 84 cm^2.
4. Area of a quadrilateral $ABCD$ in which AB = 3 cm, BC = 4cm, CD = 4 cm, DA = 5 cm and AC = 5 cm is 15 cm^2.
5. Area of the triangle whose two sides are 8m and 11m and perimeter is 32 m. is $8\sqrt{30}$ m^2.
6. The sides of a quadrilateral taken in order are 5m, 12m, 14m and 15m. If the angle between the first two sides be 90°, its area 114m^2.
7. Area of triangle with sides a, b, c is $\sqrt{(s-a)(s-b)(s-c)}$
8. In triangle, sum of two side is always greater than third side.
9. Triangle with sides 10, 10 and 20 cm is not possible.
10. Heron's formula cannot be use to calculate area of quadrilaterals.
11. If the diagonals of a quadrilateral divide it into four triangles which are equal in area, then the quadrilateral must be a parallelogram.
12. If the adjacent angles of a rhombus of side 10 cm are 120° and 60°, then its area is $25\sqrt{3}$ cm^2.
13. The three altitudes of an equilateral triangle are equal in length.
14. The areas of the triangles having a common side are proportional to their altitude to the common side.
15. If triangles of equal area have a common base, then their vertices must lie on a line parallel to the base.
16. If P is any point in the interior of a rectangle ABCD, then Area (ΔPAB) + Area (ΔPCD) = Area (ΔPBC) + Area (ΔPDA).
17. If two triangles have a common vertex and their bases lie on the same line, then their areas are proportional to the lengths of their bases.
18. A median of a triangle divides it into two triangles of equal areas.
19. The area of a triangle = 2 × Base × height
20. Parallelograms on the same base and between the same parallels are different in area.

MTC Match the Columns

DIRECTIONS: *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D, ...) in Column I have to be matched with statements (p, q, r, s, ...) in column II.*



1.

Column-I	Column-II
(A) ar (ΔBDE)	(p) ar(ΔAFD)
(B) ar (ΔBDE)	(q) $\frac{1}{4}$ar(ΔABC)
(C) ar(ΔABC)	(r) $\frac{1}{8}$ar(ΔAFC)
(D) ar(ΔBFE)	(s) 2ar(ΔBEC)
(E) ar(ΔFED)	(t) $\frac{1}{2}$ar(ΔBAE)

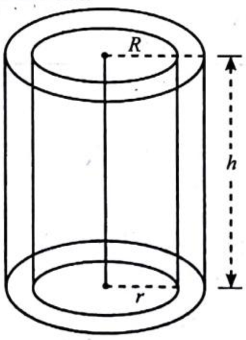
2.

Column-I	Column-II
(A) Total surface area of right circular cylinder is	(p) $\frac{2}{3}\pi r^3$
(B) Total surface area of right circular cone is	(q) $\frac{4}{3}\pi r^3$
(C) Total surface area of sphere is	(r) $l \times b \times h$
(D) Total surface area of hemisphere is	(s) $\frac{1}{3}\pi r^2 h$
(E) Volume of cuboid is	(t) $\sqrt{l^2+b^2+h^2}$
(F) Volume of cylinder is	(u) $\pi r^2 h$
(G) Diagonal of cuboid is	(v) $3\pi r^2$
(H) Volume of sphere is	(w) $4\pi r^2$
(I) Volume of cone is	(x) $\pi r(l+r)$
(J) Volume of hemisphere is	(y) $2\pi r(r+h)$

VSAQ Very Short Answer Questions

DIRECTIONS: *Give answer in one word or one sentence.*

1. Two parallelograms are no equal bases and between the same parallels. Find the ratio of their areas.
2. State formula for evaluating the area of a rhombus.
3. What is 'S' in Heron's formula?
4. Study the figure given below carefully and complete the statements based on it



(a) R and r are the ______ and ______ of a hollow cylinder respectively and h is its ______.

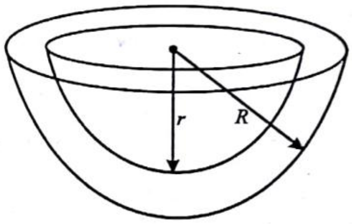
(b) Thickness of cylinder is ______. (formula)

(c) External curved surface area is ______. (formula)

(d) Total surface area = ______ + ______ + ______ = ______ ______. (formula)

(e) Area of a cross-section is ______. (formula)

5. Study the figure given below and fill the blanks based on it.



Hemispherical shell

(a) The solid enclosed between two concentric ______ is called a hemispherical shell.

(b) R and r are the radii of the ______ and the ______ hemispheres respectively.

(c) Area of base is ______. (formula)

(d) Total surface area is ______. (formula)

(e) Thickness of shell is ______. (formula)

SAQ Short Answer Questions

DIRECTIONS: *Give answer in 2-3 sentences.*

1. The total surface area of a cube is 864 cm^2. Find its edge and lateral surface area.
2. The paint in a certain container is sufficient to paint an area equal to 9.375 m^2. How many bricks of dimensions 22.5 cm × 10 cm × 7.5 cm can be painted out of paint in this container ?
3. A cylindrical pillar is 50 cm in diameter and 3.5 m in height. Find the cost of painting the curved surface of the pillar at the rate of ₹ 12.50 per m^2.
4. In a hot water heating system, there is a cylindrical pipe of length 28 m and diameter 5 cm. Find the total radiating surface in the system.

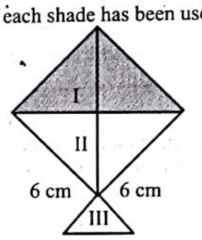
5. In the given figure, you see the frame of a lampshade. It is to be covered with a decorative cloth. The frame has a base diameter of 20 cm and height of 30 cm. A margin of 2.5 cm is to be given for folding it over the top and bottom of the frame. Find how much cloth is required for making the lampshade.

6. A right circular cylinder just encloses a sphere of radius 7 cm.
   (i) Find the curved surface area of the cylinder and the sphere.
   (ii) What is the ratio of these areas?
7. The total surface area of a cube is 486 cm^2. Find its volume.
8. A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour. How much water will fall into the sea in a minute ?
9. The total surface area of a cylinder is 231 cm^2 and its curved surface area is $\frac{2}{3}$ of the total surface area. Find the volume of the cylinder.
10. A leather football has inner radius 7 cm and the thickness of the leather is 0.25 cm. If the density of the leather is 2.5 g per cm^3, find the mass of the ball.
11. An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.
12. The perimeter of a triangular field is 450 m and its sides are in the ratio 13 : 12 : 5. Find the area of the triangle.
13. At a Ramzan mela, a stall keeper in one of the food stalls has large cylindrical vessel of base radius 15 cm filled up to a height of 32 cm with orange juice. The juice is filled in small cylindrical glasses of radius 3 cm upto a height of 8 cm and sold for ₹ 3 each. How much money does the stall keeper receive by selling the juice completely.
14. An umbrella is made by stitching 10 triangular pieces of cloth of two different colours (See Fig.), each piece measuring 20 cm, 50 cm and 50 cm. How much cloth of each colour required for the umbrella?

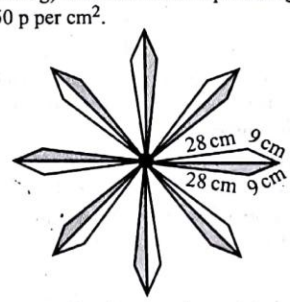
LAQ Long Answer Questions

DIRECTIONS: *Give answer in four to five sentences.*

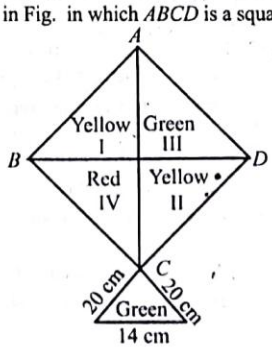
1. In the figure incircle with centre O of ΔABC touches the sides AB, BC and CA at F, D and E respectively. If radius of the incircle is 4 cm and $AF = 6$ cm, $BF = 8$ cm. Find the length of side BC and AC.
2. The sides of a triangle are 4, 5 and 6 cm. Find the area of the triangle.
3. One side of an equilateral triangle is 6 cm. Find its area by using Heron's formula. Find its altitude also.
4. An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.
5. The lengths of two adjacent sides of a parallelogram are respectively 40 cm and 50 cm. One of its diagonals is 20 cm. Find the area of the parallelogram.
6. Find the area of a quadrilateral $ABCD$ in which $AB = 3$ cm, $BC = 4$ cm, $CD = 4$ cm, $DA = 5$ cm and $AC = 5$ cm.
7. A kite in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and sides 6 cm each is to be made of three different shades as shown in figure. How much paper of each shade has been used in it?



8. A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm(see fig). Find the cost of polishing the tiles at the rate of 50 p per cm^2.



9. Find the area of a triangle whose sides are 26 cm, 28 cm and 30 cm.
10. How much paper of each shade is needed to make a kite given in Fig. in which $ABCD$ is a square with diagonal 44 cm.



EXERCISE 2

Multiple Choice Questions

DIRECTIONS: *This section contains multiple choice questions. Each questions has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. Area of the triangle whose sides are 13 cm, 9 cm and 6 cm is
(a) 23.6 cm^2 (b) 26.3 cm^2
(c) 36.34 cm^2 (d) 23.66 cm^2

2. Area of an isosceles triangle, the measure of one of its equal side being 5 cm. and the third side 4 cm is
(a) $2\sqrt{21}$ cm^2 (b) $21\sqrt{2}$ cm^2
(c) $22\sqrt{3}$ cm^2 (d) $23\sqrt{3}$ cm^2

3. The sides of a triangular plot are in the ratio of 3 : 5 : 7 and its perimeter is 300m. Its area is
(a) $1500\sqrt{2}$ cm^2 (b) $1500\sqrt{3}$ cm^2
(c) $1425\sqrt{2}$ cm^2 (d) 1500 cm^2

4. Area of the quadrilateral $ABCD$ whose diagonal $AC = 15$ cm. and sides $AB = 7$ cm, $BC = 12$ cm, $CD = 12$ cm and $DA = 9$ cm is
(a) 25.9 cm^2 (b) 29.3 cm^2
(c) 95.2 cm^2 (d) 92.5 cm^2

5. The lengths of two adjacent sides of a parallelogram are 5 cm and 3.5 cm. One of its diagonals is 6.5 cm long. Area of the parallelogram is
(a) $13\sqrt{10}$ cm^2 (b) $23\sqrt{5}$ cm^2
(c) $10\sqrt{5}$ cm^2 (d) $10\sqrt{3}$ cm^2

6 Area of a trapezium whose parallel sides are 55 cm, 40 cm and non-parallel sides are 20 cm and 25 cm respectively is
(a) 590 cm^2 (b) 950 cm^2
(c) 595 cm^2 (d) 940 cm^2

7. A floral design on a floor is made up of 16 tiles which are triangular, the sides of a triangle being 9 cm, 28 cm, 35 cm. Cost of polishing the tiles at the rate of 50p. per cm^2 is
(a) ₹ 45 (approx.) (b) ₹ 405 (approx.)
(c) ₹ 450 (approx.) (d) ₹ 706 (approx.)

8. The base of a right triangle is 8 cm and hypotenuse is 10 cm. Its area will be
(a) 24 cm^2 (b) 40 cm^2
(c) 48 cm^2 (d) 80 cm^2

9. A regular hexagon has a side 6 cm. Its perimeter and area are
(a) 35cm, $8\sqrt{3}$ cm^2 (b) 38cm, $10\sqrt{2}$ cm^2
(c) 40cm, $11\sqrt{2}$ cm^2 (d) 36cm, $54\sqrt{3}$ cm^2

10. A triangular park ABC has sides 120m, 80m and 50m. A gardener has to put a fence all around it and also plant grass inside. Area of garden and cost of fencing the garden with barbed wire at the rate of ₹ 20 per metre leaving a space 3m wide for a gate on one side are
(a) $375\sqrt{15}$ m^2, ₹ 4940 (b) $357\sqrt{10}$ m^2, ₹ 9440
(c) $573\sqrt{8}$ m^2, ₹ 4944 (d) $683\sqrt{10}$ m^2, ₹ 5490

11. If the radius of a right circular cylinder, open at both the ends, is decreased by 25% and the height of the cylinder is increased by 25%, then the surface area of the cylinder thus formed
(a) remains unaltered (b) is increased by 25%
(c) is decreased by 25% (d) is decreased by 6.25%

12. If the inner dimensions of a cuboidal box are 50 cm × 40 cm × 30 cm, then the length of the longest rod that can be placed in the box is
(a) 50 cm (b) $50\sqrt{2}$ cm
(c) $50\sqrt{3}$ cm (d) 120 cm

13. A cuboid container has the capacity to hold 50 small boxes. If all the dimensions of the container are doubled, then it can hold small boxes of the same size.
(a) 100 (b) 200
(c) 400 (d) 800

14. A cylindrical pencil of diameter 1.2 cm. has one of its end sharpened into a conical shape of height 1.4 cm. The volume of the material removed is
(a) 1.056 cm^3 (b) 4.224 cm^3
(c) 10.56 cm^3 (d) 42.24 cm^3

15. A small indoor greenhouse is made entirely of glass panes held together with tape. It is 30 cm long, 25 cm wide and 25 cm high. How much of tape is needed for all the 12 edges?
(a) 230 cm (b) 320 cm
(c) 302 cm (d) 203 cm

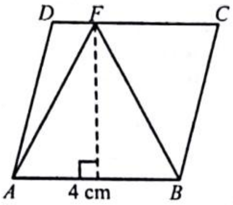
16. A conical tent has a floor area of 154 sq. m. Its height is 24 m. How much canvas is required for the tent?
(a) 500 sq. m. (b) 550 sq. m.
(c) 700 sq. m. (d) 450 sq. m.

17. If the perimeter of one face of a cube is 20 cm, then its surface area is
(a) 120 cm^2 (b) 150 cm^2
(c) 125 cm^2 (d) 400 cm^2

18. In ΔABC if D is a point on BC and divides it in the ratio 3: 5 i.e. if $BD : DC = 3 : 5$ then ar (ΔADC) : ar (ΔABC) =
(a) 3 : 5 (b) 3 : 8
(c) 5 : 8 (d) 8 : 3

19. Points D and E lie on the lines AB and AC respectively of a triangle ABC, such that $AD : BD = 1 : 2$ and $AE : EC = 1 : 2$. $\Delta\, ADE$ and trapezium $DECB$ have their areas in the ratio of :
(a) 1 : 4 (b) 1: 8
(c) 1 : 9 (d) 1 : 2

20. In the given figure, $ABCD$ is a parallelogram then ar(ΔAFB) is



(a) 16 cm^2 (b) 8 cm^2
(c) 4 cm^2 (d) 6 cm^2

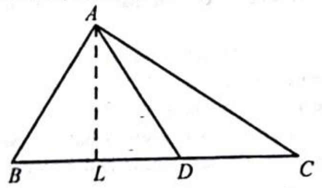
21. The figure formed by joining the consecutive mid-points of any rhombus is always :
(a) a square (b) a rhombus
(c) a parallelogram (d) None of these

22. X and Y are respectively two points on the sides DC and AD of the parallelogram $ABCD$. The area of $\Delta\, ABX$ is equal to :
(a) 1/3 × area of $\Delta\, BYC$ (b) area of $\Delta\, BYC$
(c) 1/2 × area of $\Delta\, BYC$ (d) 2 × area of $\Delta\, BYC$

23. BD is a median of a triangle ABC. F is a point on AB such that CF intersects BD at E and $BE = ED$. If $BF = 5$ cm, BA is equal to :
(a) 10 (b) 12
(c) 15 (d) 17

24. D and E are the mid points of the sides AB and AC of a triangle ABC respectively. Then the area of the triangles ADE and ABC are in the ratio :
(a) 1 : 2 (b) 1 : 3
(c) 1 : 4 (d) 2 : 3

25. AD is the median of a triangle ABC. If area of triangle $ADC = 15$ cm^2, then ar($\Delta\, ABC$) is



(a) 15 cm^2 (b) 22.5 cm^2
(c) 30 cm^2 (d) 37.5 cm^2

26. The base BC of triangle ABC is divided at D so that $BD = \frac{1}{2}DC$ Area of $\Delta\, ABD$ =
(a) 1/3 of the area of $\Delta\, ABC$
(b) 1/2 of the area of $\Delta\, ABC$
(c) 1/4 of the area of $\Delta\, ABC$
(d) 1/6 of the area of $\Delta\, ABC$

27. In the parallelogram $ABCD$, the side AB is produced to point X, so that $BX = AB$. The line DX cuts BC at E.
Area of $\Delta\, AED$ =
(a) 2 × area ($\Delta\, CEX$) (b) 1/2 × area ($\Delta\, CEX$)
(c) area ($\Delta\, CEX$) (d) 1/3 × area ($\Delta\, CEX$)

28. A hemispherical tank of radius 3 cm is full of milk. It is connected to a pipe, through which liquid is emptied at the 1/7 litre per second. The time taken to empty the tank completely?
(a) 0.302 sec (b) 0.396 sec
(c) 0.453 sec (d) 0.492 sec

29. If the radius and height of a right circular cone are 'r' and 'h' respectively, the slant height of the cone is
(a) $(h^2 + r^2)^{1/3}$ (b) $(h + r)^{1/3}$
(c) $(h^2 + r^2)^{1/2}$ (d) none of these

30. A solid consists of a circular cylinder with an exact fitting right circular cone placed at the top. The height of the cone is h. If the total volume of the solid is 3 times the volume of the cone.
(a) $2h$ (b) $\frac{2h}{3}$
(c) $\frac{3h}{2}$ (d) $4h$

MTC More than One Correct

DIRECTIONS: *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE or MORE may be correct.*

1. A match box measures 4 cm × 2.5 cm × 1.5 cm. The volume of a packet containing 12 such boxes, is
(a) 12 × 15 cm^3 (b) (12 × 4 × 2.5 × 1.5) cm^3
(c) 180 cm^3 (d) (4 × 2.5 × 1.5) cm^3

2. A solid cube is cut into two cuboids of equal volumes. The ratio of the total surface area of the given cube and that of one of the cuboids is
(a) $6a^2 : 4a^2$ (b) $4a^2 : 6a^2$
(c) 3 : 2 (d) 2 : 3

3. The weight of a lead pipe 3.5 m long, if the external diameter of the pipe is 2.4 cm and the thickness of the lead is 2 mm and 1 cubic cm of lead weighs 11 gm, is
(a) (484 × 11) gm (b) 5.324 kgs
(c) 5 kg (d) $\pi(2.2)(0.2)(350)(11)$ gm

4. The base radii of two right circular cones of the same height are in the ratio 3 : 5. The ratio of their volumes is
(a) $r_2^2 : r_1^2$ (b) $r_1^2 : r_2^2$
(c) 25 : 9 (d) 9 : 25

5. The volume of the two spheres are in the ratio 64 : 27. The difference of their surface areas, if the sum of their radii is 7, is
(a) 28 πcm^2 (b) 88 cm^2
(c) 64 πcm^2 (d) 36 πcm^2

Passage Based Questions

DIRECTIONS: *Study the given paragraph(s) and answer the following questions.*

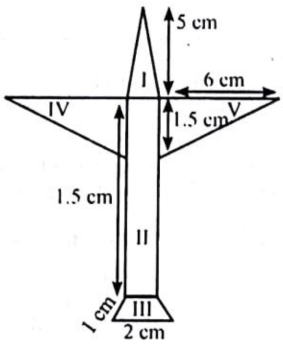
PASSAGE I

A right circular cylinder just encloses a sphere of radius r.

1. Surface area of the sphere is
   (a) $2\pi r^2$ (b) $4\pi r^2$
   (c) $2\pi r$ (d) $3\pi r^2$
2. Curved surface area of the cylinder is
   (a) $2\pi r^2$ (b) $4\pi r^2$
   (c) $2\pi r$ (d) $3\pi r^2$
3. Ratio of the areas obtained in (1) and (2) is
   (a) 1 : 2 (b) 2 : 1
   (c) 1 : 1 (d) 2 : 3

PASSAGE II

Radha made a picture of an aeroplane with coloured paper as shown in figure. Find the total area of the paper used.



1. Area of region I is
   (a) 2.5 cm^2 (b) 2 cm^2
   (c) 5 cm^2 (d) 3 cm^2
2. Area of region II is
   (a) 6 cm^2 (b) 5 cm^2
   (c) 6.5 cm^2 (d) 7 cm^2
3. Area of region III is
   (a) 1 cm^2 (b) 3 cm^2
   (c) 2 cm^2 (d) 1.3 cm^2

Assertion & Reason

DIRECTIONS: *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.
(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.
(c) If **Assertion** is **correct** but **Reason** is **incorrect**.
(d) If **Assertion** is **incorrect** but **Reason** is **correct**.

1. **Assertion:** An edge of a cube measures r cm. If the largest possible right circular cone is cut out of this cube, then the volume of the cone is $\frac{1}{6}\pi r^3$.
   Reason: Volume of the cone is given by $\frac{1}{3}\pi r^2 h$, where r is the radius of the base and h is the height of the cone.
2. **Assertion:** The area of an equilateral triangle is $16\sqrt{3}$ cm^2 whose each side is 8 cm.
   Reason: Area of an equilateral triangle is given by $\frac{\sqrt{3}}{4}(\text{side})^2$.
3. **Assertion:** In a cylinder, if radius is halved and height is doubled, the volume will be halved.
   Reason: In a cylinder, radius is doubled and height is halved, curved surface area will be same.
4. **Assertion:** The total surface area of a cone whose radius is $\frac{r}{2}$ and slant height $2l$ is $(\pi)r\left(l+\frac{r}{4}\right)$.
   Reason : Total surface area of cone is $\pi r(l + r)$ where r is radius and l is the slant height of the cone.
5. **Assertion :** Two parallelograms are on equal bases and between the same parallels. The ratio of their areas is 1 : 1.
   Reason : Two parallelograms on the same base (or equal bases) and between the same parallel lines are equal in area.
6. **Assertion :** A triangle and a rhombus are on the same base and between the same parallels. The ratio of the areas of the triangle and the rhombus is 1 : 2.
   Reason : The area of a triangle is half of the area of a parallelogram on the same base and between the same parallels.
7. **Assertion:** If the inner dimensions of a cuboidal box are 50 cm × 40 cm × 30 cm, then the length of the longest rod that can be placed in the box is $50\sqrt{2}$ cm.
   Reason: The line joining opposite corners of a cuboid is called its diagonal.
   Also, length of longest rod = length of diagonal.
   $$= \sqrt{l^2 + b^2 + h^2}$$

MMQ Multiple Matching Questions

DIRECTIONS : *Following question has four statements (A, B, C and D) given in Column I and statements (p, q, r, s, ...) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in column I with entries in column II.*

1.

	Column I		Column II
(A)	Volume of a cuboid is	(p)	perimeter of the base of the cylinder × h
(B)	Volume of a cube is	(q)	Area of circular base × height
(C)	Volume of a cylinder is	(r)	$(\text{edge})^3$
(D)	Curved surface area of the cylinder is	(s)	base × area × height
		(t)	$2\pi rh$
		(u)	length × breadth × height
		(v)	edge × edge × edge
		(w)	$\pi r^2 h$

HOTS Hots Subjective Questions

DIRECTIONS : *Answer the following questions.*

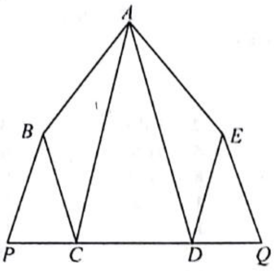
1. Find the percentage increase in the area of a triangle if its each side is doubled.

2. A triangle ABC is right angled at A, $AB = 5$ cm and $AC = 12$ cm. It revolves about its hypotenuse. Find the volume of the double cone so formed.

3. A solid toy is in the form of a hemisphere surrounded by a right circular cone. The height of the cone is 2 cm, diameter of the base is 4 cm. If a right circular cylinder circumscribes the solid, find how much more space it will cover.

4. If h, C and V respectively are the height, the curved surface and volume of cone, prove that $3\pi Vh^3 - C^2h^2 + 9V^2 = 0$.

5. In figure, $ABCDE$ is any pentagon. BP drawn parallel to AC meets DC produced at P and EQ drawn parallel to AD meet CD produced at Q. Prove that ar$(ABCDE)$ = ar(APQ)



6. The radius of a sphere is increased by 10%. Prove that the volume will be increased by 33.1% approximately.

SOLUTIONS

EXERCISE – I

FILL IN THE BLANKS

1. Volume
2. 250,000 cm^3
3. total
4. sphere
5. 220 cm^2
6. One
7. Cuboid
8. right angle
9. right circular cone
10. Spherical shell
11. cone
12. equal
13. perpendicular

TRUE/FALSE

1. True
2. False
3. True
4. False
5. True
6. True
7. False
8. True
9. True
10. False
11. True
12. False
13. True
14. False
15. True
16. True
17. True
18. True
19. False
20. False

MATCH THE COLUMNS

1. (A) $\to$ q; (B) $\to$ t; (C) $\to$ s; (D) $\to$ p; (E) $\to$ r;

(A) Let $AB = BC = CA = x$

$\Rightarrow BD = \dfrac{x}{2} = DE = BE$

We have,

$\text{ar}(\Delta ABC) = \dfrac{\sqrt{3}}{4}x^2$

$\text{ar}(\Delta BDE) = \dfrac{\sqrt{3}}{4} \times \left(\dfrac{x}{2}\right)^2 = \dfrac{\sqrt{3}}{4}x^2 \times \dfrac{1}{4}$

$\Rightarrow \quad \text{ar}(\Delta BDE) = \dfrac{1}{4}\text{ar}(\Delta ABC)$

(B) Given ΔABC and ΔBDE are equilateral triangles.

$\therefore \quad \angle ACB = \angle DBE = 60°$

$\Rightarrow \quad BE \,||\, AC$

$\Rightarrow \quad \text{ar}(\Delta BAE) = 2\,\text{ar}(\Delta BEC)$

$\Rightarrow \quad \text{ar}(\Delta BAE) = 2\text{ar}(\Delta BDE)$

(ED is median of ΔEBC)

$\Rightarrow \quad \text{ar}(\Delta BDE) = \dfrac{1}{2}\text{ar}(\Delta BAE)$

(C) Since ED is median of ΔBEC

$\therefore \quad \text{ar}(\Delta BEC) = 2\,\text{ar}(\Delta BDE)$

$\Rightarrow \quad \text{ar}(\Delta BEC) = 2 \times \dfrac{1}{4}\text{ar}(\Delta ABC)$

(from part (A) above)

$\Rightarrow \quad 2\text{ar}(\Delta BEC) = \text{ar}(\Delta ABC)$

(D) Since, ΔABC and ΔBDE are equilateral triangles

$\therefore \quad \angle ABC = 60°$ and $\angle BDE = 60°$

$\Rightarrow \quad \angle ABC = \angle BDE$

$\Rightarrow \quad$ AB || DE

$\Rightarrow \quad \text{ar}(\Delta BED) = \text{ar}(\Delta AED)$

$\Rightarrow \quad \text{ar}(\Delta BED) - \text{ar}(\Delta EFD) = \text{ar}(\Delta AED) - \text{ar}(\Delta EFD)$

$\Rightarrow \quad \text{ar}(\Delta BFE) = \text{ar}(\Delta AFD)$

(E) Let the altitude of ΔABD be h.

$\text{ar}(\Delta FED) = \dfrac{1}{2}.FD.\dfrac{h}{2} \qquad (\because \text{ altitude of } \Delta BED = \dfrac{h}{2})$

$\text{ar}(\Delta AFC) = \dfrac{1}{2}.FC.h = \dfrac{1}{2}(FD + BD)h$

$= \dfrac{1}{2}(2FD + BF)h$

$= \dfrac{1}{2}(2FD + 2FD)h \qquad (\because BF = 2FD)$

$= 2FD.h = 8\text{ar}(\Delta FED)$

$\text{ar}(\Delta FED) = \dfrac{1}{8}\text{ar}(\Delta AFC)$

2. (A) $\to y$; (B) $\to x$; (C) $\to w$; (D) $\to v$; (E) $\to r$; (F) $\to u$; (G) $\to t$; (H) $\to q$; (I) $\to s$; (J) $\to p$;

VERY SHORT ANSWER QUESTIONS

1. 1 : 1

3. $S = \dfrac{a+b+c}{2}$ where a, b, c are the sides of a triangle.

4.
(a) External, Internal, height.
(b) $R - r$.
(c) $2\pi Rh$.
(d) Total surface area = external curved area + internal curved area + area of two ends.
$= 2\pi Rh + 2\pi rh + 2\pi (R^2 - r^2)$
$= 2\pi (Rh + rh + R^2 - r^2)$
(e) $\pi (R^2 - r^2)$.

5.
(a) hemispheres
(b) outer, inner.
(c) $\pi (R^2 - r^2)$.
(d) $2\pi R^2 + 2\pi r^2 + \pi (R^2 - r^2) = \pi (3R^2 + r^2)$
(e) $R - r$.

SHORT ANSWER QUESTIONS

1. The total surface area of the cube = 864 cm^2

Let each side of the cube be a cm.

Then $6a^2 = 864\ cm^2$

$$a^2 = \frac{864}{6} cm^2 = 144\ cm^2$$

$\therefore$ edge $a = \sqrt{144}$ cm = 12 cm

Lateral surface area of the cube = $4a^2 = 4 \times (12)^2\ cm^2$
$= 4 \times 144\ cm^2 = 576\ cm^2$

2. Dimensions of each brick is length (l) = 22.5 cm, breadth (b) = 10 cm, height (h) = 7.5 cm

Surface area of each brick

$= 2(l \times b + b \times h + h \times l)$

$= 2$ (22.5 cm × 10 cm + 10 cm × 7.5 cm + 7.5 cm × 22.5 cm)

$= 2\ (225\ cm^2 + 75\ cm^2 + 168.75\ cm^2)$

$= 2 \times 468.75\ cm^2 = 937.5\ cm^2$

Let n bricks will be painted.

Then the surface area of n bricks = 9.375 m^2

$n \times$ area of each brick = $9.375 \times 10000\ cm^2$ $\left[\because 1m^2 = 10000cm^2\right]$

$n \times 937.5\ cm^2 = 93750\ cm^2$

$$n = \frac{93750 cm^2}{937.5 cm^2} = \frac{93750 \times 10}{937.5 \times 10} = \frac{937500}{9375}$$

= 100 bricks.

3. Diameter of the pillar d = 50 cm, d = 0.50 m

radius of the pillar $r = \frac{d}{2} = \frac{0.50}{2} m = 0.25m$

height h = 3.5 m

Curved surface area of the pillar = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 0.25 \times 3.5 m^2 = 5.5\ m^2$$

Cost of painting the pillar at the rate of ₹ 12.50 per m^2 = 5.50 m^2 × ₹ 12.50 per m^2 = ₹ 68.75.

4. Diameter of the pipe d = 5 cm = 0.05 m

radius $r = \frac{d}{2} = \frac{0.05}{2} m$

Length of the pipe h = 28 m

Total radiating surface = $2\pi rh$

$$= 2 \times \frac{22}{7} \times \frac{0.05}{2} \times 28 m^2 = 4.4\ m^2$$

5. Diameter of the base of lampshade, d = 20 cm

radius $r = \frac{d}{2} = \frac{20}{2} cm = 10 cm$; height = 30 cm

Margin over the top and bottom = 2.5 cm + 2.5 cm = 5 cm

Hence total length of the cloth used

h = 30 cm + 5 cm

h = 35 cm

The cloth required for covering the lampshade = $2\pi rh$

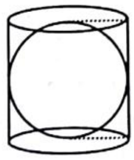
$$= 2 \times \frac{22}{7} \times 10 \times 35 = 2200\ cm^2$$

6. Base radius of the cylinder = radius of the sphere = 7 cm

Height of the cylinder h = diameter of the sphere

$= 2r = 2 \times 7$ cm, h = 14 cm

The curved surface area of the cylinder = $2\pi rh$



$$= 2 \times \frac{22}{7} \times 7 \times 14 = 616\ cm^2$$

Curved surface area of the sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times 7 \times 7 cm^2 = 616\ cm^2$$

Ratio of these areas $= \frac{616 cm^2}{616 cm^2} = 1 : 1$

7. Let each side of the cube be a cm.

Total surface area = 486 cm^2

$\Rightarrow$ $6a^2 = 486\ cm^2$

$\Rightarrow$ $a^2 = \frac{486}{6} cm^2$

$\Rightarrow$ $a^2 = 81\ cm^2$

$\Rightarrow$ $a = \sqrt{81} = 9 cm$

Volume of the cube = $a^3 = 9 \times 9 \times 9\ cm^3 = 729\ cm^3$

8. Width of the river b = 40 m

depth h = 3 m

Speed of the flowing of the river = 2 km per h

$$= \frac{2 \times 1000}{60} \text{ m per min} = \frac{100}{3} \text{ m per min}$$

So, length of the water falling into the sea in a minute

$$= l = \frac{100}{3} m$$

Volume of the water falling into the sea in a minute

$$= l \times b \times h = \frac{100}{3} \times 40 \times 3 m^3 = 4000\ m^3$$

= 4,000 × 1,000 lit. [$\because$ 1 m^3 = 1000 lit]

= 4,000,000 lit.

9. Total surface area of the cylinder = 231 cm^2

$$2\pi r(h + r) = 231 \Rightarrow 2 \times \frac{22}{7} r(h + r) = 231$$

$$r(h + r) = \frac{231 \times 7}{2 \times 22}$$

$$rh + r^2 = \frac{21 \times 7}{2 \times 2} \quad \text{... (i)}$$

Curved surface area of the cylinder

$= 2\pi rh = \frac{2}{3} \times 231 \text{cm}^2$

$2 \times \frac{22}{7} r h = 154 \text{cm}^2 \Rightarrow rh = \frac{154 \times 7}{2 \times 22}$

$rh = \frac{7 \times 7}{2}$ ...(ii)

Putting the value of rh in equation (i), We get

$\frac{7 \times 7}{2} + r^2 = \frac{21 \times 7}{2 \times 2}$

$\Rightarrow r^2 = \frac{21 \times 7}{2 \times 2} - \frac{7 \times 7}{2} = \frac{7 \times 7}{2}\left[\frac{3}{2} - 1\right] = \frac{7 \times 7}{2}\left(\frac{3-2}{2}\right)$

$\Rightarrow r^2 = \frac{7 \times 7}{2} \times \frac{1}{2} \Rightarrow r = \sqrt{\frac{7 \times 7}{2 \times 2}} = \frac{7}{2} \text{cm}$

Putting the value of r in equation (ii), we get

$\frac{7}{2} h = \frac{7 \times 7}{2} \Rightarrow h = 7 \text{ cm}$

Hence, the volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 7$

$= 22 \times \frac{7}{2} \times \frac{7}{2} = \frac{539}{2} = 269.5 \text{cm}^3$

10. Inner radius of the ball $r_1 = 7$cm, thickness of the leather $= 0.25$ cm

Then outer radius of the ball $r_2 = 7.25$ cm

So, the volume of the leather used in the football

$= \frac{4}{3}\pi(r_2^3 - r_1^3)$

$= \frac{4}{3} \times \frac{22}{7}[(7.25^3) - (7)^3] = \frac{88}{21} \times (381.07 - 343)$

$= \frac{88}{21} \times 38.07 \text{cm}^3 = 159.53 \text{ cm}^3$ approx

Mass of the football = volume × density

$= 159.53 \text{ cm}^3 \times 2.5$ g per cm^3

$= 398.83$ g approx.

11. Let the length of unequal side be x cm.

Then perimeter = 30 cm.

$\Rightarrow x + 12 + 12 = 30$

$\Rightarrow x + 24 = 30$

$\Rightarrow x = 6$ cm

We have, $2s = 30$ cm

$\therefore s = 15$ cm

Hence, area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{15 \times (15-12) \times (15-12) \times (15-6)}$

$= \sqrt{15 \times 3 \times 3 \times 9} = 9\sqrt{15} \text{ cm}^2$.

12. It is given that the sides a, b, c of the triangle are in the ratio 13 : 12 : 5 i.e. $a : b : c = 13 : 12 : 5$

$\Rightarrow a = 13x, b = 12x$ and $c = 5x$

$\therefore$ perimeter $= a + b + c = 450$

$13x + 12x + 5x = 450$

$x = \frac{450}{30} = 15$

$\therefore a = 13 \times 15 = 195$ m; $b = 12 \times 15 = 180$ m

$c = 5 \times 15 = 75$ m

It is given that perimeter $= 450 \Rightarrow 2s = 450$

$\Rightarrow s = 225$ m

Hence, area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{225 \times (225-195) \times (225-180) \times (225-75)}$

$= \sqrt{225 \times 30 \times 45 \times 150} = \sqrt{5^6 \times 3^6 \times 2^2}$

$= 5^3 \times 3^3 \times 2 = 6750 \text{ m}^2$

13. Volume of the juice in vessel $= \pi \times (15)^2 \times 32 \text{ cm}^3$

volume of the juice in each glass $= \pi \times (3)^2 \times 8 \text{ cm}^3$

$\therefore$ number of glasses of juice prepared $= \frac{\pi \times (15)^2 \times 32}{\pi \times (3)^2 \times 8}$

$= 100$

$\therefore$ total amount received by the stall keeper

$= ₹ (3 \times 100) = ₹ 300$

14. Sides of one triangular piece of cloth are of lengths $a = 20$ cm, $b = 50$ cm and $c = 50$ cm.

Let s be the semi-perimeter of the triangular piece. Then,

$2s = a + b + c \Rightarrow 2s = 20 + 50 + 50 \Rightarrow s = 60$

$\therefore \Delta =$ Area of one triangular piece

$= \sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow \Delta = \sqrt{60 \times (60-20) \times (60-50) \times (60-50)} \text{ cm}^2$

$\Rightarrow \Delta = \sqrt{60 \times 40 \times 10 \times 10} \text{ cm}^2$

$= \sqrt{6 \times 4 \times 10 \times 10 \times 10 \times 10} \text{ cm}^2$

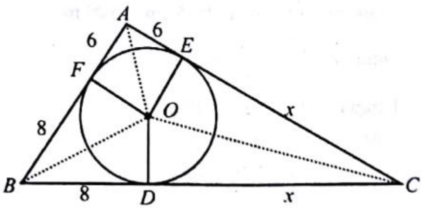
$= 200\sqrt{6} \text{ cm}^2$

$\therefore$ Area of cloth of each colour $= 5 \times 200\sqrt{6} \text{ cm}^2$

$= 1000\sqrt{6} \text{ cm}^2$

LONG ANSWER QUESTIONS

1. Let $CD = CE = x$ cm



Now using fact that

Area of ΔABC = ar. ΔAOB + ar. ΔBOC + ar. ΔAOC

For ΔABC,

Then $s = \frac{14 + 8 + x + 6 + x}{2} = (14 + x)$ cm

L.H.S.: ar. $\Delta ABC = \sqrt{(14+x).x.(6)(8)}$ ..... (i)

R.H.S.: ar. ΔAOB + ar. ΔBOC + ar.Δ AOC

$\therefore$ $= \frac{1}{2}\times 14\times 4 + \frac{1}{2}\times(8+x).4 + \frac{1}{2}\times(6+x).4$

$= 2(14+8+x+6+x) = 2.(28+2x)$

$= 4\,(14+x)$ ..... (ii)

Now according to the question, equating (i) and (ii)

$\sqrt{(14+x).x.48} = 4(14+x)$

Squaring both sides $48x\,(14+x) = 16(14+x)^2$

$\Rightarrow$ $3x = 14+x$

$\Rightarrow$ $2x = 14$

$\Rightarrow$ $x = 7$

2. We have, $a = 4$ cm, $b = 5$ cm, $c = 6$ cm.

Semi perimeter $(s) = \frac{a+b+c}{2} = \frac{4+5+6}{2} = \frac{15}{2}$cm

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{\frac{15}{2}\left(\frac{15}{2}-4\right)\left(\frac{15}{2}-5\right)\left(\frac{15}{2}-6\right)} = \frac{15}{4}\sqrt{7}\text{cm}^2$

3. $a = 6$ cm, $b = 6$ cm, $c = 6$ cm

$\therefore$ $s = \frac{a+b+c}{2} = \frac{6+6+6}{2} = 9$cm

$\therefore$ Area of the equilateral triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{9(9-6)(9-6)(9-6)}$

$= \sqrt{9(3)(3)(3)} = 9\sqrt{3}$ cm^2

Area $= \frac{1}{2}$ × Base × Altitude

$\Rightarrow$ $9\sqrt{3} = \frac{1}{2}\times 6\times$ Altitude $\Rightarrow$ $9\sqrt{3} = 3\times$ Altitude

$\Rightarrow$ Altitude $= \frac{9\sqrt{3}}{3} = 3\sqrt{3}$cm.

4. Let a, b, c be the sides of isoscele Δ.

Given, $a = 12$ cm, $b = 12$ cm

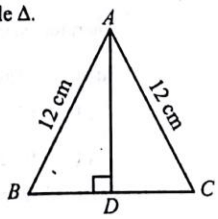
Perimeter = 30 cm

$\Rightarrow$ $a+b+c = 30$

$\Rightarrow 12+12+c = 30$

$\Rightarrow c = 30-24$

$\Rightarrow$ $c = 6$ cm



Now, $s = \frac{a+b+c}{2} = \frac{12+12+6}{2}$

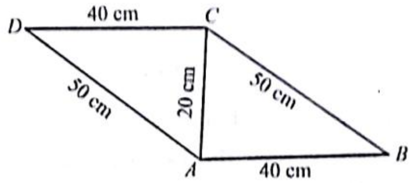
$\Rightarrow$ $s = \frac{30}{2}$cm = 15cm

$\therefore$ Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{15(15-12)(15-12)(15-6)}$

$= \sqrt{15(3)(3)(9)} = 9\sqrt{15}\,\text{cm}^2$.

5. Side of ΔABC are given as $a = 50$ cm, $b = 20$ cm, $c = 40$ cm



Now, $s = \frac{a+b+c}{2}$

$\Rightarrow s = \frac{50+20+40}{2} = \frac{110}{2} = 55$ cm

$\therefore$ Area of ΔABC $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{55(55-50)(55-20)(55-40)} = \sqrt{55(5)(35)(15)}$

$= 5\sqrt{5775}\,\text{cm}^2$

$\therefore$ Area of the parallelogram $ABCD$ = 2 Area of ΔABC

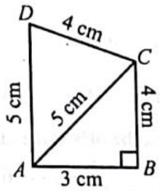
$= 2\times 5\sqrt{5775} = 10\sqrt{5775}$ cm^2

6. Let $ABCD$ be the quadrilateral such that $AB = 3$ cm, $BC = 4$ cm, $CD = 4$ cm, $DA = 5$ cm and $AC = 5$ cm.

Let AC be the diagonal which divides quadrilateral in 2Δs ABC and ACD.

For ΔABC, we have, $a = 4$ cm, $b = 5$ cm, $c = 3$ cm

Since, $a^2 + c^2 = b^2$



Therefore, ΔABC is right angled with $\angle B = 90°$.

$\therefore$ Area of right angled triangle ABC

$= \frac{1}{2}$ × Base × Height $= \frac{1}{2}\times 3\times 4 = 6$ cm^2

But, for ΔACD, the sides are $a = 4$ cm, $b = 5$ cm, $c = 5$ cm

. We have $s = \frac{a+b+c}{2}$

$\therefore$ $s = \frac{4+5+5}{2} = \frac{14}{2} = 7$cm

$\therefore$ Area of the $\Delta ACD = \sqrt{s(s-a)(s-b)(s-c)}$

(By Herson's formula)

$= \sqrt{7(7-4)(7-5)(7-5)} = \sqrt{7(3)(2)(2)} = 2\sqrt{21}\,\text{cm}^2$

$= 2\times 4.6$ cm^2 (approx.) = 9.2 cm^2 (approx.)

$\therefore$ Area of the quadrilateral $ABCD$ = Area of ΔABC + Area of ΔACD = 6 cm^2 + 9.2 cm^2 = 15.2 cm^2. (approx.)

7. Area of paper of shade $I = \frac{1}{2}\left[\frac{1}{2}d_1d_2\right]$ Where d_1 and d_2 denotes the diagonals

$$= \frac{1}{2}\left(\frac{1}{2}\times 32\times 32\right) = 256\text{ cm}^2$$

Area of paper of shade II = 256 cm^2

For paper of shade III, sides are given as $a = 8$ cm, $b = 6$ cm, $c = 6$ cm

$\therefore \quad \frac{1}{2}\left(\frac{1}{2}\times 32\times 32\right) = 256\text{ cm}^2$

$\therefore$ Area of paper of shade III

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{10(10-8)(10-6)(10-6)}$$

$$= \sqrt{10(2)(4)(4)} = 8\sqrt{5} = 17.92\text{ cm}^2$$

8. Let the sides of one tile are given as $a = 9$ cm, $b = 28$ cm, $c = 35$ cm

we know $s = \frac{a+b+c}{2} = \frac{9+28+35}{2} = 36$ cm

$\therefore$ Area of one tile = $\sqrt{s(s-a)(s-b)(s-c)}$

(By Heron's formula)

$$= \sqrt{36(36-9)(36-28)(36-35)} = \sqrt{36(27)(8)(1)}$$

$$= 6\times 3\times 2\sqrt{6} = 36\sqrt{6}\text{ cm}^2$$

$\therefore$ Area of 16 tiles = $36\sqrt{6}\times 16 = 576\sqrt{6}\text{ cm}^2$

$\therefore$ cost of polishing the tiles at the rate of 50 paise per cm^2.

$$= 576\sqrt{6}\times 50\text{p} = ₹\frac{576\sqrt{6}\times 50}{100}$$

$$= ₹\ 288\sqrt{6} = ₹\ 705.60$$

9. Here $a = 26$ cm, $b = 28$ cm, $c = 30$ cm.

$$s = \frac{a+b+c}{2} = \frac{26+28+30}{2} = 42\text{ cm}$$

Area = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{42(42-26)(42-28)(42-30)}$$

$$= \sqrt{42\times 16\times 14\times 12}$$

$$= \sqrt{7\times 2\times 3\times 2\times 2\times 2\times 2\times 7\times 2\times 2\times 2\times 3}$$

$$= 7\times 2\times 2\times 2\times 2\times 3$$

$$= 336\text{ cm}^2$$

10. Yellow : 484 m^2; Red : 242 m^2 Green : 373.04 m^2.

EXERCISE - 2

MULTIPLE CHOICE QUESTIONS

1. (d) The sides of the triangle are $a = 13$ cm, $b = 9$ cm and $c = 6$ cm.

$$s = \frac{a+b+c}{2} = \frac{13+9+6}{2} = 14\text{ cm.}$$

$\therefore$ Area of the triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$ [Heron's formula]

$$= \sqrt{14(14-13)(14-9)(14-6)}\text{ cm}^2$$

$$= \sqrt{14\times 1\times 5\times 8}\text{ cm}^2 = \sqrt{560}\text{ cm}^2$$

$$= 23.66\text{ cm}^2$$

2. (a) Let a = 5 cm, b = 4 cm.

Therefore area of an isosceles triangle

$$= \frac{b}{4}\times\sqrt{4a^2-b^2} = \frac{4}{4}\sqrt{4\times 25-16}\text{ sq. cm.}$$

$$= \sqrt{84}\text{ cm.} = 2\sqrt{21}\text{ sq. cm.}$$

3. (b) Suppose that the sides, in metres, are $3x$, $5x$ and $7x$

Then, we know that $3x + 5x + 7x = 300$ (perimeter of the triangle)

Therefore, $15x = 300$, which gives $x = 20$.

So the sides of the triangle are 3×20m, 5×20m and 7×20m

i.e., 60m, 100m and 140m

We have $s = \frac{60+100+140}{2}\text{m} = 150\text{ m}$, and area will be $\sqrt{150(150-60)(150-100)(150-140)}\text{ m}^2$

$$= \sqrt{150\times 90\times 50\times 10}\text{ m}^2 = 1500\sqrt{3}\text{ m}^2$$

4. (c) We know that the diagonal of a quadrilateral divides it into two triangles.

As per figure, in ΔABC

$AB = 7$ cm, $BC = 12$ cm, $AC = 15$ cm

Therefore semi perimeter $s = \frac{7+12+15}{2} = 17$ cm

Area of ΔABC

$$= \sqrt{17\times(17-7)\times(17-12)\times(17-15)}$$

$$= \sqrt{17\times 10\times 5\times 2}\text{ sq. cm.}$$

$$= 10\sqrt{17}\text{ sq. cm.} = 10\times 4.12 = 41.2\text{ sq. cm.}$$

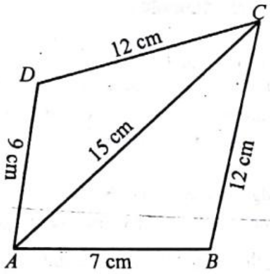
Similarly for ΔACD, $AC = 15$ cm, $CD = 12$ cm and $DA = 9$ cm

Therefore $s = \frac{15+12+9}{2} = \frac{36}{2} = 18$ cm.

Area of ΔACD

$$= \sqrt{18\times(18-15)\times(18-12)\times 18-9)}$$

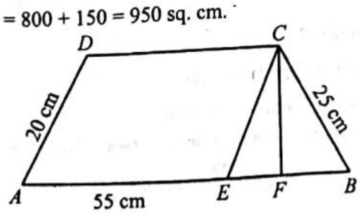
$$= \sqrt{18\times 3\times 6\times 9} = 54\text{ sq. cm.}$$



Area of quadrilateral $ABCD$
$= \text{Area of } \Delta\, ABC + \text{Area of } \Delta\, ACD$
$= 41.2 + 54 = 95.2$ sq. cm.

5. **(d)** Area of the parallelogram $ABCD$
$= 2 \times (\text{Area of } \Delta\, ABC)$
In $\Delta\, ABC$, $AB = 5$ cm, $BC = 3.5$ cm and $AC = 6.5$ cm.
So its perimeter $s = \dfrac{5+3.5+6.5}{2} = 7.5$ cm.
$\because$ Area of $\Delta\, ABC$ (By Heron's formula)
$= \sqrt{s(s-a)(s-b)(s-c)}$
$= \sqrt{7.5\times(7.5-5)\times(7.5-3.5)\times(7.5-6.5)}$ sq. cm.
$= \sqrt{7.5\times2.5\times4\times1}$ sq. cm. $= \sqrt{75}$ sq. cm. $= 5\sqrt{3}$ sq.cm.
Area of parallelogram
$= 2\times5\sqrt{3}$ sq. cm. $= 10\sqrt{3}$ sq.cm.

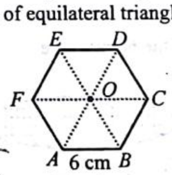
6. **(b)** In figure, $ABCD$ is a trapezium in which parallel sides $AB = 55$ cm, $DC = 40$ cm and non-parallel sides $AD = 20$ cm and $BC = 25$ cm.
Draw $AD \parallel EC$ and $CF \perp AB$
Therefore, $EB = AB - AE = 55 - 40 = 15$ cm. and $EC = 20$ cm.
$\because$ Area of $\Delta\, BCE$
$= \sqrt{30\times(30-15)\times(30-20)\times(30-25)}$
where $s = \dfrac{15+20+25}{2} = 30$ cm.
$= \sqrt{30\times15\times10\times5} = \sqrt{22500} = 150$ sq. cm.
Area of $\Delta\, BCE$
$= \dfrac{1}{2}\times BE\times CF \Rightarrow CF = \dfrac{2\times150}{15} = 20$ cm.
Area of parallelogram $AECD = AE \times CF = 40 \times 20$
$= 800$ sq. cm.
$\therefore$ Area of the trapezium $ABCD$
$= \text{Area of parallelogram } AECD + \text{Area of } \Delta\, EBC$
$= 800 + 150 = 950$ sq. cm.



7. **(d)** First we find the area of 16 triangular tiles.
For area of 1 triangular tile
$s = \dfrac{9+28+35}{2} = \dfrac{72}{2} = 36$ cm.
$\therefore$ Area $= \sqrt{36(36-9)(36-28)(36-35)}$ cm^2
$= \sqrt{36\times27\times8\times1}$ cm^2 $= 36\sqrt{6}$ cm^2
$\therefore$ Area of 16 tiles $= 16\times36\sqrt{6}$ cm^2
$= 576\sqrt{6}$ cm^2
Cost of polishing $= ₹\left(\dfrac{50}{100}\times576\sqrt{6}\right)$
$= ₹\,705.45 = ₹\,706$ (approx.)

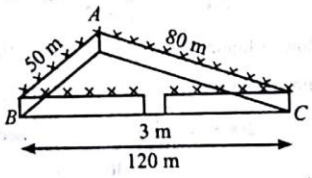
8. **(a)** Third side $= \sqrt{(\text{Hypotenuse})^2 - (\text{Base})^2}$
$= \sqrt{100-64} = 6$ cm
$s = \dfrac{10+8+6}{2}$ cm $= 12$ cm
Area $= \sqrt{s(s-a)(s-b)(s-c)}$
$= \sqrt{12(12-10)(12-8)(12-6)}$
$= \sqrt{12\times2\times4\times6}$ cm^2 $= 24$ cm^2

9. **(d)** Side $= 6$ cm
$\therefore$ Perimeter of regular hexagon $= 6 \times 6 = 36$ cm
$ABCDEF$ is a regular hexagon. Join diagonals AD, BE and CF. The three diagonal divides the hexagonal in six. congruent equilateral triangle with side 6cm.
Area of one such triangle
$= \sqrt{s(s-a)(s-b)(s-c)}$
$= \sqrt{9(9-6)(9-6)(9-6)} = \sqrt{9\times3\times3\times3} = 9\sqrt{3}$
$\therefore$ Area of the regular hexagon
$= 6 \times$ Area of equilateral triangle OAB



$= 6\times9\sqrt{3} = 54\sqrt{3}$ cm^2.

10. **(a)** For area of the park, we have
$2s = 50\text{m} + 80\text{m} + 120\text{ m} = 250$ m.
$s = 125$ m
i.e., $s - a = (125 - 120)$ m $= 5$ m,
$s - b = (125 - 80)$ m $= 45$ m,



$s - c = (125 - 50)$ m $= 75$ m.

Therefore, area of the park

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{125 \times 5 \times 45 \times 75}\ \text{m}^2 = 375\sqrt{15}\text{m}^2$

Also, perimeter of the park = $AB + BC + CA$ = 250 m

Therefore, length of the wire needed for fencing

= 250 m – 3 m (to be left for gate) = 247 m

And so the cost of fencing = ₹ 20 × 247 = ₹ 4940

11. **(d)**

12. **(b)** length of longest rod = length of diagonal

$= \sqrt{\ell^2 + b^2 + h^2} = \sqrt{(50)^2 + (40)^2 + (30)^2} = 50\sqrt{2}$ cm

13. **(c)** Let the dimensions of the cuboidal container be l, b and h. Then its capacity = $l \times b \times h$

When all dimensions are doubled, its capacity = $2l \times 2b \times 2h$ = 8 (previous capacity)

∴ The number of small boxes which the new container can hold = 8 × 50 = 400

14. **(a)**

15. **(b)** length of tape = $(4l + 4b + 4h)$ cm

= (4 × 30 + 4 × 25 + 4 × 25) cm = 320 cm

16. **(b)**

17. **(b)** Edge of cube = $\frac{20}{4}$ cm = 5 cm,

surface area = 6×5^2 cm^2 = 150 cm^2

18. **(c)** **19.** **(a)**

20. (b) $\text{ar}(\Delta AFB) = \frac{1}{2}\text{ar}(\|\text{ gm } ABCD) = \frac{1}{2} \times 4 \times 4$

= 8 cm^2

21. (e) **22.** (b) **23.** (c) **24.** (c)

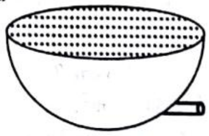
25. (c) AD is median of ΔABC.

$\text{ar}(\Delta ABD) = \text{ar}(\Delta ADC) = \frac{1}{2}\text{ar}(\Delta ABC)$

26. **(a)** **27.** **(a)**

28. (b) Radius of the hemisphere

h = 3 cm.



Volume of milk in the sphere = $\frac{2}{3}\pi r^3$

$= \frac{2}{3}p \times 3^3 = \frac{22}{7} \times 2 \times 3^2$ cm^3

$= \frac{22}{7} \times 2 \times 3^2 \times \frac{1}{1000}$ litre.

Flow of liquid through the pipe = $\frac{1}{7}$ litre per second.

Time taken to empty the hemisphere

$= \frac{22}{7} \times 2 \times 3^2 \times \frac{1}{1000} \times 7 = \frac{396}{1000}$ sec.

29. **(c)** **30.** **(b)**

MORE THAN ONE CORRECT

1. **(a, b, c)**

Volume of one match box

= (4 × 2.5 × 1.5) cm^3 = 15 cm^3

∴ Required volume = Volume of 12 match boxes

= 12 × 15 cm^3 = 180 cm^3

2. **(a, c)**

Let the edge of the solid cube = a

Since, cube is cut into two cuboids of equal volumes.

∴ Length = a, breadth = a, height = $\frac{a}{2}$

Total surface area of cube = $6a^2$ sq. unit

Total surface area of one cuboid

$= 2\left(a \times a + a \times \frac{a}{2} + \frac{a}{2} \times a\right) = 4a^2$ sq. unit

Required ratio = $6a^2 : 4a^2 = 3 : 2$

3. **(a, b, d)**

R = external radius of pipe = 1.2 cm

r = internal radius = 1.2 – 0.2 = 1 cm

h = length of pipe = 3.5 m

∴ Volume of lead = $\pi(R^2 - r^2)\,h$

$= \left[\frac{22}{7}(1.2+1)(1.2-1) \times 350\right]\text{cm}^3$

$= \frac{22}{7} \times 2.2 \times 0.2 \times 350\ \text{cm}^3 = 484\ \text{cm}^3$

Since, one cubic centimeter of lead weighs 11 gm.

∴ Weight of the pipe = (484 × 11) gms

$= \frac{484 \times 11}{1000}$ kgs = 5.324 kgs

4. **(b, d)**

Let r_1 and r_2 be the radii of two cones and V_1 and V_2 be their volumes.

Let h be the height of the two cones, then

$V_1 = \frac{1}{3}\pi r_1^2 h$ and $V_2 = \frac{1}{3}\pi r_2^2 h$

$\frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h}{\frac{1}{3}\pi r_2^2 h} = \frac{r_1^2}{r_2^2} = \frac{9}{25}$ ($\because$ $r_1 : r_2 = 3 : 5$)

5. **(a, b)**

Let the radii of two spheres be r_1 cm and r_2 cm respectively.

Let the volumes be V_1 and V_2 respectively.

Now, $\frac{V_1}{V_2} = \frac{64}{27} \Rightarrow \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27} \Rightarrow \frac{r_1}{r_2} = \frac{4}{3}$ ...(1)

Also, given $r_1 + r_2 = 7$ ...(2)

Using (1) and (2), we get

$r_1 = 4$ cm and $r_2 = 3$ cm

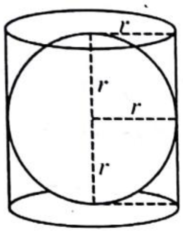
So, $S_1 - S_2 = 4\pi r_1^2 - 4\pi r_2^2 = 64\pi - 36\pi = 28\ \pi\text{cm}^2$

$= 28 \times \frac{22}{7} = 88\ \text{cm}^2$

PASSAGE BASED QUESTIONS

Passage-I

1-3. Radius of the sphere = r,
Radius of the cylinder = r,
height of the cylinder $h = 2r$



1. (b) Surface area of the sphere $= 4\pi r^2$

2. (b) Curved surface area of the cylinder

$= 2\pi rh = 2\pi r \times 2r = 4\pi r^2$

3. (c) Ratio of the areas in (1) and (2)

$= \frac{4\pi r^2}{4\pi r^2} = 1:1$

Passage-II

1-3. For Triangular Area I, we have

$a = 5$ cm, $b = 5$ cm, $c = 1$ cm

as the sides of the triangle

$\therefore \quad s = \frac{a+b+c}{2}, = \frac{5+5+1}{2} = \frac{11}{2} = 5.5\text{cm}$

$\therefore$ Area of shaded region I $= \sqrt{s(s-a)(s-b)(s-c)}$.

$= \sqrt{5.5(5.5-5)(5.5-5)(5.5-1)}$

$= \sqrt{5.5(.5)(.5)(4.5)} = (.5)\sqrt{(5.5)(4.5)}$

$= (.5)\sqrt{(.5)(11)(.5)(9)} = (.5)(.5)(3)\sqrt{11}$

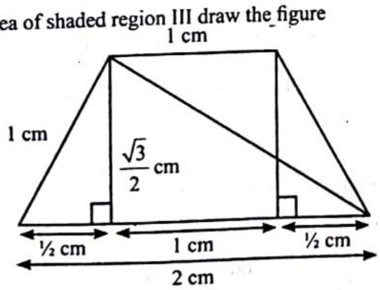
$= 0.75\sqrt{11} = 0.75(3.3)$ (approx.)

$= 2.5\text{ cm}^2$ (approx.),

Area of shaded region II = Area of rectangle

$= 6.5 \times 1 = 6.5\text{ cm}^2$

For Area of shaded region III draw the figure



Area of region III

$= \frac{1}{2} \times 2 \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times 1 \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4}$

$= \frac{3\sqrt{3}}{4} = \frac{3 \times 1.732}{4}$ (Approx.)

$= \frac{5.196}{4} = 1.3\text{ cm}^2$ (approx.)

Thus, area of region IV = Area of right angled Δ

$= \frac{6 \times 1.5}{2} = 4.5\text{ cm}^2$

Similarly, Area $V = \frac{6 \times 1.5}{2} = 4.5\text{ cm}^2$

$\therefore$ Total area of the paper used

= Area I + Area II + Area III + Area IV + Area V

$= 2.5\text{ cm}^2 + 6.5\text{ cm}^2 + 1.3\text{ cm}^2 + 4.5\text{ cm}^2 + 4.5\text{ cm}^2$

$= 19.3\text{ cm}^2$. (approx)

1. (a) **2.** (c) **3.** (d)

ASSERTION & REASON

1. (d) Assertion is false but reason is correct.

2. (a) Area of triangle $= \frac{\sqrt{3}}{4}(8)^2\text{ cm}^2$

$= \frac{\sqrt{3}}{4} \times 8 \times 8 = 16\sqrt{3}\text{ cm}^2.$

3. (b) Both Assertion and Reason is correct.

4. (a) **Assertion :** T.S.A. $= \pi r(l + r)$

$= \pi\frac{r}{2}\left[2l + \frac{r}{2}\right] = \pi r\left[l + \frac{r}{4}\right]$

5. (a) Both assertion and reason is correct.
Also, reason is the correct explanation for assertion.

6. **Assertion:** Area of $\Delta = \frac{1}{2}$ area of rhombus

$\Rightarrow \quad \frac{\text{Area of }\Delta}{\text{Area of rhombus}} = \frac{1}{2} \equiv 1:2$

7. (a) Length of the diagonal is the length of the longest rod that can be placed in the cuboid.

MULTIPLE MATCHING QUESTIONS

1. (A) → s, u; (B) → r, v; (C) → q, w; (D) → p, t

HOTS SUBJECTIVE QUESTIONS

1. Let a, b, c be the sides of the old triangle and 's' be its semi-perimeter.

Then, $s = \frac{1}{2}(a + b + c)$

The sides of the new triangle are $2a$, $2b$ and $2c$. Let's be its semi-perimeter. Then,

$s' = \frac{1}{2} \times (2a + 2b + 2c) = (a + b + c) = 2s.$

Let Δ and Δ's be the area of the old and new triangles respectively. Then,

$$\Delta' = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\Delta' = \sqrt{s'(s'-2a)(s'-2b)(s'-2c)}$$

$$\Rightarrow \quad \Delta' = \sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$$

$$= 4\sqrt{s(s-a)(s-b)(s-c)} = 4\Delta$$

Increase in the area of triangle $= \Delta' - \Delta$

$$= 4\Delta - \Delta = 3\Delta.$$

Hence, percentage increase in area

$$= \left(\frac{3\Delta}{\Delta} \times 100\right) = 300\%$$

2. In ΔABC, right angled at A, $AB = 5$ cm and $AC = 12$ cm.

Therefore hypotensue $BC = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$ cm

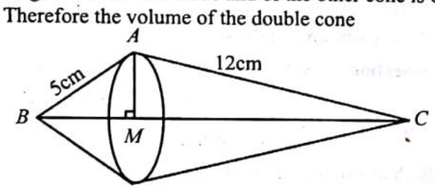
Let $AM \perp BC$.

Now, $\frac{1}{2}\ BC \times AM =$ area $(\Delta ABC) = \frac{1}{2} \times AC \times AB$

or, $\frac{1}{2} \times 13 \times AM = \frac{1}{2} \times 12 \times 5.$

So, $AM = \frac{60}{13}$ cm.

As ΔABC revolves about BC, AM becomes the radius of the each of the two cones forming the double cone. The height of one cone is BM and of the other cone is CM. Therefore the volume of the double cone



$$= \frac{\pi}{3}\left(\frac{60}{13}\right)^2 \times BM + \frac{\pi}{3}\left(\frac{60}{13}\right)^2 \times CM$$

$$= \frac{\pi}{3} \cdot \frac{3600}{169}(BM + CM) = \frac{22}{7}\frac{3600}{169} \cdot \frac{13}{3} CM^3$$

$$= \frac{22 \times 3600}{91 \times 3}\ \text{cm}^3 = \frac{870.33}{3}\ \text{cm}^3 = 290.11\ \text{cm}^3$$

3. Let BPC, be the hemisphere and ABC be the cone standing on the base of hemisphere.

Radius BO of hemisphere $= \frac{1}{2} \times 4$ cm $= 2$ cm.

Now, let the right circular cylinder $EFGH$ circumscribe the given solid.

Radius of base of the right circular cylinder

$= HP = BO = 2$ cm.

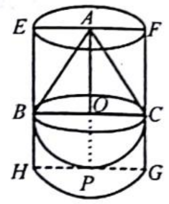
Height of the cylinder $= AP = AO + OP$

$= 2$ cm $+ 2$ cm $= 4$ cm.

Now, the volume of the right circular cylinder – volume of the solid

$$= \left[\pi \times 2^2 \times 4 - \left(\frac{2}{3} \times \pi \times 2^3 + \frac{1}{3} \times \pi \times 2^3\right)\right] \text{cm}^3$$

$= (16\pi - 8\pi)\ \text{cm}^3 = 8\pi\ \text{cm}^3$.



Hence, the right circular cylinder covers 8π cm^3 more space than the solid.

4.
$$V = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow V^2 = \frac{1}{9}\pi^2 r^4 h^2$$

$$\Rightarrow 9V^2 = \pi^2 r^2(\ell^2 - h^2)h^2$$

$$\Rightarrow 9V^2 = (\pi^2 r^2 \ell^2)h^2 - \pi^2 r^2 \ell^4$$

$$\Rightarrow 9V^2 = C^2 h^2 - 3\left(\frac{1}{3}\pi r^2 h\right)\pi h^3$$

$$\Rightarrow 9V^2 = C^2 h^2 - 3V\pi h^3$$

$$\Rightarrow 3\pi V h^3 - C^2 h^2 + 9V^2 = 0$$

5. As $BP \parallel AC$ and $AD \parallel EQ$

$\therefore\quad$ ar$(\Delta ABC) =$ ar(ΔAPC)

[Δs on the same base and between the same parallels]... (i)

$AD \parallel EQ$ [Given]

$\therefore\quad$ ar$(\Delta ADE) =$ ar(ΔADQ)

[Δs on the same base and between the same parallels]... (ii)

$\therefore\quad$ ar$(\Delta ACD) =$ ar(ΔACD) ...(iii)

Adding (i), (ii) and (iii)

ar(ΔABC) + ar(ΔADE) + ar(ΔACD)

= ar(ΔAPC) + ar(ΔADQ) + ar(ΔACD)

ar$(\Delta ABCD) =$ ar(ΔAPQ)

Hence proved.

6. The volume of a sphere $= \frac{4}{3}\pi r^3$

10% increase in radius = 10% r

Increased radius $= r + \frac{1}{10}r = \frac{11}{10}r$

The volume of the sphere now becomes

$$\frac{4}{3}\pi\left(\frac{11}{10}r\right)^3 = \frac{4}{3}\pi \times \frac{1331}{1000}r^3 = \frac{4}{3}\pi \times 1.331 r^3$$

Increase in volume $= \frac{4}{3}\pi \times 1.331 r^3 - \frac{4}{3}\pi r^3$

$$= \frac{4}{3}\pi r^3(1.331 - 1) = \frac{4}{3}\pi r^3 \times .331$$

Percentage increase in volume

$$= \left[\frac{\frac{4}{3}\pi r^3 \times .331}{\frac{4}{3}\pi r^3} \times 100\right] = 33.1\%$$

The probability of getting number "3" with one throw $= \frac{1}{6}$

The probability of getting number "3" with double throw

$$= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

10

CHAPTER

Statistics and Probability

INTRODUCTION

Statistics is the science which deals with the collection of numerical facts called raw data and classification, tabulation, interpretation of raw data. Expressing facts with the help of a data is of great importance in our day-to-day life just as we collect bricks, stones, rods, cement etc. before construction of a house. Data in fact is the fundamental unit of statistics. We can understand and solve even most complicated problems logically by making proper use of data.

In this chapter, we study the types of data, grouping or classification of data, graphical representation of data and central tendency (i.e. mean, median & mode) of raw data.

Probability is a branch of mathematics which evolved out of practical considerations. It is used practically in almost every field. Probability theory can be thought as the science of uncertainty. In this chapter, you will learn how to find the probability of occurrence or non-occurrence of an outcome in simple situations.

STATISTICS

PRIMARY AND SECONDARY DATA (OR OBSERVATIONS)

Primary Data : The data which is collected by the investigator with a definite plan or design in mind is called Primary Data.

Secondary Data : When the data is gathered from some sources which already had stored for some purpose, then the data is called secondary data.

UNGROUPED (OR RAW) DATA

The data which is collected for specific purpose and put as it is (without any arrangement) is called raw data. Each entry in raw data is known as an observation.

For example : runs scored by batsmen in a T-20 cricket match are :

17, 20, 15, 42, 25, 17, 23, 18, 5, 15

By observation we can find the lowest and highest scores.

RANGE OF RAW DATA

The difference between the highest and lowest values of given data is called range. In the above case,

Highest Score = 42, Lowest score = 5

$\therefore$ Range = Highest Score – Lowest score

Range = 42 – 5 = 37

FREQUENCY

The number of times a particular observation occurs is called its frequency.

For example : Marks obtained by 20 students of a class in a unit test are:

8, 5, 10, 6, 12, 18, 10, 5, 11, 13, 4, 3, 9, 11, 2, 10, 10, 19, 5, 7

By observation, we can see that 5, 10 and 11 come three times, four times and two times respectively. Hence, frequency of 5, 10 and 11 are 3, 4 and 2 respectively. Each other number comes only once. Hence, Frequency of each other number is 1.

FREQUENCY DISTRIBUTION TABLE

There are two types of frequency distribution table.

(I) DISCRETE (OR UNGROUPED) FREQUENCY DISTRIBUTION TABLE

Consider the marks obtained by 20 students in mathematics annual examination of class VIII (Marks are out of 100) :

60, 70, 59, 60, 50, 58, 62, 56, 59, 59, 58, 70, 58, 62, 50, 58, 58, 50, 62, 56

To make the data easily understandable we write it as in the following table.

Marks obtained	50	56	58	59	60	62	70
No. of students (Frequency)	3	2	5	3	2	3	2

The above table is called Discrete (or Ungrouped) Frequency Distribution Table or simply a Frequency Distribution Table.

(II) GROUPED FREQUENCY DISTRIBUTION TABLE

When our data is large e.g. we are given marks out of 100 of 60 students. To make the data more sensible, we condense the data into groups like 50 – 55, 56 – 60, ..., 90 – 95 (Assuming that our data is from 50 to 92) as given below.

Marks obtained (Groups or class Intervals)	No. of Students (frequency)
50 – 55	8
55 – 60	4
60 – 65	12
65 – 70	5
70 – 75	2
75 – 80	6
80 – 85	8
85 – 90	5
90 – 95	10
Total	60

The above type of table is called Grouped Frequency Distribution Table.

Least number of any class interval is called the lower limit of that class interval. And greatest number of any class interval is called the upper limit of that class interval. For example 50 and 55 are the lower and upper limit respectively of the class interval 50 – 55.

The difference of upper limit and lower limit of any class interval is called class size or class width of the whole grouped frequency distribution. Class size of the above grouped frequency distribution is (55 – 50 = 5).

Note : Upper limit of any class is not included in that class. Hence the class interval 60 – 65 means 'equal and greater than 60 but less than 65.'

CUMULATIVE FREQUENCY DISTRIBUTION TABLE

There are two types of cumulative frequency distribution table.

(i) DESCRETE FREQUENCY DISTRIBUTION

In a descrete frequency distribution, the cumulative frequency of a particular value of the variable is the total of all the frequencies of the values of the variable which are less than or equal to the particular value.

For example : Consider marks obtained by 45 students of class VIII in an assessment is given below:

Marks	0	5	10	14	16	18	20
No. of students (Frequency)	1	4	8	9	12	6	5

From the table we conclude that
1 student secured zero marks
4 students secured 5 marks
8 students secured 10 marks and so on.
Now, how many students secured 10 marks or less
To answer this question we have to add all the students who secured 10 or less marks i.e. (1 + 4 + 8) = 13 students.
13 is termed as cumulative frequency of markes 10. Similarly cumulative frequency of marks 16 is (1 + 4 + 8 + 9 + 12) = 34.
See the following table :

No. of children	No. of families (frequency)	Cumulative frequency
1	5	5
2	6	11 (= 5 + 6)
3	4	15 (= 11 + 4)
4	3	18 (= 15 + 3)
5	2	20 (= 18 + 2)
Total	20	

The above table is called Discrete Cumulative Frequency Distribution Table.

(ii) GROUPED FREQUENCY DISTRIBUTION

In a grouped frequency distribution the cumulative frequency of a class is the total of all frequencies up to that particular class.
To calculate cumulative frequencies, the classes should be written in ascending order.

For Example : The following table shows the number of patients getting medical treatment in a hospital on a day.

Age (in years [Class Interval]	No. of Patients [Frequency]	Cumulative Frequency
10 – 20	90	90
20 – 30	50	140 (= 90 + 50)
30 – 40	60	200 (= 140 + 60)
40 – 50	80	280 (= 200 + 80)
50 – 60	50	330 (= 280 + 50)
60 – 70	30	360 (= 330 + 30)
Total	360	

The above table showing the cumulative frequency with class interval is called grouped frequency distribution table.
The above grouped cumulative frequency distribution table can be presented in two other following ways.

(a) Less Than Type Grouped Cumulative Frequency Distribution Table

Age (in years)	No. of Patients (cumulative frequency)
Less than 20	90
Less than 30	140
Less than 40	200
Less than 50	280
Less than 60	330
Less than 70	360

(b) More Than Type Grouped Frequency Distribution Table

Age (in years)	No. of Patients (cumulative frequency)
Equal and more than 10	360
Equal and more than 20	270 (= 360 – 90)
Equal and more than 30	220 (= 270 – 50)
Equal and more than 40	160 (= 220 – 60)
Equal and more than 50	80 (= 160 – 80)
Equal and more than 60	30 (= 80 – 50)

EXCLUSIVE AND INCLUSIVE CLASS INTERVAL

Class interval of the form 10 – 20, 20 – 30, 30 – 40, ...; in which upper limit of any class interval coincides with the lower limit of the just next class interval, is called **Exclusive class Interval**.

Class interval of the form 10 – 19, 20 – 29, 30 – 39, ...; in which upper limit of any class interval does not coincides with the lower limit of the just next class interval, is called **Inclusive class Interval.** In the inclusive class interval, the difference between lower limit of any class interval and upper limit of just previous class interval in always 1.

To use inclusive class interval frequency or cumulative frequency distribution, first of all we convert it into exclusive class interval frequency distribution. For this, we only decrease the lower limit of each class interval by 0.5 and increase the upper limit of each class interval by 0.5 of inclusive class interval. But frequency or cumulative frequency remains the same.

For Example:

Marks obtained (Inclusive class interval)	No. of students (Frequency)	Cumulative Frequency
10 – 19	5	5
20 – 29	2	7
30 – 39	7	14
40 – 49	3	17
50 – 49	8	25
60 – 69	10	35
70 – 79	5	40
80 – 89	10	50
Total	**50**	

The above table is the inclusive class interval cumulative frequency distribution. Its corresponding exclusive class interval cumulative frequency distribution is given below.

Marks obtained (exclusive class interval)	No. of students (frequency)	Cumulative frequency
9.5 – 19.5	5	5
19.5 – 29.5	2	7
29.5 – 39.5	7	14
39.5 – 49.5	3	17
49.5 – 59.5	8	25
59.5 – 69.5	10	35
69.5 – 79.5	5	40
79.5 – 89.5	10	50
Total	**50**	

CONVERSION OF LESS THAN TYPE GROUPED CUMULATIVE FREQUENCY DISTRIBUTION TABLE INTO GENERAL TYPE GROUPED CUMULATIVE FREQUENCY DISTRIBUTION TABLE

Age (in years)	No. of Patients (Cumulative Frequency)
Less than 20	90
Less than 30	140
Less than 40	200
Less than 50	280
Less than 60	330
Less than 70	360

The above table is the less than type grouped cumulative frequency distribution table. Its corresponding general type grouped cumulative frequency distribution table is

Age (in years)	No. of Patients (frequency)	Cumulative frequency
10 – 20	90	90
20 – 30	50 (= 140 – 90)	140
30 – 40	60 (= 200 – 140)	200
40 – 50	80 (= 280 – 200)	280
50 – 60	50 (= 330 – 280)	330
60 – 70	30 (= 360 – 300)	360
Total	360	

CONVERSION OF MORE THAN TYPE GROUPED CUMULATIVE FREQUENCY DISTRIBUTION TABLE IN TO GENERAL TYPE GROUPED CUMULATIVE FREQUENCY DISTRIBUTION TABLE

Age (in years)	No. of Patients (cumulative frequency)
Equal and more than 10	360
Equal and more than 20	270
Equal and more than 30	220
Equal and more than 40	160
Equal and more than 50	80
Equal and more than 60	30

The above table is the more than type grouped cumulative frequency distribution table. Its corresponding general type grouped cumulative frequency distribution table is

Age (in years)	No. of Patients (frequency)	Cumulative frequency
10 – 20	90 (= 360 – 270)	90
20 – 30	50 (= 270 – 220)	140
30 – 40	60 (= 220 – 160)	200
40 – 50	80 (= 160 – 80)	280
50 – 60	50 (= 80 – 30)	330
60 – 70	30	360
Total		

GRAPHICAL REPRESENTATION OF DATA

To analyse and draw quick conclusion from the numerical data, data is suitably represented through graphs.

In this class, we study only three types of graphical representation of data. The three types of graph are

(A) Bar graph

(3) Histogram and

(C) Frequency polygon

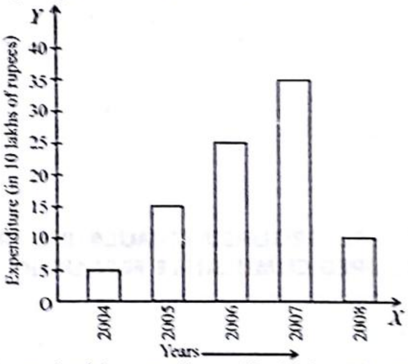
(A) BAR GRAPH

Bar graphs is the simple most and popular graph to shows ungrouped (or descrete) frequency distribution graphically. To draw bar graph, rectangular bars of uniform width with equal spaces in between them are drawn on a horizontal ray OX. Data (or items) are taken on OX ray. Each bar represent a data (or items). Height of each bar represents frequency of the corresponding data. Suitable scale of height of bars are shown on a vertical ray OY. See the following table :

Years	Expenditure (in 10 lakh of rupees) [frequency]
2004	5
2005	15
2006	25
2007	35
2008	10

The above table is the discrete frequency distribution table, which shows expenditure on health by ABC Pvt. Ltd. during five years (2004 to 2008).

Bar graph of the above discrete frequency distribution is shown below:



Bar graph of the expenditure of health by ABC Pvt. Ltd.

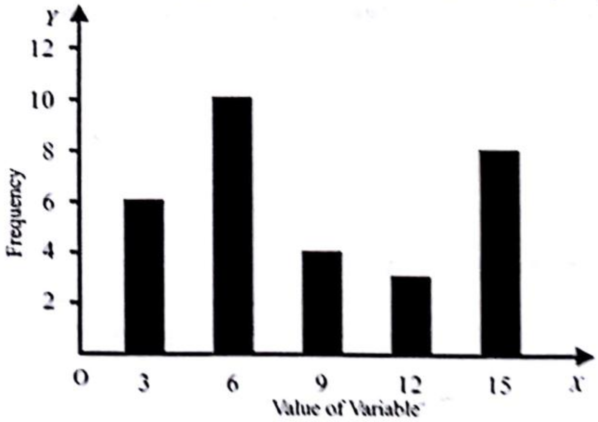
Note : Sometimes for the shake of convenient, to draw bar graph of some discrete frequency distribution, data (or items) are taken on vertical OY-ray and frequency represented by length of the bars are measured on suitable scale on horizontal OX-ray.

Illustration 1 : Represent the following frequency distribution by bar graph :

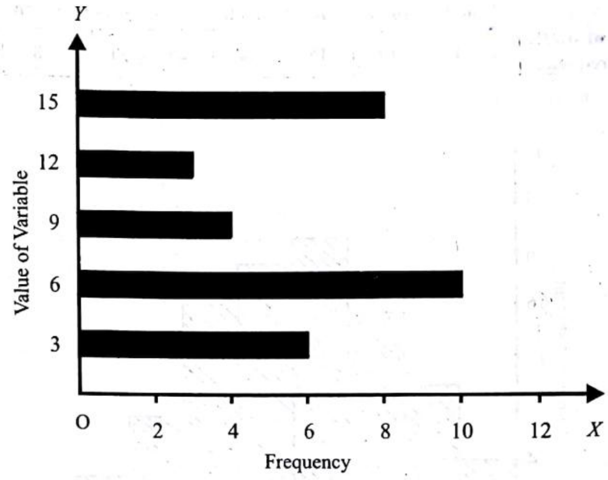
Value of variable	3	6	9	12	15
Frequency	6	10	4	3	8

SOLUTION:

(i) Either of the following bar graphs (fig (i) or fig (ii) may be used to represent the above frequency distribution.



(fig. (i))



(fig. (ii))

(B) HISTOGRAM

A histogram is special type of bar graph which shows the cumulative frequency distributions graphically. See the following table.

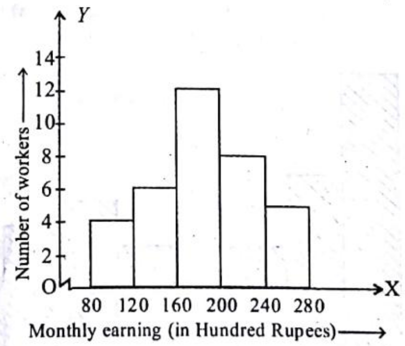
Monthly Earnings (in hundred rupees)	No. of Workers
80-120	4
120-160	6
160-200	12
200-240	8
240-280	5

To draw the histogram of the above frequency distribution we follow the following steps :

(a) On the horizontal axis, mark the class intervals with a uniform scale.

(b) On the vertical axis mark a scale to measure the height of bars which is equal to the frequencies, with a uniform scale.

(c) Construct rectangles with class intervals as bases and the corresponding frequencies as heights



The above graph is the graph of histogram of the given grouped frequency distribution.

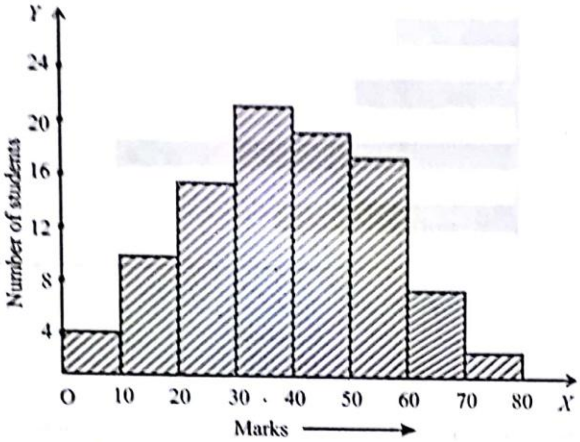
Note : Since the first class interval is start from 80 not from 0, hence we use a 'kink' or a break (z) on the x-ray just after 0.

Illustration 2 : The following table gives the marks scored by 100 students in an entrance examination.

Mark :	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of students (Frequency)	4	10	16	22	20	18	8	2

Represent this data in the form of a histogram.

SOLUTION:



We represent the class limits along X-axis on a suitable scale and the frequencies along Y-axis on a suitable scale.

Taking class-intervals as bases and the corresponding frequencies as heights, we construct rectangles to obtain the histogram of the given frequency distribution as shown in figure.

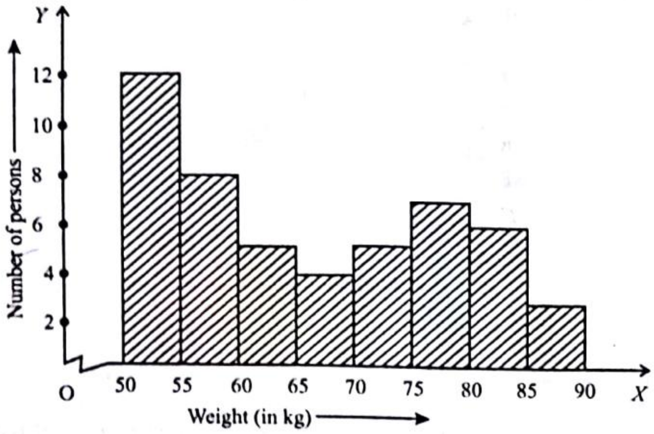
Illustration 3 : The following is the distribution of weights (in kg) of 50 persons:

Weight (in kg) :	50-55	55-60	60-65	65-70	70-75	75-80	80-85	85-90
Number of persons :	12	8	5	4	5	7	6	3

Draw a histogram for the above data.

SOLUTION : We represent the class limits along X-axis on a suitable scale and the frequencies along Y-axis on a suitable scale.

Since the scale on X-axis starts at 50, a kink (break) is indicated near the origin to signify that the graph is drawn to scale beginning at 50, and not at the origin.



(C) FREQUENCY POLYGON

It is a line graph of grouped frequency distribution plotted between class marks and frequencies. It can be obtained in two ways

(a) By first drawing Histogram and
(b) Without drawing Histogram

(a) Steps of Drawing Frequency Polygon (By First Drawing Histogram):

Following are the steps

(i) Draw the histogram from the given data

(ii) Obtain the mid-points of the upper horizontal sides of each rectangle.

(iii) Join these mid-points of the adjacent rectangles by dotted line segments.

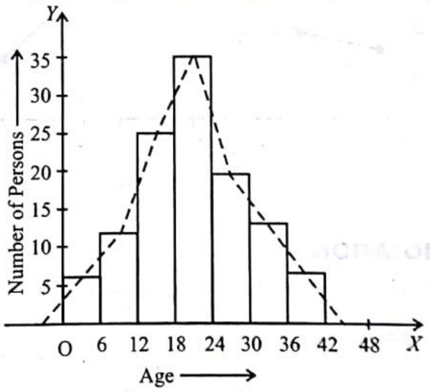
(iv) Obtain the mid-point of two assumed class intervals of zero frequency. One before the first and other after the last class interval.

(v) Complete the polygon by joining the mid-point of class first to mid-point of its left adjacent class and mid-point of last class intervals to the mid point of its right adjacent class interval.

Illustration 4 : Draw a histogram and a frequency polygon of the following data :

Age in years	0-6	6-12	12-18	18-24	24-30	30-36	36-42
No. of persons	6	11	25	35	18	12	6

SOLUTION : First we draw histogram of the given data, then we will locate mid-points of top horizontal side of rectangles.



Now, join these mid points by dotted line segments. Complete the polygon by joining the mid points of first interval to the mid-point of its left adjacent assumed class interval of zero frequency and mid-point of last class interval to its adjacent right assumed class interval of zero frequency.

(b) Steps of Drawing Frequency Polygon (Without Drawing Histogram)

(i) First calculate the class marks (mid points) $x_1, x_2, x_3, \ldots, x_n$ of the given class intervals.

(ii) Mark $x_1, x_2, x_3, \ldots, x_n$ along X-axis.

(iii) Mark respective frequencies $f_1, f_2, f_3, \ldots, f_n$ along Y-axis.

(iv) Plot the points $(x_1, f_1), (x_2, f_2), (x_3, f_3), \ldots (x_n, f_n)$

(v) Join points plotted in step (iv) by line segments.

(vi) Take two class intervals of zero frequency, one just before the first and other just after the last class interval given. Locate their mid points.

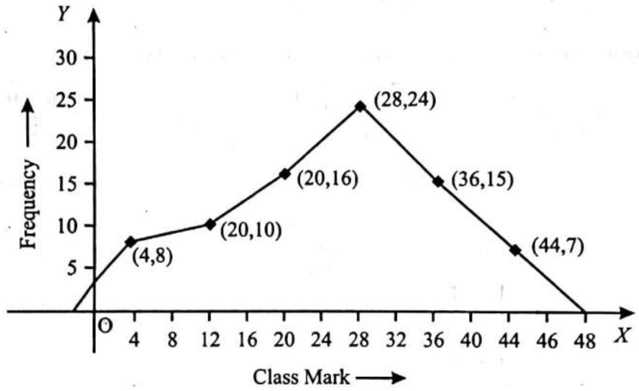
(vii) Complete the frequency polygon by joining the point (x_1, f_1) to the mid-point of the assumed class interval of zero frequency just before the first class interval and (x_n, f_n) to the assumed classed interval of zero frequency just right to the last class interval.

Illustration 5 : Construct a frequency polygon for the following data without drawing the histogram:

Class Interval	0-8	8-16	16-24	24-32	32-40	40-48
Frequency	8	10	16	24	15	7

SOLUTION : Calculate class marks of given frequency distribution

Class Interval	Class Mark (or mid-point)	Frequency
0-8	4	8
8-16	12	10
16-24	20	16
24-32	28	24
32-40	36	15
40-48	44	7



USE OF SUMMATION (Σ) NOTATION

The symbol Σ (read: sigma) means summation. If $x_1, x_2, x_3, ..., x_n$ are the 'n' values of a variable x, then their sum $x_1 + x_2 + x_3 + ... + x_n$ is denoted by $\sum_{i=1}^{n} x_i$ or simply Σx.

Similarly the sum $k_1x_1 + k_2x_2 + \cdots\cdots + k_nx_n$ is denoted by $\sum_{i=1}^{n} k_i x_i$ or simply $\sum kx$

Note : that $\sum_{i=1}^{n}(ax_i + b) = a\sum_{i=1}^{n} x_i + nb$

MEASURES OF CENTRAL TENDENCIES

If the data is very large, the user cannot get much information from these data or its associated frequency distribution. In this case, the information contained in data are represented by some numerical values, called averages. These averages are also called measures of central tendency or measures of location because they also give an idea about the concentration of the values in the central part of the distribution of data which describes the characteristics fo the entire data or its associated frequency distribution.

The most commonly used averages are

(a) Arithmetic mean simply called mean

(b) Median and

(c) Mode

MEAN

(I) MEAN OF UNGROUPED DATA

Mean of a set of observations is their sum divided by number of observations. Mean is denoted by $\bar{x}$. The mean $\bar{x}$ of n observations $x_1, x_2, x_3, ..., x_n$ is given by

$$\bar{x} = \frac{x_1 + x_2 + x_3 + ... + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Illustration 6 : Find mean of 21, 22, 24, 26, 32.

SOLUTION: $\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = \frac{21+22+24+26+32}{5}$

$\bar{x} = \frac{125}{5} = 25$

Illustration 7 : The mean of 6, 10, x and 12 is 8. Find the value of x.

SOLUTION: $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{6+10+x+12}{4} = \frac{28+x}{4}$

$\Rightarrow 8 = \frac{28+x}{4} \qquad (\because \bar{x} = 8)$

$\Rightarrow 28 + x = 32 \Rightarrow x = 4, \quad \therefore$ value of x is 4

Some Important Results About Mean

(i) If each observation is increased by 'a', then the mean is also increased by 'a'. If $\bar{x}$ is the mean of n observations $x_1, x_2, ..., x_n$, then the mean of observations $(x_1 + a), (x_2 + a), (x_3 + a), ..., (x_n + a)$ is $(\bar{x} + a)$.

(ii) If each observation is decreased by 'a', then mean is also decreased by 'a'. If $\bar{x}$ is the mean of n observations $x_1, x_2, ..., x_n$; then mean of observations
$(x_1 - a), (x_2 - a), ..., (x_n - a)$ is $(\bar{x} - a)$.

(iii) If each observation is multiplied by a non-zero number 'a'. Then, mean is also multiplied by 'a'. If $\bar{x}$ is mean of n observations $x_1, x_2, ..., x_n$; then mean of $ax_1, ax_2, ..., ax_n$ is $\overline{ax}$.

(iv) If each observation is divided by a non-zero number 'a', then mean is also divided by the non-zero number 'a'. If $\bar{x}$ is the mean of n observations $x_1, x_2, ..., x_n$, then mean of $\frac{x_1}{a}, \frac{x_2}{a}, ... \frac{x_n}{a}$, is $\frac{\bar{x}}{a}$.

(a) Combined Mean : If $\bar{x}_1, \bar{x}_2, ..., \bar{x}_k$ are the mean of k series having number of observations (or data) $n_1, n_2, ..., n_k$ respectively then the mean $\bar{x}$ of the composite series is given by

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + ... + n_k\bar{x}_k}{n_1 + n_2 + ... + n_k}$$

Illustration 8 : The mean income of a group of persons is ₹ 400. Another group of persons has mean income ₹ 480. If the mean income of all the persons in the two groups together is ₹ 430, then find ratio of the number of persons in the group.

SOLUTION : $\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} \qquad \because \bar{x}_1 = 400, \ \bar{x}_2 = 480, \ \bar{x} = 430$

$\therefore \ 430 = \frac{n_1(400) + n_2(480)}{n_1 + n_2} \Rightarrow 30n_1 = 50n_2$

$\Rightarrow \frac{n_1}{n_2} = \frac{5}{3} \Rightarrow n_1 : n_2 = 5 : 3$

Hence, required ratio = 5 : 3

(b) Weighted Mean: When the variable $x_1, x_2, \ldots, x_n$, do not have same importance, and the weights $w_1, w_2, \ldots, w_n$, are given to each of the variables, the weighted arithmetic is given by $\bar{X}_w = \frac{\Sigma x_i w_i}{\Sigma w_i}$.

Illustration 9 : The following table contains the marks obtained by a student of class XI and the approved weightage for every subject prescribed by the selection committee of a professional colleges.

S.No.	Subject	Weightage	Marks Obtained
1	English	1	60
2	Mathamatics	3	85
3	Physics	3	79
4	Chemistry	2	75

Compare the arithmetic mean and weighted mean of the marks obtained.

SOLUTION : Calculation of mean and weighted mean

S.No.	Subject	Marks Obtained (x_i)	Weight (w_i)	$w_i x_i$
1	English	60	1	60
2	Mathamatics	85	3	255
3	Physics	79	3	237
4	Chemistry	75	2	150
		$\Sigma x_i = 299$	$\Sigma w_i = 9$	$\Sigma w_i x_i = 702$

Thus Mean $= \frac{\sum x_i}{n} = \frac{299}{4} = 74.75$ marks

and weighted mean $= \frac{\sum w_i x_i}{\sum w_i} = \frac{702}{9} = 78$ marks.

(ii) MEAN OF UNGROUPED (OR DISCRETE) FREQUENCY DISTRIBUTION

Let $f_1, f_2, f_3, \ldots, f_n$ are the frequencies of $x_1, x_2, x_3, \ldots, x_n$ then

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + f_3x_3 + \ldots\ldots + f_nx_n}{f_1 + f_2 + f_3 + \ldots\ldots + f_n}$$

or $$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} = \frac{1}{N}\sum_{i=1}^{n} f_i x_i \text{, where } N = \sum_{i=1}^{n} f_i$$

Illustration 10 : Find the mean of the following :

Number (x)	8	10	15	20
Frequency (f)	5	8	8	4

SOLUTION :

x	f	fx
8	5	40
10	8	80
15	8	120
20	4	80
Total	25	320

$$\therefore \quad \bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

$\bar{x} = \frac{320}{25} = 12.8$, $\quad \therefore$ mean = 12.8

MEDIAN

(i) MEDIAN OF UNGROUPED DATA

Median of a given number of observations arranged in ascending or descending order, is the middle most observation. If there are two middle most observation, then median is the average of these two observations.

When the data is arranged in ascending or descending order then median is calculated as follows:

(a) When the number of observations (n) is odd, the median is the $\left(\frac{n+1}{2}\right)^{th}$ observation. For example if there are 21 observations, then $\left(\frac{21+1}{2} = 11\right)^{th}$ observation will be the median of given observations.

(b) If the number of observation is even, then median is the mean of $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ observations. For example if number of observations are 20 then

$$\text{Median} = \frac{\left(\frac{20}{2}\right)^{th} \text{observation} + \left(\frac{20}{2}+1\right)^{th} \text{observation}}{2}$$

$$\text{Median} = \frac{10^{th}\text{observation} + 11^{th}\text{observation}}{2}$$

Thus median can be calculated as

(i) $\text{median} = \left(\frac{n+1}{2}\right)^{th}$ observation, if 'n' is odd

(ii) $\text{median} = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} \text{observation} + \left(\frac{n}{2}+1\right)^{th} \text{observation}\right]$, if '$n$' is even

Illustration 11 : The monthly salaries (in ₹) of 10 employees of a factory are: 12000, 8500, 9200, 7400, 11300, 12700, 7800, 11500, 10320, 8100. Find the median salary.

SOLUTION: Arranging the observation in ascending order :

7400, 7800, 8100, 8500, 9200, 10320, 11300, 11500, 12000, 12700

Total number of observations (n) = 10 (even)

$$\therefore \quad \text{median} = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} \text{observation} + \left(\frac{n}{2}+1\right)^{th} \text{observation}\right]$$

$$= \frac{1}{2}\left[\left(\frac{10}{2}\right)^{th} \text{observation} + \left(\frac{10}{2}+1\right)^{th} \text{observation}\right]$$

$$= \frac{1}{2}\left[5^{th}\text{observation} + 6^{th}\text{observation}\right]$$

$$\text{Median} = \frac{1}{2}[9200 + 10320] = \frac{19520}{2}$$

Median = 9760

Median Salary = ₹ 9760

(ii) MEDIAN OF UNGROUPED (OR DISCRETE) FREQUENCY DISTRIBUTION

Consider the following ungrouped (or discrete) frequency distribution :

x_i	x_1	x_2	x_3	....	x_n
f_i	f_1	f_2	f_3	....	f_n

From its cumulative frequency distribution table in which observations are in either ascending or descending order. If required cumulative frequency distribution table is the following table :

x_i	f_i	Cumulative frequency (F_i)
x_1	f_1	F_1
x_1	f_1	F_2
x_1	f_1	F_3
|	|	|
|	|	|
|	|	|
x_n	f_n	F_n
Total	F_n	

We can find any observation from the above discrete frequency distribution table. Let us find the rth observation from the above discrete frequency distribution.

(a) If $r = F_t$, where t is a natural number then x_t is the r^{th} observation.

(b) If $F_s < r < F_{s+1}$, where s is a natural number then x_{s+1} is the r^{th} observation

(c) If $r < F_1$, then x_1 is the r^{th} observation.

From the above table, it is clear that total number of observations = F_n

If F_n is odd, then median $= \left(\frac{F_n+1}{2}\right)^{th}$ observation

If F_n is even, then median $= \frac{1}{2}\left[\left(\frac{F_n}{2}\right)^{th} \text{observation} + \left(\frac{F_n}{2}+1\right)^{th} \text{observation}\right]$

Illustration 12 : Find the median of the following discrete frequency distribution :

x_i	2	5	6	10	15	18	20
f_i	3	1	4	2	3	2	5

SOLUTION:

x_i	f_i	Cumulative frequency (F_i)
2	3	3
5	1	4
6	4	8
10	2	10
15	3	13
18	2	15
20	5	20
Total	20	

Here total number of observation = 20, which is even

Hence Median $= \frac{1}{2}\left[\left(\frac{20}{2}\right)^{th} \text{observation} + \left(\frac{20}{2}+1\right)\text{observation}\right]$

$= \frac{1}{2}\left[10^{th}\text{observation} + 11^{th}\text{ observation}\right]$

$= \frac{1}{2}(10+15) = 12.5$

MODE

(i) MODE OF UNGROUPED DATA

Mode of a set of observation is the observation(s) which comes maximum number of times i.e. the observation(s) of whose frequency is/are maximum.

Illustration 13 : Find the value of mode of the following data
50, 70, 50, 70, 80, 70, 70, 80, 70, 50

SOLUTION : To find mode, we prepare ungrouped (or discrete) frequency table.

Observation	Frequency
50	3
70	5
80	2

In the above table we see that observation 70 is repeating maximum number of times i.e. frequency of 70 is maximum. Hence the mode of the given set of observation is 70.

(ii) MODE OF DISCRETE FREQUENCY DISTRIBUTION

Mode of discrete frequency distribution is the observation(s) whose frequency is/are maximum.

Illustration 14 : For what value of x, the mode of following data 17, 15, 16, 17, 14, 17, 16, 13, x, 16, 15, 15 is 17.

SOLUTION: The frequency table of data except 'x' is shown below

Observation	13	14	15	16	17
Frequency	1	1	3	3	3

Frequency of three observation 15, 16 and 17 are 3.

Mode of given observation = 17 (given)

$\therefore$ frequency of 17 should be more than that of 15 & 16 which is possible only if $x = 17$

$\therefore$ for $x = 17$, the mode of given distribution is 17.

Properties of Mode : It is not effected by presence of extremely large or small observation.

RELATIONSHIP BETWEEN MEAN, MODE AND MEDIAN

$$\boxed{\text{Mode} = 3\text{ Median} - 2\text{ Mean}}$$

Illustration 15 : If the value of mode and mean is 60 and 66 respectively, then find the value of median.

SOLUTION: Mode = 3 Median – 2 Mean

$\therefore$ Median $= \frac{1}{3}$ (mode + 2 mean) $= \frac{1}{3}$ $(60 + 2 \times 66) = 64$

PROBABILITY

If you go to buy 10 kg of sugar at ₹ 30 per kg, you can easily find the exact price of your purchase is ₹ 300. On the other hand, the shopkeeper may have a good estimate of the number of kg of sugar that will be sold during the day, but it is impossible to predict the exact amount. The number of kg of sugar that customers will purchase during a day is random because the quantity can not be predicted exactly.

So, it is best for the shopkeeper to determine the sale, whose probability is highest.

To describe the situation, where we are not able to find the exact outcome, we generally use the word chance, probably or most probably.

For example :

(a) Probably India may win the cricket match against Pakistan.

(b) Chances of Neha passing the MBA examination is very low.

In all the cases, where it is not possible to find the exact outcome, we find the probability of occurrence of the outcome.

SOME BASIC TERMS

Random Experiments : An experiment whose outcome has to be among a set of events that are completely known but whose exact outcome is unknown is called a random experiment. For example, throwing of a dice, tossing of a coin are some examples of random experiments because their outcomes are among the set {1, 2, 3, 4, 5, 6} and {H, T} respectively but exact outcome is unknown.

Note : $\{a, b, c\}$ represent a set whose elements are a, b and c.

Sample Space : It is the set of all possible outcomes of a random experiment. For example, when a coin is tossed, sample space is {Head, Tail}. Sample space when a dice is thrown, is {1, 2, 3, 4, 5, 6}

An Event of a Random Experiment : An event is a subset of a sample space of a random experiment. For example, the subset {2, 3, 5} is an event of throwing a dice whose sample space is {1, 2, 3, 4, 5, 6}.

Trial : Performing an experiment once is called a trial.

Equally Likely Outcomes: Different outcomes of a random experiment are said to be equally likely if the different outcomes have the same or equal chances of occurrence. For example, when a dice is thrown, each of the faces i.e., 1, 2, 3, 4, 5 or 6 are equally likely to appear. Hence, they are equally likely outcomes of throwing a dice.

Favourable Outcome(s) of An Event : The outcome(s) which ensure the occurrence of an event are called favourable outcome(s) to that event. For example, the favourable outcomes to the occurrence of an even number when a die is thrown are 2, 4 or 6.

MEASUREMENT OF PROBABILITY

Suppose in a random experiment there are n outcomes, which are equally likely. Also suppose E be an event of the random experiment and there are m outcomes favourable to the happening of the event E. Then the probability $P(E)$ of the happening of the event E of the random experiment is defined as

$$P(E) = \frac{\text{Number of outcomes favourable to happening the event}}{\text{Number of all possible outcomes of the random experiment}}$$

$$\Rightarrow \quad P(E) = \frac{m}{n}$$

The probability of not happening the event E of the random experiment,

$$P(\text{not } E) \text{ or } P(\bar{E}) = 1 - P(E) \quad = 1 - \frac{m}{n} = \frac{n-m}{n}$$

Clearly $P(E) + P(\bar{E}) = 1$

Probability of any event is equal or greater than 0 (zero) but equal or less than 1. i.e. $0 \le P(E) \le 1$.

For example: On tossing a coin, there are two possibilities either head may come up or tail may come up. Therefore, total no. of possible outcomes = 2.

The number of favourable outcomes for getting a head = 1

$\therefore$ Probability of getting a head, $P(H) = \frac{1}{2}$

Similarly, probability of getting a tail, $P(T) = \frac{1}{2}$

IMPOSSIBLE EVENT

In throwing a die, there are only six possible outcomes 1, 2, 3, 4, 5 and 6. Let we are interested in getting a number 7 on throwing a die. Since no face of the die is marked with 7, so 7 cannot come under any circumstances. Hence getting a 7 is impossible, this type of event is called an impossible event.

$$P(7) = \frac{0}{6} = 0$$

Probability of an impossible event is always zero.

SURE EVENT

Suppose, we want to find the probability of getting a number less than 7 in a single throw of a die having numbers 1 to 6 on its six faces. We are sure that we shall always get a number less than 7 whenever we throw a die. So the above event is sure event

P (Getting a number less than 7) = $\frac{6}{6} = 1$

Probability of a sure event is alway 1.

Illustration 16 : A card is drawn from a well shuffled deck of 52 cards. Find the probability of (i) getting an ace (ii) getting a queen of red suit (iii) getting a club card.

Note : 52 cards are divided into 4 suits of 13 cards each. The suits are : spades (♠), hearts (♥), diamonds (♦) and clubs (♣). In each suit, there is an ace, a king, a queen, a jack, 10, 9, 8, 7, 6, 5, 4, 3 and 2.

SOLUTION : Total number of possible outcomes = 52 (i.e., 52 different cards.)

(i) There are 4 ace cards in the pack. Event E_1 is to get an ace card. So, 4 outcomes favour the event E_1.

Therefore, $P(E_1)$, i.e., P (getting an ace) = $\frac{4}{52} = \frac{1}{13}$

(ii) There are 2 queens of red colour, i.e., red suits. Event E_2 is to get a queen of red suit. So, 2 outcomes favour the event E_2.

Therefore, $P(E_2)$, i.e., P (getting a queen of red suit) = $\frac{2}{52} = \frac{1}{26}$

(iii) Event E_3 is to get a club card and there are 13 club cards.

Therefore, $P(E_3)$, i.e., P (getting a club card) = $\frac{13}{52} = \frac{1}{4}$

MISCELLANEOUS

Solved Examples

Example 1 : **The mean of discrete observations $y_1, y_2, \dots\dots\dots, y_n$ is given by –**

(a) $\dfrac{\sum_{i=1}^{n} y_i}{n}$ (b) $\dfrac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} i}$ (c) $\dfrac{\sum_{i=1}^{n} y_i f_i}{n}$ (d) $\dfrac{\sum_{i=1}^{n} y_i f_i}{\sum_{i=1}^{n} f_i}$

SOLUTION : (a)

Example 2 : **If the mean of the numbers $27 + x$, $31 + x$, $89 + x$, $107 + x$, $156 + x$ is 82, then the mean of $130 + x$, $126 + x$, $68 + x$, $50 + x$, $1 + x$ is –**

(a) 75 (b) 157 (c) 82 (d) 80

SOLUTION : (a)

Given, $82 = \dfrac{(27+x)+(31+x)+(89+x)+(107+x)+(156+x)}{5}$

$\Rightarrow \quad 82 \times 5 = 410 + 5x \quad \Rightarrow \quad 410 - 410 = 5x \quad \Rightarrow \quad x = 0$

$\therefore$ Required mean is, $\bar{x} = \dfrac{130 + x + 126 + x + 68 + x + 50 + x + 1 + x}{5}$

$$\bar{x} = \frac{375 + 5x}{5} = \frac{375 + 0}{5} = \frac{375}{5} = 75$$

Example 3 : The number of observations in a group is 40. If the mean of first 10 is 4.5 and that of the remaining 30 is 3.5, then the mean of the whole group is –

(a) $\frac{1}{5}$ **(b)** $\frac{15}{4}$ **(c)** 4 **(d)** 8

SOLUTION : (b)

$$\frac{x_1 + x_2 + \cdots + x_{10}}{10} = 4.5$$

$\Rightarrow$ $x_1 + x_2 + \cdots + x_{10} = 45$ and $x_{11} + x_{12} + \cdots + x_{40} = 105$

$\therefore$ $x_1 + x_2 + \cdots + x_{40} = 150$

$\therefore$ $\dfrac{x_1 + x_2 + \cdots + x_{40}}{40} = \dfrac{150}{40} = \dfrac{15}{4}$

Example 4 : In a class of 100 students there are 70 boys whose average marks in a subject are 75. If the average marks of the complete class are 72, then the average marks of the girls is –

(a) 73 **(b)** 65 **(c)** 68 **(d)** 74

SOLUTION : (b)

Let the average marks of the girls students be x, then

$72 = \dfrac{70 \times 75 + 30 \times x}{100}$ (Number of girls = 100 – 70 = 30) i.e., $\dfrac{7200 - 5250}{30} = x$, $\therefore$ $x = 65$

Example 5 : The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observation of the set is increased by 2, then the median of the new set –

(a) Is increased by 2 **(b)** Is decreased by 2

(c) Is two times the original median **(d)** Remains the same as that of the original set

SOLUTION : (d)

Since $n = 9$, then median $= \left(\dfrac{9+1}{2}\right)^{\text{th}} = 5^{\text{th}}$ observation

Now, last four observations are increased by 2.

$\because$ The median is 5th observation, which is remaining unchanged.

$\therefore$ There will be no change in median.

Example 6 : A set of numbers consists of three 4's, five 5's, six 6's, eight 8's and seven 10's. The mode of this set of numbers is–

(a) 6 **(b)** 7

(c) 8 **(d)** 10

SOLUTION : (c) Mode of the data is 8 as it repeated maximum number of times.

Example 7 : Mean of 25 observations was found to be 78.4. But later on it was found that 96 was misread 69. Find the correct mean.

SOLUTION : Mean $\bar{x} = \dfrac{\Sigma x}{n}$ or $\Sigma x = n\bar{x}$ = Sum of 25 observations

$\Sigma x = 25 \times 78.4 = 1960$

= sum of 25 observations

Sum of 24 obs. + 69 = 1960
As 69 was misread in place of 96
Sum of 24 obs. + 96 = 1960 − 69 + 69
Sum of 25 obs. = 1987

$\therefore$ Correct mean = $\frac{1987}{25} = 79.48$

Example 8 : **The values of (x) in ascending order as follows :**

8 11 12 16 $16+x$ 20 25 30

If the median is 18, then find the value of x.

SOLUTION : Here total number of variate is 8 therefore the two middle terms are 16 and $16+x$ respectively.

Therefore median = $\frac{(16)+(16+x)}{2} = 18$ (Given)

$\Rightarrow$ $32 + x = 36$ $\Rightarrow$ $x = 4$

Therefore value of $x = 4$.

Example 9 : **Draw a histogram for the following data :**

Class Mark	12.5	17.5	22.5	27.5	32.5	37.5
Frequency	7	13	20	29	10	5

SOLUTION: h = Class-size = Difference between two consecutive class-marks = (17.5 − 12.5) = 5

$\therefore$ $\frac{h}{2} = \frac{5}{2} = 2.5$

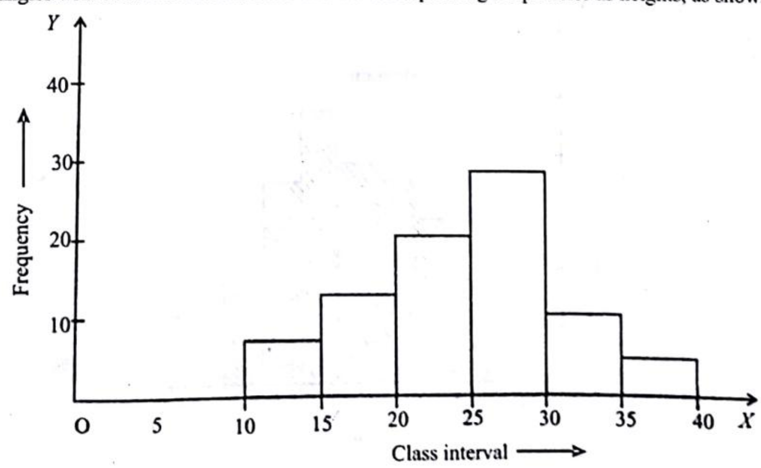
$\therefore$ Subtract 2.5 from each class-mark to get lower-limit. And, add 2.5 to the class-mark to get upper-limit.

Thus, the given frequency distribution in exclusive form will be as given below :

Class Interval	Frequency
10-15	7
15-20	13
20-25	20
25-30	29
30-35	10
35-40	5

Now, along x-axis, mark the points 10, 15, 20, 25, 30, 35 and 40 and along y-axis, mark the corresponding points 7, 13, 20, 29, 10, 5.

Construct rectangles with class-intervals as bases and the corresponding frequencies as heights, as shown.



Example 10 : The marks obtained by a set of students in an examination are given below :

Marks	5	10	15	20	25	30
No. of Students	6	4	6	12	x	4

If the mean of the above data is 18, calculate the numerical value of x.

SOLUTION : From the above data, we may prepare the table given below :

Marks (x_i)	No. of students (frequency) (f_i)	$f_i x_i$
5	6	30
10	4	40
15	6	90
20	12	240
25	x	25x
30	4	120
	$\Sigma f_i = (32 + x)$	$\Sigma f_i x_i = (520 + 25x)$

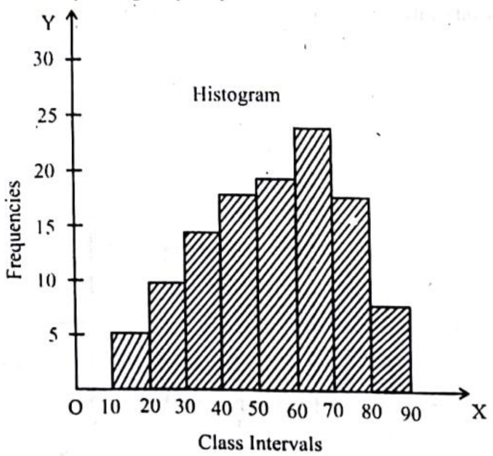
$\therefore$ Mean $= \dfrac{\Sigma f_i x_i}{\Sigma f_i} = \dfrac{(520+25x)}{(32+x)}$ But, mean = 18

$\therefore$ $\dfrac{520+25x}{32+x} = 18 \Rightarrow 520 + 25x = 18(32 + x) \Rightarrow 7x = 56 \Rightarrow x = 8$, Hence, $x = 8$

Example 11 : For the following frequency table, draw a histogram

Class	Frequency
10-20	5
20-30	10
30-40	15
40-50	17
50-60	20
60-70	24
70-80	16
80-90	8

SOLUTION: In the given histogram, class-intervals are taken on the x-axis and the corresponding frequencies on the y-axis. The height of each rectangle is equal to the corresponding frequency in that interval.



Example 12 : **Calculate the median score for the following scores :**

(a) **2, 27, 15, 10, 8, 13, 30** **(b)** **2, 22, 25, 8, 12, 17, 3, 10**

SOLUTION :

(a) The scores in ascending order :
2, 8, 10, 13, 15, 27, 30
Since, there are 7 scores
$\therefore$ median = 4th score = 13

(b) The scores in ascending order :
2, 3, 8, 10, 12, 17, 22, 25
Since, there are 8 scores

$\therefore$ median $= \frac{1}{2}$(4th score + 5th score) $= \frac{1}{2}(10 + 12) = 11$

Example 13 : **Find the value of k from the following data if mean of the given data is 16.**

x	5	10	15	20	25
f	2	8	k	10	5

SOLUTION

x	f	fx
5	2	10
10	8	80
15	k	$15k$
20	10	200
25	5	125
Total	$25 + k$	$415 + 15k$

Mean $= \frac{\Sigma fx}{\Sigma f}$

$$16 = \frac{415+15k}{25+k}$$

$(25 + k)\,16 = 415 + 15k$

$400 + 16k = 415 + 15k$

$16k - 15k = 415 - 400$

$k = 15$

Example 14 : **Construct a frequency polygon for the following data:**

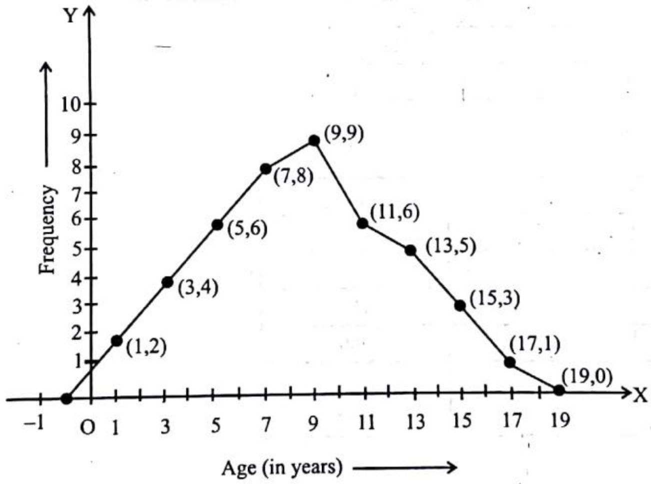
Age (in years)	0–2	2–4	4–6	6–8	8–10	10–12	12–14	14–16	16–18
Frequency	2	4	6	8	9	6	5	3	1

SOLUTION: First we obtain the class marks as given in the following table:

Age (in years)	Class Marks	Frequency
0–2	1	2
2–4	3	4
4–6	5	6
6–8	7	8
8–10	9	9
10–12	11	6
12–14	13	5
14–16	15	3
16–18	17	1

Now, we plot the points (1, 2), (3, 4), (5, 6), (7, 8), (9, 9), (11, 6), (13, 5), (15, 3), and (17, 1).

Now, we join the plotted points by line segments. The end points (1, 2) and (17, 1) are joined to the mid-points (–1, 0) and (19, 0) respectively of imagined class-intervals to obtain the frequency polygon.



Example 15 : **There are 52 students in class 10th of a school out of which there are 35 girls and 17 boys. The class teacher has to select one student as a class representative. She writes the name of each student on a separate card, the cards being identical. Then she puts the cards in a bag and stirs them thoroughly. She then draws one card from the bag. What is the probability that the name written on the card is the name of (i) a girl? (ii) a boy?**

SOLUTION:

Total number of cards with different names written on them = 52

The number of the cards on which the names of the girl students written = 35

The number of the cards having boy names = 17

We write G for the event of the card drawn with name of a girl and write B for the event of the card drawn with name of a boy.

35 outcomes favour the event B and 17 outcomes favour the event B.

Then $P(G) = \frac{35}{52}$ and $P(B) = \frac{17}{52}$

Also, we note that the two events are complementary and $P(G) + P(B) = 1$.

Example 16 : **A bag contains 7 red balls and 6 green balls. A ball is drawn at random from the bag. What is the probability of getting (i) a red ball? (ii) a green ball? (iii) a black ball? (iv) a coloured ball?**

SOLUTION:

Number of red balls = 7
Number of green balls = 6
Total number of balls in the bag = 7 + 6 = 13
All the 13 balls are coloured balls
No ball in the bag is black.

(i) P (a red ball) = $\frac{7}{13}$

(ii) P (a green ball) = $\frac{6}{13}$

(iii) P (a black ball) = $\frac{0}{13} = 0$

(iv) P (a coloured ball) = $\frac{13}{13} = 1$

Example 17 : **Cards each marked with one of the numbers 8, 9, 10, 11, 12, ..., 30 are placed in a box and mixed thoroughly. One card is drawn at random from the box. What is the probability of getting (i) an even number? (ii) an odd number? (iii) a prime number? (iv) a number multiple of 5? (v) a number divisible by 3?**

SOLUTION:

The possible outcomes 8, 9, 10, 11, 12, ..., 30 are in all 23 (total number of possible outcomes)

(i) P (an even number) = P (8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30) = $\frac{12}{23}$

(ii) P (an odd number) = P (9, 11, 13, 15, ..., 29) = $\frac{11}{23}$

(iii) P (a prime number) = P (11, 13, 17, 19, 23, 29) = $\frac{6}{23}$

(iv) P (a number multiple of 5) = P (10, 15, 20, 25, 30) = $\frac{5}{23}$

(v) P (a number divisible by 3) = P (9, 12, 15, 18, 21, 24, 27, 30) = $\frac{8}{23}$

Example 18 : **The population of 50 villages in a state is given below:**

Population	Number of Villages
6000	8
7000	10
9000	12
10000	5
11000	7
13000	6
15000	2
Total	**50**

Find the mean population of the villages of the state.

SOLUTION: The mean $\bar{x}$ is given by

$$\bar{x} = \frac{(6000 \times 8) + (7000 \times 10) + (9000 \times 12) + (10000 \times 5) + (11000 \times 7) + (13000 \times 6) + (15000 \times 2)}{8 + 10 + 12 + 5 + 7 + 6 + 2}$$

$$= \frac{461000}{50}$$

$$\bar{x} = 9220$$

i.e., the mean population of the village is 9220.

EXERCISE 1

Fill in the Blanks

DIRECTIONS: *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

1. ____________ is the value of the middle most observation (s).
2. The__________ is the most frequently occurring observation.
3. ____________ is found by adding all the values of the observations and dividing this by the total number of observations.
4. ____________ can also be drawn independently without drawing a histogram.
5. If n is an odd number, the median = value of the __________ observation.
6. An activity which results in a well defined end is called an ______________.
7. Total number of results are called ______________.
8. Probability of an event can be any ________________ from 0 to 1.
9. A ______________ is an action which results in one of several outcomes.
10. Number of favourable outcomes for an event cannot be ______________ than the number of total outcomes.
11. The ______________ probability of an event E can be calculated by $P(E)$ =
12. An experiment is called a ______________ experiment if all the possible outcomes are predecided.
13. Probability is a measure of ______________.
14. When the outcome of an experiment satisfies the condition mentioned in the event, then we say that event has __________ (or happened).
15. An______________ for an experiment is the collection of some outcomes of the experiment.

True / False

DIRECTIONS: *Read the following statements and write your answer as true or false.*

1. If a coin is tossed thrice, the total number of outcomes is 8.
2. Probability of the occurrence of an event always lies between1 and –1.
3. Total number of outcomes in a throw of two dice is 12.
4. The sum of probabilities of occurrence and not occurrence of an event is always equal to 1
5. $P(E) = \dfrac{\text{Total number of trials}}{\text{Number of trials in which } E \text{ has happened}}$
6. The experimental probability of an event is a negative number.
7. The experimental probability of an event is greater than 1.
8. Tossing a coin 50 times is called an experiment.
9. When a die is rolled, the number of outcomes for getting a composite number is 2.
10. When a die is rolled, the probability that the number on the face showing up is greater than 6 is 0.

Match the Column

DIRECTIONS: *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D...) in Column I have to be matched with statements (p, q, r, s, ...) in column II.*

1.

	Column I		Column II
(A)	Mode of the data 15, 14, 19, 20, 14, 15, 16, 14, 15, 18, 14, 19, 20, 15, 17, 15 is	(p)	64
(B)	The range of the data 25, 18, 20, 22, 16, 6, 17, 15, 12, 30, 32, 10, 19, 8, 11, 20 is	(q)	105
(C)	The class mark of the class 90 – 120 is	(r)	17.5 – 22.5
(D)	The class marks of a frequency distribution are given as follows: 15, 20, 25, ... The class corresponding to the class mark 20 is	(s)	15
(E)	The following observations are arranged in ascending order: 26, 29, 42, 53, x, x + 2, 70, 75, 82, 93. If the median is 65, then the value of x is	(t)	26

2. **Column I** **Column II**

Column I		Column II	
(A)	The probability of getting one or more tails in the toss of two coins simultaneously is	(p)	$\frac{2}{13}$
(B)	During rainy season of 90 days, it was observed that it rained on 20 days only. Then the probability that it did not rain on a day is	(q)	0.8
(C)	The probability of not getting a prime number in a single throw of a die is	(r)	$\frac{7}{10}$
(D)	In a one-day cricket match Kohli faced 30 balls and hit 9 boundaries. The probability that he did not hit a boundary on a ball is	(s)	$\frac{1}{2}$
(E)	The sum of all the probabilities is	(t)	$\frac{3}{6}$
(F)	Probability of getting an even number on a die will be	(u)	$\frac{3}{10}$
(G)	A bag contains 12 pencils, 3 sharpeners and 7 pens. If we take out one item from the bag at random, probability of drawing a pencil is	(v)	$\frac{6}{11}$
(H)	In a medical examination of students of a class, the following blood groups are recorded. A student is selected at random from the class. The probability that the student has blood group B is	(w)	$\frac{7}{9}$
(I)	In a sample study of 642 people, it was found that 514 people have a high school certificate. If a person is selected at random, the probability that the person has a high school certificate is	(x)	$\frac{3}{4}$
(J)	The probability of getting a king or a queen, if a card is drawn from a well shuffled pack of cards, is	(y)	1

VSAQ Very Short Answer Questions

DIRECTIONS: *Give answer in one word or one sentence.*

1. A coin is tossed 50 times with 17 head. Find the probability of getting a tail.
2. What is the sum of probabilities of different events in an experiment?
3. Consider the data : 2, 3, 9, 16, 9, 3, 9. Since 16 is the highest value in the observations, is it correct to say that it is the mode of the data? Give reason.
4. Define the term Median.
5. Find the probability of drawing a jack or an ace from a pack of playing cards.
6. A die is thrown once. Find the probability that 3 or greater than 3 turns up.
7. A coin is tossed twice. What is the probability that head will appear twice ?
8. What do you mean by primary data?
9. Give five examples of data that you can collect from your day-to-day life.
10. Explain the term 'Frequency'.
11. Write a formula to calculate the class-mark.
12. The class marks of a distribution are 6, 10, 14, 18, 22, 26, 30. Find the class size and the class intervals.

SAQ Short Answer Questions

DIRECTIONS: *Give answer in 2-3 sentences.*

1. If 6, 4, 8 and 3 occur with frequencies 4, 2, 5 and 1 respectively then find the arithmetic mean.
2. Find the mean of thirty numbers where mean of ten numbes is 12 and that of the remaining 20 is 9.
3. 100 surnames were randomly picked up from a local telephone directory and a frequency distribution of the number of letters in the English alphabet in the surnames was found as follows :

Number of letters	Number of surnames
1 – 4	6
4 – 6	30
6 – 8	44
8 – 12	16
12 – 20	4

(i) Draw a histogram to depict the given information.
(ii) Write the class interval in which the maximum number of surnames lie.

4. If two dice are tossed, find the probability of throwing a total of ten or more.

5. One hundred cards are numbered from 1 to 100. Find the probability that a card chosen at random has the digit 5.

6. Find the mean of the factors of 12.

7. The following is the distribution of weights (in kg) of 50 persons :

Weight (in kg)	Number of persons
50-55	12
55-60	8
60-65	5
65-70	4
70-75	5
75-80	7
80-85	6
85-90	3
	Total = 50

Draw a histogram for the above data.

8. Draw a histogram for the following data :

Class interval	25-29	30-34	35-39	40-44	45-49	50-54
Frequency	5	15	23	20	10	7

9. The scores of two groups of class IV students on a test of reading ability are given below

Class interval	50-52	47-49	44-46	41-43	38-40	35-37	32-34	Total
Group-A	4	10	15	18	20	12	13	92
Group-B	2	3	4	8	12	17	22	68

Construct a frequency polygon for each of these groups on the same graph.

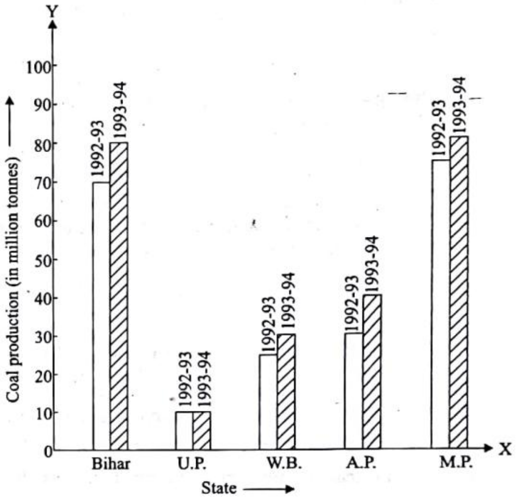
10. At a shooting competition, the scores of a competitor were as given below :

Score	1	2	3	4	5
Number of shots	3	6	4	7	5

(i) What was his modal score?
(ii) What was his total score?
(iii) What was his mean score?

11. The mean of the age of three students Reema, Dipanshu and Bhavya is 15 years. If their ages are in the ratio 4 : 5 : 6 respectively, then find their respective ages.

12. Read the following bar graph given in fig. answer the following questions :



(i) What information is given by this bar graph?
(ii) Which two states have same production in 1993-94?
(iii) Name the state having same production in both the years?
(iv) Which state has minimum production?

Long Answer Questions

DIRECTIONS: *Give answer in four to five sentences.*

1. The following table gives the distribution of students of two sections according to the marks obtained by them:

Marks (class)	Section A (frequency)	Section B (frequency)
0-10	3	5
10-20	9	19
20-30	17	15
30-40	12	10
40-50	9	1

Represent the marks of the students of both the sections on the same graph by two frequency polygons. From the two polygons compare the performance of the two sections.

2. The mean of 1, 7, 5, 3, 4, and 4 is m. The observations 3, 2, 4, 2, 3, 3 and p have mean $(m - 1)$ and median q. Find p and q.

3. Find value of p if the mean of the following data is 40.4.

Variable (x)	Frequency (f)
10	3
20	8
30	12
40	5
50	p
60	7
70	5

4. Find the mean of the observations 25, 27, 19, 29, 21, 23, 25, 33, 28, 20 and show that sum of the deviations of the mean from the observations is zero.

5. 30 children were asked about the number of hours they watched TV last week. The results are recorded as under:

Number of hours	0-5	5-10	10-15	15-20
Frequency	8	16	4	2

Can we say that the number of children who watched TV for 10 or more hours a week is 22? Justify your answer.

6. Draw a histogram for the following data:

Height (in cm)	150-160	160-170	170-180	180-190	190-200
Number of student	8	3	4	10	1

7. The length of 40 leaves of a plant are measured correct to one millimetre, and the data obtained is represented in the following table :

Length (in mm)	Number of leave)
118-126	3
127-135	5
136-144	9
145-153	12
154-162	5
163-171	4
172-180	2

(i) Draw a histogram to represent the given data.

(ii) Is there any other suitable graphical representation for the same data?

(iii) Is it correct to conclude that the maixmum number of leaves are 153 mm long? Why?

8. Following is the distribution of ages (in years) of teachers working in a primary school :

Age (in years)	Number of teacher
21-25	70
26-30	110
31-35	165
36-40	320
41-45	200
46-50	135

(a) Determine the class limit of third class interval.

(b) Determine the class size.

(c) Determine the class marks of fifth class intervals.

(d) How many teachers are in the age group 26 to 45 years.

EXERCISE 2

Multiple Choice Questions

DIRECTIONS: *This section contains multiple choice questions. Each questions has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. In a frequency distribution, the mid value of a class is 10 and the width of the class is 6. The lower limit of the class is

(a) 6 (b) 7

(c) 8 (d) 12

2. If the mean of the observations $x, x+3, x+5, x+7$ and $x+10$ is 9, the mean of the last three observations is

(a) $10\frac{1}{3}$ (b) $10\frac{2}{3}$

(c) $11\frac{1}{3}$ (d) $11\frac{2}{3}$

3. If $\bar{x}$ represents the mean of n observations $x_1, x_2, \ldots\ldots, x_n$, then value of $\sum_{i=1}^{n}(x_i-\bar{x})$ is

(a) -1 (b) 0

(c) 1 (d) $n-1$

4. Let $\bar{x}$ be the mean of $x_1, x_2, \ldots, x_n$ and $\bar{y}$ the mean of $y_1, y_2, \ldots, y_n$. If $\bar{z}$ is the mean of $x_1, x_2, \ldots, x_n, y_1, \ldots y_n$ then $\bar{z}$ is equal to

(a) $\bar{x}+\bar{y}$ (b) $\frac{\bar{x}+\bar{y}}{2}$

(c) $\frac{\bar{x}+\bar{y}}{n}$ (d) $\frac{\bar{x}+\bar{y}}{2n}$

5. If $\bar{x}$ is the mean of $x_1, x_2, \ldots, x_n$ then for $a \neq 0$, the mean of $ax_1, ax_2, \ldots, ax_n, \frac{x_1}{a}, \frac{x_2}{a}, \ldots, \frac{x_n}{a}$ is

(a) $\left(a+\frac{1}{a}\right)\bar{x}$ (b) $\left(a+\frac{1}{a}\right)\frac{\bar{x}}{2}$

(c) $\left(a+\frac{1}{a}\right)\frac{\bar{x}}{n}$ (d) $\frac{\left(a+\frac{1}{a}\right)\bar{x}}{2n}$

6. The mean of 100 observations is 50. If one of the observations which was 50 is replaced by 150, the resulting mean will be

(a) 50.5 (b) 51

(c) 51.5 (d) 52

7. Mean of 20 observations is 15.5 Later it was found that the observation 24 was misread as 42. The corrected mean is:

(a) 14.2 (b) 14.8

(c) 14.0 (d) 14.6

8. In some given data, some variables are given with particular values. We want to represent these graphically. Then we can represent these, using

(a) histogram (b) frequency polygons

(c) bar graph (horizontal) (d) Tally marks

9. In a morning walk, I took 20 rounds of a park. During this period I came across person A, person B, person C and person D, 11 times, 7 times, 10 times and 5 times respectively. I want to represent this data graphically. Which of the following is the best representation?

(a) Bar graph

(b) Histogram with unequal widths

(c) Histogram with equal widths

(d) Frequency polygon

10. The mean of six numbers is 30. If one number is excluded, the mean of the remaining numbers is 29. The excluded number is

(a) 29 (b) 30

(c) 35 (d) 45

11. Probability of an event can be

(a) -0.7 (b) $\frac{11}{9}$

(c) 1.001 (d) 0.6

12. In a class of 40 students there are 120% boys. Then the number of boys is

(a) 48 (b) 24

(c) 80 (d) None of these

13. A coin is tossed 40 times and it showed tail 24 times. The probability of getting a head was

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$

(c) $\frac{1}{2}$ (d) $\frac{17}{40}$

14. A bag contains 10 balls, out of which 4 balls are white and the others are non-white. The probability of getting a non-white ball is

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$

(c) $\frac{1}{2}$ (d) $\frac{2}{3}$

15. If E is an event, then
(a) $0 < P(E) < 1$ (b) $0 \le P(E) < 1$
(c) $0 \le P(E) \le 1$ (d) $0 < P(E) \le 1$

16. In an experiment, the sum of probabilities of different events is
(a) 1 (b) 0.5
(c) -2 (d) 0

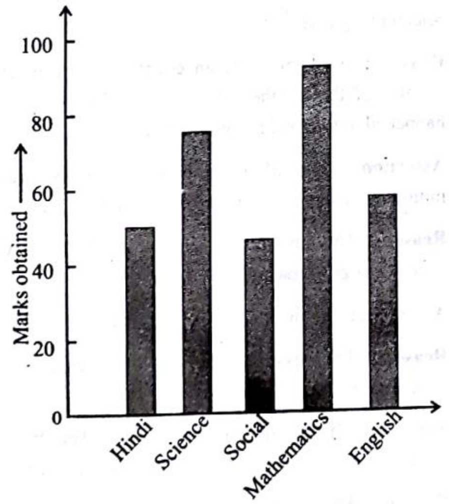
17. The probability of happening of an event is 37%. Then probability of the event is.
(a) 37 (b) 0.037
(c) 3.7 (d) 0.37

18. If a coin was tossed 100 times, out of which 65 times we got head and 35 times tail. Then the probability of not getting a tail is
(a) 6.5 (b) 7.5
(c) 65 (d) 35

19. In rolling a dice, the probability of getting number 8 is
(a) 0 (b) 1
(c) -1 (d) $\frac{1}{2}$

20. Two dice are rolled simultaneously. Find the probability that they show different faces.
(a) $\frac{6}{5}$ (b) $\frac{1}{6}$
(c) $\frac{1}{3}$ (d) $\frac{5}{6}$

21. Marks scored by Reema in an examination (out of 100 marks) in different subjects are shown by the bar graph given below.



(i) In which subject is Reema the best?
(a) Science (b) English
(c) Mathematics (d) Social

(ii) In which subject is Reema weakest?
(a) Hindi (b) Social
(c) English (d) Science

(iii) The ratio of the highest marks to the lowest marks are
(a) 2 : 1 (b) 1 : 2
(c) 3 : 2 (d) 2 : 3

(iv) The average marks obtained by Reema are
(a) 33 (b) 50
(c) 66 (d) 75

(v) The percentage of marks obtained by Reema are
(a) 75% (b) 66%
(c) 50% (d) 46%

MTOC More than One Correct

DIRECTION: *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

1. Which of the following is/are measure of central tendency?
(a) mean (b) median
(c) Variance (d) mode

2. Given below are the seats won by different political parties in the polling outcome of a state assembly election :

Political party	A	B	C	D
Seats Won	75	55	37	29

The graphical representation will be
(a) a histogram
(b) a vertical bar graph
(c) a horizontal bar graph
(d) a frequency polygon curve

3. Which of the following is/are experiment?
(a) Tossing a coin
(b) Rolling a single 6-sided die
(c) Choosing a marble from a jar.
(d) None of these

4. Which of the following is/are not outcome?
(a) Rolling a pair of dice.
(b) Landing on red.
(c) Choosing two marbles from a jar.
(d) Tossing a coin.

5. Which of the following is not the sample space when two coins are tossed?
(a) {H, T, H, T} (b) {H, T}
(c) {HH, HT, TH, TT} (d) {HH, TT}

6. A frequency polygon can be
- (a) drawn using histogram
- (b) drawn using bar graph
- (c) drawn independently
- (d) drawn using tally marks

7. Measure of central tendency represents
- (a) averages
- (b) central value
- (c) mean, median and mode
- (d) None of these

PBQ Passage Based Questions

DIRECTIONS : *Study the given paragraph(s) and answer the following questions.*

PASSAGE-I

A tyre manufacturing company kept a record of the distance covered before a tyre needed to be replaced. The table shows the results of 1000 cases.

Table-I

Distance (in km)	<4000	4000 to 9000	9001 to 14000	>14000
Frequency	20	210	325	445

If you buy a tyre of this company, what is the probability that:

(i) It will need to be replaced before it has covered 4000 km?

(ii) it will last more than 9000 km?

(iii) It will need to be replaced after it has covered somewhere between 4000 km and 14000 km?

PASSAGE-II

Over the past 200 working days, the number of defective parts produced by a machine is given in the following table:

Table-I

No. of defective parts	0	1	2	3	4	5	6	7	8	9	10	11	12	13
No. of days on which the defective parts produce	50	32	22	18	12	12	10	10	10	8	6	6	2	2

Find the probability that tomorrow's output will have

(i) no defective part

(ii) at least one defective part

(iii) not more than 5 defective parts

(iv) more than 13 defective parts

A&R Assertion & Reason

DIRECTIONS: *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.

(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.

(c) If **Assertion** is **correct** but **Reason** is **incorrect**.

(d) If **Assertion** is **incorrect** but **Reason** is **correct**.

1. **Assertion :** Mode of the given data 110, 120, 130, 120, 110, 140, 130, 120, 140, 120, is 120.

Reason : The observation that occurs most frequently, *i.e.,* the observation with maximum frequency is called mode.

2. **Assertion :** Median of the given data 34, 31, 42, 43, 46, 25, 39, 45, 32, is 39.

Reason : When the number of observations (n) is even, the median is the mean of the $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ observations.

3. **Assertion :** In a class there are x boys. and y girls, A student is selected at random, then the probability of selecting a girl is $\frac{y}{x}$.

Reason : Probability of an event E of an experiment is ratio of the number of trials in which event E has happened to the total number of trials.

4. **Assertion :** The difference between the maximum and minimum values of a variable is called its range.

Reason : The number of times a variate (observation) occurs in a given data is called range.

5. **Assertion :** Tossing a coin 50 times is called an event.

Reason : The possible outcomes of an experiment are called events.

6. **Assertion :** If the median of the given data 26, 29, 42, 53, x, $x+2$, 70, 75, 82, 93, is 65 then the value of x is 64.

Reason : When the number of observations (n) is odd the median is the value of the $\left(\frac{n+1}{2}\right)^{th}$ observation.

HOTS Hots Subjective Questions

DIRECTIONS: *Answer the following questions.*

1. The mean of 121 numbers is 59. If each number is multiplied by 4. What will be the new means?
   (a) 234 (b) 235
   (c) 236 (d) 237

2. Consider a small unit of a factory where there are 5 employees : a supervisor and four labourers. The labourers draw a salary of ₹ Rs 5,000 per month each while the supervisor gets ₹ 15,000 per month. Calculate the mean, median and mode of the salaries of this unit of the factory.

3. Prove that $\sum_{i=1}^{n}(x_i - \overline{x}) = 0$ where $\overline{x}$ is the mean of the n observations $x_1, x_2, \ldots x_n$

4. In a data, 10 numbers are arranged in ascending order. If the 8th entry is increased by 6, then what will be the change in median.

5. There are six numbers. Combinations of 3 numbers are selected and their mean was calculated. The resulting means were 2, 4, 6, ... 36, 38, 40. What is the average of the original six numbers?

6. The mean of 10 numbers is 20. If 5 is subtracted from every number, what will be the new mean?

7. Out of ten students, who appeared in a test, three secured less than 35 marks and three secured more than 70 marks. The marks secured by the remaining four students are 39, 51, 69 and 43. Determine the median score of the whole group.

SOLUTIONS

EXERCISE - 1

FILL IN THE BLANKS

1. Median
2. mode
3. Mean
4. Frequency polygon
5. $\left(\frac{n+1}{2}\right)^{th}$
6. Experiment
7. Outcomes
8. Fraction
9. Trial
10. Greater
11. Emperical
12. Random
13. Uncertainty
14. Occurred
15. Event

TRUE/FALSE

1. True
2. False
3. False
4. True
5. False
6. False
7. False
8. True
9. True
10. True

MATCH THE COLUMNS

1. (A) → (s); (B) → (t); (C) → (q); (D) → (r); (E) → (p)

(A) Most occuring observation is 15.

(B) Highest data value is 32 and the lowest is 6

$\therefore$ Range = highest value – lowest value

= 32 – 6 = 26.

(C) Class-mark $= \frac{90+120}{2} = \frac{210}{2} = 105$

(D) The class corresponding to the class mark 20 is given as $\frac{15+20}{2} = \frac{35}{2} = 17.5$ and $\frac{20+25}{2} = \frac{45}{2} = 22.5$

(E) Since the number of observations is 10 (even)

$$\therefore \text{ Median} = \frac{5^{th}\text{ obs} + 6^{th}\text{ obs}}{2}$$

$$65 = \frac{x+x+2}{2} = \frac{2x+2}{2} = x+1$$

$\Rightarrow x = 64.$

2. (A) → (x); (B) → (w); (C) → (t); (D) → (r); (E) → (y); (F) → (s); (G) → (v); (H) → (u); (I) → (q); (J) → (p)

VERY SHORT ANSWER QUESTIONS

1. No. of head = 17

$\therefore$ No. of tail = 50 – 17 = 33

Total no. of trials = 50

$\therefore$ Required prob $= \frac{33}{50}$

2. Required sum = 1

3. 16 is not the mode of the data. The mode of a given data is the observation with highest frequency and not the observation with highest value.

4. The median is that value of the given number of observations, which divides it into exactly two parts.

5. Total number of cards = 52 as there are four jacks and four aces, the number of favourable cases = 8

$\therefore$ The required probability $= p = \frac{8}{52} = \frac{2}{13}$

6. n = Number of all cases = 6

m = Number of favourable cases = 4 (since the numbers that appear are 3, 4, 5, 6)

$\therefore$ The required probability $= p = \frac{m}{n} = \frac{4}{6} = \frac{2}{3}$

7. Here, the total number of cases = 4 i.e. HH, HT, TH, TT. The number of favourable cases = 1 i.e. HH

$\therefore$ Required probability $= p = \frac{m}{n} = \frac{1}{4}$

8. When the information was collected by the investigator herself or himself with a definite objective, the data obtained is called primary data.

9. (i) Heights of students in our class.

(ii) Number of class-rooms in our school.

(iii) Water bills of our house for last three years.

(iv) Election results obtained from television or newspaper.

(v) Literacy rate figures obtained from educational survey.

10. **Frequency :** It is a number, which tells that how many times does a particular data appear in a given set of data.

E.g. consider the following set of data : 1, 2, 0, 3, 2, 1, 5, 4, 3, 2, 1, 2

In the given set of data, 1 appears 3 times, $\therefore$ frequency of 1 is 3.

Similarly, 2 appears 4 times, therefore frequency of 2 is 4 and so on.

11. Class-mark $= \frac{\text{upper class limit + lower class limit}}{2}$

12. Class size = 10 – 6 = 4

Class intervals are : 4–8, 8–12, 12–16, 16–20, 20–24, 24–28, 28–32.

SHORT ANSWER QUESTIONS

1. Frequency table

x_i	f_i	f_ix_i
6	4	24
4	2	8
8	5	40
3	1	3
Total	12	75

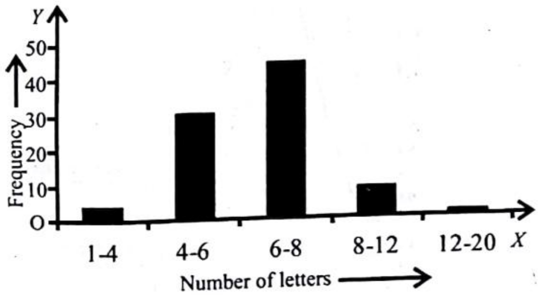
$\therefore$ Required mean $= \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{75}{12} = 6.25$

2. Required mean $= \frac{10\times12+20\times9}{30} = \frac{120+180}{30} = 10$

3. (i) Above given table can be rewritten in modified manner as :

[Minimum class-size = 2]

Number of letters	Numbers of surnames	Width of the class	Length of the rectangle
1-4	6	3	$\frac{6}{3}\times2=4$
4-6	30	2	$\frac{30}{2}\times2=30$
6-8	44	2	$\frac{44}{2}\times2=44$
8-12	16	4	$\frac{16}{4}\times2=8$
12-20	4	8	$\frac{4}{8}\times2=1$



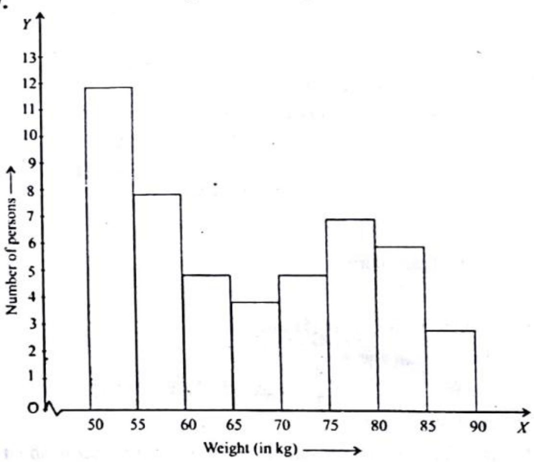
4. Here the number of favourable cases, consists of throwing 10,11 or 12 with the two dice. The number of ways in which a sum of 10 can be thrown are (4,6), (5,5), (6,4) i.e. 3 ways.
The number of ways in which a total of 11 can be thrown are (5, 6), (6,5) i.e. 2 ways.
The number of ways in which a total of 12 can be thrown in (6, 6) i.e. 1 way.
m = number of favourable cases $= 3+2+1=6$
n = Total number of cases $= 6\times6=36$
$\therefore$ Probability $= p = \frac{m}{n} = \frac{6}{36} = \frac{1}{6}$

5. Total number of cases $= n = 100$
All the numbers from 50 to 59 have the digit 5. They are 10 in number. Besides these numbers the numbers 5, 15, 25, 35, 45, 65, 75, 85, 95 have the digit 5. These are 9 in number.
Number of favourable cases $= m$ = Number of numbers which have the digit $5 = 10+9=19$
$\therefore$ Probability $= p = \frac{m}{n} = \frac{19}{100}$

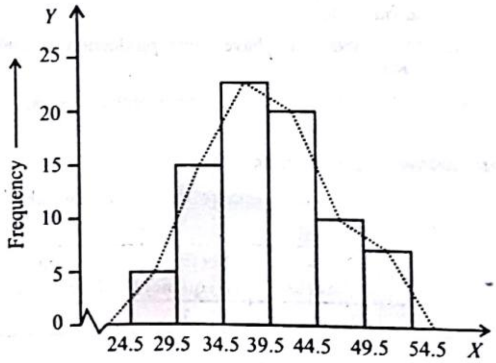
6. Factors of 12 are 1, 2, 3, 4, 6, 12

Mean $= \frac{1+2+3+4+6+12}{6} = \frac{28}{6} = \frac{14}{3}$

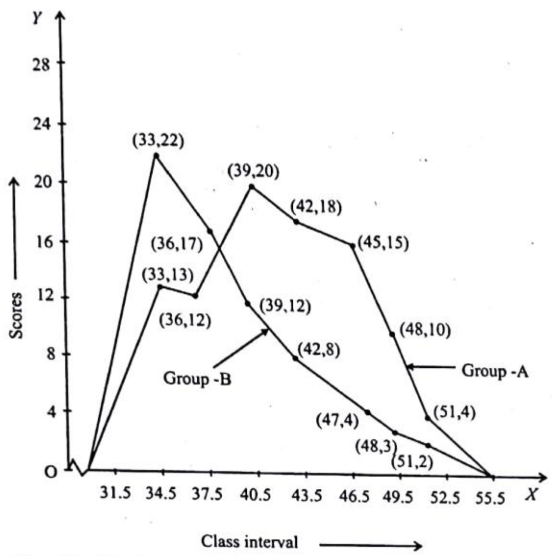
7.



8. The histogram on the above frequency distribution is given in the figure.



9. Frequency polygon for groups A and B of class IV students in a test of reading ability.



10. (i) Modal score = 4
because 4 occurs seven time.
(ii) Total score = $(1 \times 3) + (2 \times 6) + (3 \times 4) + (4 \times 7) + (5 \times 5)$
$= 3 + 12 + 12 + 28 + 25$
$= 80$
(iii) Mean score $= \frac{80}{25} = 3.2$

11. Let their ages be $4x$, $5x$ and $6x$.
Their mean age $= \frac{4x+5x+6x}{3}$
$15 = \frac{15x}{3} \Rightarrow 5x = 15 \Rightarrow x = 3$
So, their ages are 12, 15, 18 year.

12. (i) The given bar graph gives the information about the production of coal in million tonnes in two consecutive years, namely 1992-93 and 1993-94 in various states.
(ii) M.P. and Bihar have same production in 1993-1994.
(iii) U.P. has the same production in both the years.
(iv) U.P.

LONG ANSWER QUESTIONS

1. First we find the class marks and make new table as follows:

Class	Class marks	Section-A (frequency)	Section-B (frequency)
0-10	5	3	5
10-20	15	9	19
20-30	25	17	15
30-40	35	12	10
40-50	45	9	1

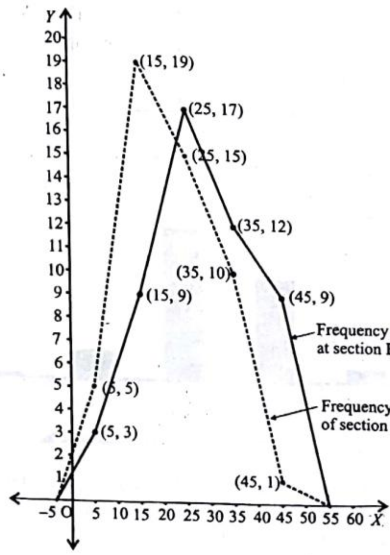
Steps to draw frequency polygon:

For Section-A

(1) First plot the points (5,3), (15, 9), (25, 17), (35, 12) and (45, 9) on graph paper.
(2) Join consecutive points by thick line segments.
(3) Join the first end point with the mid-point of class (– 10 to 0) with zero frequency, and join the other end point with the mid-point of class (50 – 60) with zero frequency.
(4) The required frequency polygon is shown by thick line segments in the figure.

For Section-B

(1) First plot the points (5, 5), (15, 19), (25, 15), (35, 10) and (45, 1)
(2) Join consecutive points with dotted line segment.
(3) Join the first end point with the mid-point of class (– 10 to 0) with zero frequency, and join the other end point with the mid-point of class (50 – 60) with zero frequency.
(4) The required frequency polygon is shown by dotted line segments in the same figure.



Performance of section A is better than performance of section B.

2. Mean $= \frac{1+7+5-3+4+4}{6} \Rightarrow m = \frac{24}{6}$;

Also, $m-1 = \frac{3+2+4+2+3+3+p}{7}$

$\Rightarrow \quad 21 = 17 + p$

$\Rightarrow \quad p = 4$

First we arrange the observations in ascending order.
i.e. 2, 2, 3, 3, 3, 4, 4.
Since, no. of observations is odd

$\therefore$ Median $= \left(\frac{7+1}{2}\right)^{th}$ observation.

$q = 4^{th}$ observation.
$q = 3$.

3. Mean $= \frac{\Sigma f(x)}{\Sigma f} = \frac{30+160+360+200+420+350+50p}{40+p}$

$40.4 = \frac{1520+50p}{40+p} \Rightarrow 9.6\text{ p} = 96 \Rightarrow \text{p} = 10$

4. Mean $= \frac{25+27+19+29+21+23+25+33+28+20}{10}$

$= \frac{250}{10} = 25$

Further, Required sum

$= (25-25) + (27-25) + (19-25) + (29-25) + (21-25) + (23-25) + (25-25) + (33-25) + (28-25) + (20-25)$

$= 0 + 2 + (-6) + (4) + (-4) + (-2) + 0 + 8 + 3 + (-5)$

$= -6 + 8 + 3 - 5 = 0$

5. No, we cannot say that the number of children who watched TV for 10 or more hours a week is 22.

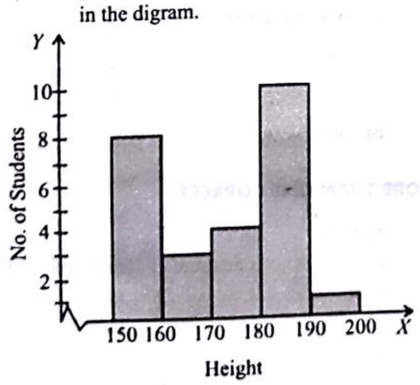
It is (4 + 2) = 6.

6. **Steps** (i) Since the scale on x-axis starts at 150, a break is shown near the origin along x-axis to indicate that the graph is drawn to scale beginning at 150 and not at the origin itself.

(ii) Take 1 cm on x-axis = 10 cm (height)

(iii) Take 1 cm on y-axis = 2 (no. of students)

(iv) Construct rectangles corresponding to the given data. The required histogram is shown in the digram.



7. (i) The given frequency distribution is discontinuous, to make it continuous, adjustment factor

$= \frac{127-126}{2} = \frac{1}{2} = 0.5$

The continuous frequency distribution for the given data is

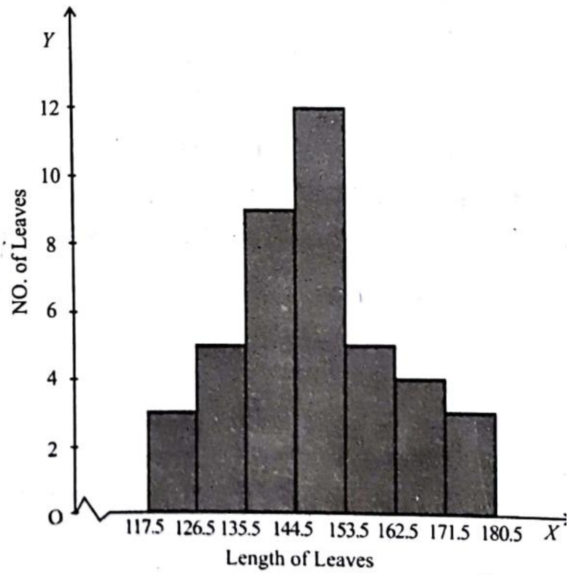
Classes before adjustment	Class after adjustment	Frequency
118-126	117.5 - 126.5	3
127-135	126.5 - 135.5	5
136-144	135.5 - 144.5	9
145-153	144.5 - 153.5	12
154-162	153.5 - 162.5	5
163-171	162.5 - 171.5	4
172-180	171.5 - 180.5	3

Steps to construct histogram

1. Since the scale on x - axis starts at 117.5 a break is shown near the origin.
2. Take 1 cm along x-axis = 9 mm (length).
3. Take 1 cm along y-axis = 2 leaves.
4. Construct rectangles corresponding to the above continuous distribution. The required histogram is shown in the adjacent figure.

(ii) Frequency polygon is the other suitable graphical representation.

(iii) No; in fact, maximum number of leaves area of length between 145 mm and 153 mm (both inclusive).



8. $x_1 = 12, x_2 + 12 = 25$

$\Rightarrow \quad x_2 = 13$

$12 + x_2 + 10 = x_3$

$\Rightarrow \quad x_3 = 35$

$12 + x_2 + 10 + x_4 = 43$

$\Rightarrow \quad x_4 = 8$

$12 + x_2 + 10 + x_4 + x_5 = 48$

$\Rightarrow \quad x_5 = 5$

(a) 31 – 35 (b) 5

(c) 43 (d) 795

EXERCISE-2

MULTIPLE CHOICE QUESTIONS

1. (b) Let x be the upper limit and y be the lower limit.

Since the mid value of the class is 10

$\therefore \frac{x+y}{2} = 10 \quad \Rightarrow \quad x + y = 20. \quad (1)$

and $x - y = 6$ (width of the class = 6) (2)

By solving (1) and (2), we get $y = 7$.

Hence, lower limit of the class is 7.

2. (c) We know, mean $= \frac{\text{Sum of all the observations}}{\text{Total no. of observation}}$

$\Rightarrow \text{Mean} = \frac{x - x + 3 - x + 5 - x + 7 + x - 10}{5}$

$9 = \frac{5x + 25}{5} \Rightarrow x = 4$

So, mean of last three observations is

$\frac{3x+22}{?} - \frac{12+22}{3} = \frac{34}{3} = 11\frac{1}{3}$

3. (b) **4.** (b)

5. (b) Given $\frac{x_1 + \ldots + x_n}{n} = \bar{x}$ .....(1)

$\Rightarrow \frac{ax_1 + \ldots + ax_n}{an} = \bar{x} \Rightarrow \frac{ax_1 + \ldots + ax_n}{n} = a\bar{x}$ .....(2)

Also, from (1), we have

$\frac{\frac{1}{a}x_1 + \ldots + \frac{1}{a}x_n}{\frac{1}{a}n} = \bar{x}$

$\Rightarrow \frac{\frac{x_1}{a} + \ldots + \frac{x_n}{a}}{n} = \frac{\bar{x}}{a}$ .....(3)

So, the mean of $ax_1, \ldots\ldots, ax_n, \frac{x_1}{a}, \ldots\ldots, \frac{x_n}{a}$

is $\frac{a\bar{x} - \frac{\bar{x}}{a}}{2} = \frac{\bar{x}}{2}\left(a + \frac{1}{a}\right)$ (from Ques. 4)

6. (b) 51

7. (d) Sum of 20 observations = 20 × 15.5 = 310

Corrected sum = 310 – 42 + 24 = 292

So, corrected Mean $= \frac{292}{20} = 14.6$

8. (c)

9. (a)

10. (c) Sum of 6 numbers = 30 × 6 = 180

Sum of remaining 5 numbers = 29 × 5 = 145

∴ Excluded number = 180 – 145 = 35.

11. (d) Probability of an event always lies between 0 and 1. (both inclusive)

12. (d) 120% is not valid.

13. (a) P (getting a head) $= \frac{40-24}{40} = \frac{16}{40} = \frac{2}{5}$

14. (b) Total no. of balls = 10

No. of white balls = 4

No. of non-white balls = 10 – 4 = 6

So, Required prob $= \frac{6}{10} = \frac{3}{5}$

15. (c) $0 \le P(E) \le 1$

16. (a)

17. (d) Required prob = 37% $= \frac{37}{100} = .37$

18. (c) Required probability $= \frac{100-35}{100} = \frac{65}{100} = .65$

19. (a)

20. (d)

21.

(i) (c) Mathematics

(ii) (b) Social

(iii) (a) Required Ratio $= \frac{92}{46} = \frac{2}{1} = 2:1$

(iv) (c) Average marks $= \frac{92 + 75 + 67 + 50 + 46}{5}$

$= \frac{330}{5} = 66\%$

(v) (b) 66%

MORE THAN ONE CORRECT

1. (a, b, d)

Variance is not a measure of central tendency.

2. (b, c) **3.** (a, b, c)

4. (a, c, d) **5.** (a, b, d)

6. (a, c) **7.** (a, b, c)

PASSAGE BASED QUESTIONS

PASSAGE-I

(i) The total number of trials = 1000. From the table, it is clear that P (tyre to be replaced before it covers 4000 km)

$= \frac{20}{1000} = 0.02$

(ii) The frequency of a tyre that will last more than 9000 km is $325 + 445 = 770$.

So, Required probability $= \frac{770}{1000} = .77$

(iii) Required frequency is $210 + 325 = 535$

So, required probability $= \frac{535}{1000} = .535$

PASSAGE-II

(i) P (no defective part) $= \frac{50}{200} = \frac{1}{4} = .25$

(ii) P (at least one defective part)

$= P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7) + P(8) + P(9) + P(10) + P(11) + P(12) + P(13)$

$= \frac{32+22+18+12+12+10+10+10+8+6+6+2+2}{200}$

$= \frac{150}{200} = \frac{3}{4} = .75$

(iii) P (not more than 5 defective parts)

$= P(0) + P(1) + P(2) + P(3) + P(4) + P(5)$

$= \frac{50+32+22-18+12-12}{200} = \frac{146}{200} = .73$

(iv) P (more than 13 defective parts) = 0

ASSERTION AND REASON

1. (a) Since the value 120 occurs maximum number of times.
2. (b) Both Assertion And Reason is correct but Reason is not correct explanation for Assertion.
3. (d) Assertion is false.

   P (selecting a girl) $= \frac{y}{x+y}$
4. (c) Assertion is correct. But reason is false.

   The number of times a variate (observation) occurs in a given data is called frequency of that variate.
5. (d) Assertion is false. Tossing a coin 50 times is called an experiment.
6. (b) Assertion : Given no. of observation is 10 (even)

   $\therefore$ Median $= \frac{x+(x+2)}{2} = \frac{2x+2}{2} = x+1$

   $65 = x+1 \Rightarrow x = 64$

HOTS SUBJECTIVE QUESTIONS

1. Here $n = 121$, $\bar{x} = 59$.

   $\therefore$ The sum of 121 numbers $= n\bar{x} = (121 \times 59) = 7139$.

   When each number is multiplied by 4,

   total = 4 × (previous total) = 4 × 7139 = 28556.

   $\therefore$ New mean $= \left(\frac{28556}{121}\right) = 236$

2. Mean $= \frac{5000+5000+5000+5000+15000}{5}$

   $= \frac{35000}{5} = 7000$

   So, the mean salary is ₹ 7000 per month.

   To find the median, we arrange the salaries in ascending order :

   5000, 5000, 5000, 5000, 15000

   Since the number of employees in the factory is 5, the median is given by the $\frac{5+1}{2}$th $- \frac{6}{2}$th = 3rd observation.

   Therefore, the median is ₹ 5000 per month.

   To find the mode of the salaries, we see that 5000 occurs the maximum number of times in the data 5000, 5000, 5000, 5000, 15000. So, the modal salary is ₹ 5000 per month.

3. By defn of mean

   $\bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$

   $\Rightarrow x_1 + x_2 + x_3 + \dots + x_n = n\bar{x}$ ... (i)

   Now, consider

   $\sum_{i=1}^{n}(x_i - \bar{x}) = (x_1 - \bar{x}) - (x_2 - \bar{x}) + (x_3 - \bar{x}) + \dots + (x_n - \bar{x})$

   $= (x_1 - x_2 + x_3 - \dots + x_n) - n\bar{x}$

   $= n\bar{x} - n\bar{x} = 0$ [(From (i)]

4. As the median depends only on 5th and 6th entries and there is no change in these entries therefore there is no change in the value of the median.

5. Let the six numbers be $x_1, x_2, x_3, x_4, x_5, x_6$.

   Number of combinations of $3 = {}^6C_3 = \frac{6 \times 5 \times 4}{1 \times 2 \times 3} = 20$

The mean of these 20 combinations are 2, 4, 6, ..., 36, 38, 40.

Sum of all these means $= \frac{2+40}{2} \times 20 = 420$

In the above sum, each number from the original six is repeated 10 times.

$$\Rightarrow \frac{10(x_1+x_2+x_3+x_4+x_5+x_6)}{3} = 420$$

$$\Rightarrow \frac{x_1+x_2+x_3+x_4+x_5+x_6}{3} = 42$$

$$\Rightarrow \frac{x_1+x_2+x_3+x_4+x_5+x_6}{6} - \frac{42}{2} = 0$$

$$\Rightarrow \frac{x_1+x_2+x_3+x_4+x_5+x_6}{6} = \frac{42}{2} = 21$$

6. Let $x_1, x_2, \ldots, x_{10}$ be 10 numbers with their mean equal to 20. Then,

$$\bar{X} = \frac{1}{n}(\Sigma x_i)$$

$$\Rightarrow \quad 20 = \frac{x_1+x_2+\cdots+x_{10}}{10}$$

$$\Rightarrow \quad x_1+x_2+\cdots\cdots+x_{10} = 200$$

New numbers are $x_1-5, x_2-5, \ldots, x_{10}-5$. Let $\bar{X}'$ be the mean of new numbers. Then,

$$\bar{X}' = \frac{(x_1-5)+(x_2-5)+\cdots+(x_{10}-5)}{10}$$

$$\bar{X}' = \frac{(x_1+x_2+\cdots+x_{10})-5\times 10}{10} = \frac{200-50}{10} \quad \text{[Using (i)]}$$

$$\bar{X}' = 15$$

7. 47